Zetetiké

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# Professional knowledge to teach mathematics in the context of STEM education

## Conhecimento Profissional para ensinar matemática num contexto de educação em STEM

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**Abstract**

This paper contributes to the literature by presenting new developments about knowledge to teach and practices of primary teachers who participated in a teachers' professional development program focused on STEM (Science, Technology, Engineering and Mathematics) hands-on practices. This training context brings added challenges for teachers to innovate their practices in class, namely mathematical practices related to STEM. In particular, we research what knowledge promotes the development and implementation of mathematical interdisciplinary practices within the referred context. With a qualitative methodology and an interpretative approach, and based on data collected for three school years, we conclude that there exists specialized knowledge that is crucial for teachers to be able to develop and implement mathematical interdisciplinary practices in class, which must be taken into account, with a view to the effectiveness of teachers' professional development programs.

**Keywords:** Professional development; Teachers' knowledge; STEM; Hands-on; Primary school.

**Resumo**

Este artigo pretende contribuir para a literatura apresentando novos desenvolvimentos sobre o conhecimento e práticas de professores do ensino básico que participaram num programa de desenvolvimento profissional com foco em atividades práticas *hands-on* relacionadas com as STEM (Science, Technology, Engineering and Mathematics). Este contexto de formação traz desafios acrescidos aos professores para inovarem as suas práticas em aula, nomeadamente as práticas de matemática relacionadas com as STEM. Em particular, procura-se investigar que conhecimento promove o desenvolvimento e a implementação de práticas de matemática interdisciplinares no âmbito do referido contexto. Com uma metodologia qualitativa de natureza interpretativa, e com base em dados recolhidos durante três anos letivos, conclui-se existir conhecimento especializado que é essencial para os professores conseguirem desenvolver e implementar práticas de matemática interdisciplinares em aula, o qual deve ser tido em conta, com vista à eficácia da formação dos professores.

**Palavras-chave:** Desenvolvimento profissional; Conhecimento para ensinar; STEM; Hands-on; Ensino básico.

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# Introduction

International literature increasingly advocates the importance of promoting interdisciplinary education, drawing attention to the need to respond to the increasingly complex and demanding challenges of modern societies. In this sense, there have been increasing calls for the introduction of STEM (Science, Technology, Engineering and Mathematics) education, with the aim of developing skills in students, considered essential for the 21st century, in order to contribute to the scientific and economic development of nations (e.g., Baker & Galanti, 2017; European Schoolnet, 2018; Office of the Chief Scientist, 2016). In fact, STEM has been gaining more and more prominence and already integrates the curriculum of schools in several countries (e.g., Kim & Bolger, 2017). Although there is no consensus on a definition of STEM (Baker & Galanti, 2017), this article considers an integrated approach involving the various areas that make up the acronym. In this context, hands-on practical activities should be introduced, where students have the opportunity to manipulate materials with the aim of drawing conclusions under the guidance of the teacher, in order to motivate them to learn in these areas (Abrahams, Reiss & Sharpe, 2014).

The previous recommendations present new challenges to teachers, since they imply the need for them to innovate their classroom practices. Thus, it is crucial to promote their professional development so that they acquire the knowledge and capacity to meet the above calls (English, 2017; Fitzallen, 2015; Kim & Bolger, 2017). In this sense, it is crucial to investigate what knowledge for teaching teachers need to acquire to be able to implement the proposed approaches effectively. In particular, to understand how to provide teachers with this knowledge within the framework of professional development programs.

This paper is part of a larger project involving a Professional Development Program (PDP) for elementary school teachers. This program was designed to meet the needs of the region and aims to enable teachers to develop and implement hands-on practical activities in class that promote the integration of STEM (Costa et al., 2020). This is a training context that brings added challenges to teachers and trainers, namely on how to develop interdisciplinary mathematics practices that involve STEM content. In this sense, it becomes fundamental to understand what knowledge promotes the implementation of this type of practices in class. Thus, the following research question is posed: what is the knowledge that is necessary for teachers to be able to innovate their mathematics practices in the context of a PDP that promotes the integration of STEM? We seek to answer this question on the basis of an empirical study that took place over 3 school years in the context of such a PDP. A good characterization of this knowledge is not only essential for teachers but also for trainers who need to implement a PDP in a way that is effective, i.e., so that teachers acquire this knowledge and it is reflected in their classroom practices, as suggested by Desimone (2009).

Regarding teachers' professional knowledge, the literature presents several studies, one of the best known being Shulman's (1986), which is related to Pedagogical Content Knowledge (PCK). Other authors have developed PCK by adapting it to areas such as

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mathematics (Ball, Thames and Phelps, 2008), science (Park & Oliver, 2008; Magnusson, Krajcik & Borko, 1999) or technology (Mishra & Koehler, 2006). However, there is a gap in the literature on studies that characterize the knowledge needed to implement STEM in class. In the particular case of this article, further research is sought with a focus on the innovation of mathematics practices from hands-on STEM activities.

The results of this research could be important for teachers, trainers, and anyone interested in implementing this kind of approach or implementing this professional development context. This paper is organized as follows. The next section reviews the literature, followed by the methodology. Then the data analysis and discussion are done and, finally, the conclusions of this study are presented.

## Literature Review

The literature review starts by referring to the importance of promoting STEM education and then moves on to teacher professional development, in particular the knowledge needed to teach.

In recent years there have been increasing calls to promote interdisciplinarity between the various curriculum areas. Indeed, to solve real-life problems, interdisciplinary teams are needed, rather than specialists in just a single discipline (Baker & Galanti, 2017; Rennie, Venville & Wallace, 2012). In order to better prepare students for the increasingly demanding scientific and technological challenges of a rapidly changing world, knowledge in the area of STEM subject matters is needed (English, 2017; European Schoolnet, 2018; Office of the Chief Scientist, 2016).

Mathematics plays a crucial role in STEM education, with more and more authors arguing that it should be given more prominence in this context (Stohlmann, 2018). However, it is also accused of contributing to the lack of professionals in STEM, due to the fact that its teaching is uninspiring (Beswick & Fraser, 2019). Thus, STEM education can be a way to innovate mathematics education (Fitzallen, 2015), as well as to improve performance in this discipline (Stohlmann, 2018). Becker and Park (2011) report that approaches that involve the integration of STEM-related topics have positive effects on student performance, with improved outcomes at the elementary school level.

Teachers play a key role in any process of pedagogical renewal, which makes it a priority to promote PDP to innovate their teaching practices (Costa & Domingos, 2017; Costa et al., 2020). According to Desimone (2009), the effectiveness of professional development should be measured through the characteristics that lead to the intended outcomes, that is, those that result in changes in teachers' knowledge, skills, and classroom practices.

In this sense, it is essential to characterize the knowledge that is necessary for teachers to implement the approaches introduced in PDP. Several authors have presented work on knowledge for teaching, and Shulman's (1986) contributions in this area are crucial. This

DOI: 10.20396/zet.v30i00.8661697

author proposes to distinguish three categories of Content Knowledge (CK): Subject Matter Content Knowledge (SMCK), Pedagogical Content Knowledge (PCK) and Curriculum Knowledge (CuK).

From the SMCK perspective, Shulman (1986) states that a teacher's understanding of the subject matter to be taught includes "not only understanding that something is so; beyond that the teacher must understand why it is so, what the bases for it are, and under what circumstances our beliefs in its justification may be weakened or even denied" (p. 9). Regarding Pedagogical Content Knowledge (PCK), Shulman refers to "the most useful ways of representing ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in short, the ways of representing and formulating matter that make it comprehensible to others" (p. 9). Regarding knowledge of the curriculum, he states that:

> The curriculum is represented by the set of programs designated for teaching a particular subject or topic at a given level, the variety of teaching materials available for those programs, and the set of characteristics that serve as both indications and contraindications for the use of a particular curriculum, or program materials in particular circumstances. (Shulman, 1986, p. 10).

Based on Shulman's work on content knowledge, Ball et al. (2008) have investigated what skills are needed to teach mathematics. In this empirical study, the same authors highlight and classify different types of knowledge, related to subject matter knowledge and pedagogical knowledge (Figure 1). Regarding knowledge of the subject matter to be taught, Ball distinguishes Common Sense Content Knowledge, Horizon Content Knowledge, and Specialized Content Knowledge. With regard to Pedagogical Content Knowledge (PCK), these authors distinguish: Knowledge of Content and Students; Knowledge of Content and Teaching; and Knowledge of Content and Curriculum.

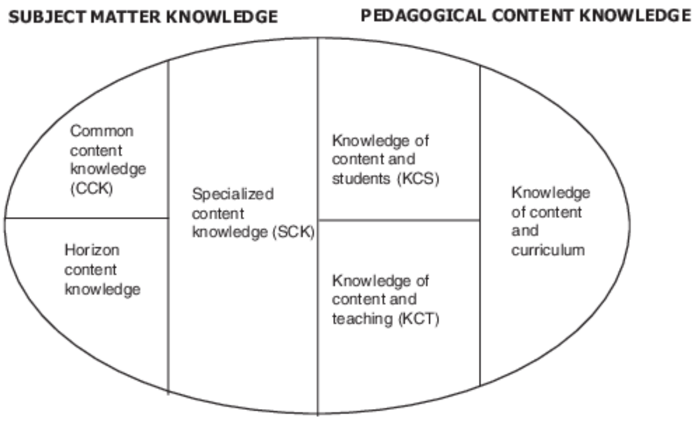


Figure 1 - Knowledge domains for teaching mathematics
Source: Ball, Thames and Phelps (2008, p. 403)

In the area of science education, based on the foundations of Shulman (1986) and other authors, Park and Oliver (2008, p. 263) present a summary of knowledge for teaching. Figure 2 represents the following subdomains: Subject Matter Knowledge; Pedagogical Knowledge; Pedagogical Content Knowledge (PCK); and Context Knowledge. In this

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framework, Context Knowledge has to do with the environment where the school is located, which means that the teacher must take into account the context, where the student is located, in order to teach science. As can be seen in the scheme of the figure above, PCK is the result of Knowledge of the subject to be taught and Pedagogical Knowledge. In turn, PCK is also influenced by Contextual Knowledge.

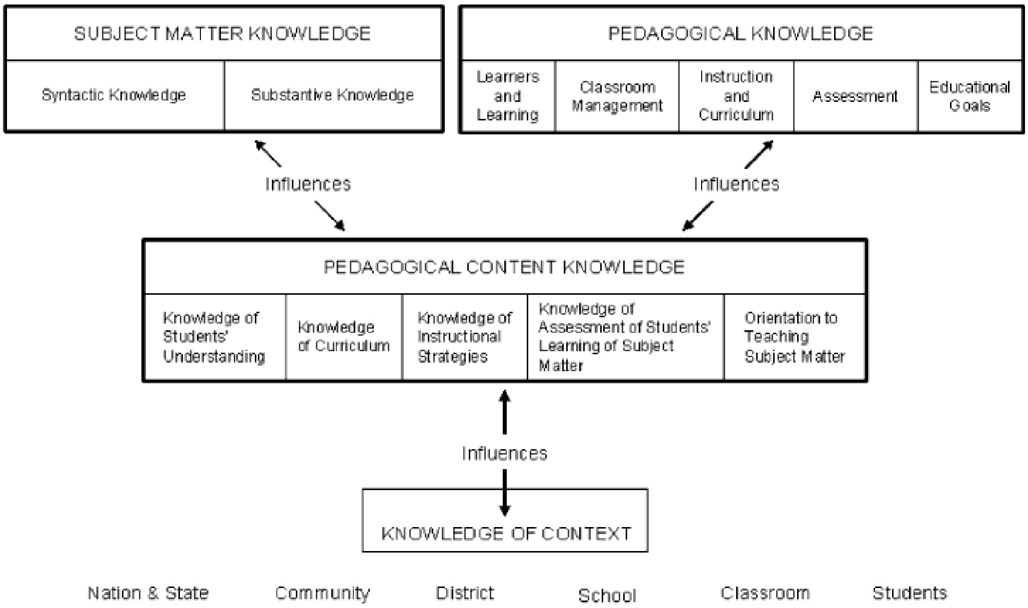


Figure 2 - Knowledge bases for teaching ()
Source: Park and Oliver (2008, p. 263)

In the area of technology education, developed from Shulman's (1986) PCK, Koehler, Mishra, and Cain (2013) report that teaching depends on knowledge of several domains: knowledge about student learning and reasoning, curriculum content, and the growing knowledge of technology. In this regard, they argue that the interaction between various forms of knowledge-such as Content Knowledge (CK), Pedagogical Knowledge (PK), and Technological Knowledge (TK)-results in Technological Pedagogical Content Knowledge (TPACK), which they advocate as the knowledge needed to integrate Technology effectively (Figure 3).

DOI: 10.20396/zet.v30i00.8661697

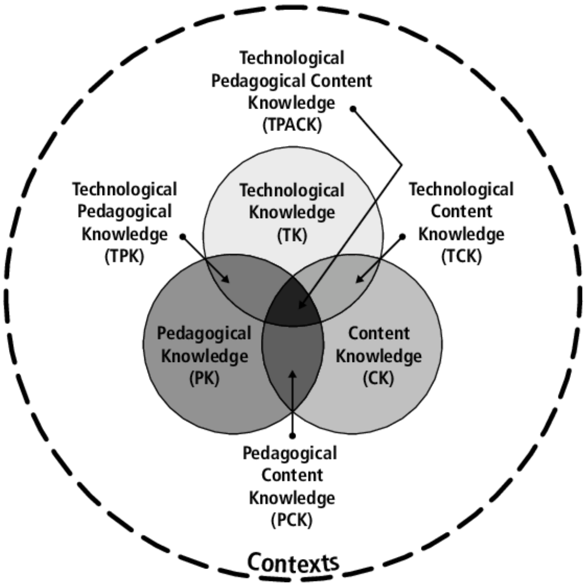


Figure 3 -Domains of technological knowledge
Source: Koehler, Mishra and Cain (2013, p. 15)

In view of what has been said in this section, there is a consensus regarding the specificity of the knowledge required to teach, namely about a specialized knowledge that needs to be acquired by teachers to be transmitted to students. On the other hand, teachers have to have the ability to modify and adapt this knowledge in order to make it accessible to their students, which implies the existence of PCK. For example, the knowledge of mathematics required for an engineer is not the same knowledge that is required to teach mathematics. The same reasoning applies in the area of science education and education with technology. In fact, the PCK:

> results from the intersection of knowledge and pedagogy. (...) PCK is how subject matter is transformed to teach. This occurs when the teacher interprets the subject matter and finds different ways to represent it and make it accessible to students. (Mishra et al., 2006, p. 1021).
>
> offers a way to build bridges between the academic world of disciplinary knowledge and the practice world of teaching (...) by identifying an amalgam of knowledge that combines content knowledge with knowledge of students and pedagogy" (Ball, 2008, p. 398).

According to the authors, mentioned above, it is not only crucial to consider the importance of Content Knowledge of the subjects to be taught, but also Pedagogical Knowledge to make these same subjects accessible to students.

## Methodology

This article uses a qualitative methodology of interpretative nature, using document analysis and participant observation (Cohen, Lawrence & Keith, 2007). The data for the empirical study were collected during three school years: 2015/2016, 2016/2017 and 2017/2018, within the framework of PDPs.

DOI: 10.20396/zet.v30i00.8661697

Each PDP lasts for a full academic year and consists of several workshops lasting 3 to 4 hours each, led by higher education teachers (university and polytechnic) and researchers in the fields of mathematics, physics, chemistry, biology, computer engineering, electrical and computer engineering, and information and communication technologies. The trainees are elementary school teachers who have enrolled voluntarily in the PDP. At the end of each school year, the teachers present a portfolio with a critical reflection on the training in which they participated, the proposed tasks to implement in class, as well as evidence of the practical activities developed with their students. During the three school years, more than 90 teachers from 15 schools, aged between 35 and 61 years and with more than 10 years of service, participated in the PDP.

Data collection was mainly the result of participant observation and document analysis of the portfolios written by the teachers as part of their participation in the PDP. Participant observation took place primarily in the face-to-face training workshops with teachers (to learn and practice what they are expected to implement in class) and in visits to their classrooms (to support them and also to observe them in action). Triangulation of the data was carried out together with the second author of this article, and several meetings were held to cross-check the perceptions of both authors, in order to achieve the greatest possible accuracy in interpreting the data.

The participants in this study are primary school teachers who participated in the PDP in at least one of the school years mentioned above. In order to exemplify hands-on practical STEM-related tasks, and emphasizing mathematics, teacher Josephine was selected to show how she created and implemented the tasks in class and what kind of knowledge is present in this approach. All names given are fictitious to preserve the anonymity of the participants.

## Data Analysis and Discussion

In this section we begin by discussing Professional Knowledge for integrating STEM, based on the literature review. Next, we analyze the reports presented by the teachers at the end of each PDP. Finally, we show how teacher Josephine implemented hands-on practical tasks related to STEM, emphasizing mathematics.

### *Professional knowledge to integrate STEM*

Within the PDP, described above, one of the main goals is the promotion of STEM education, through the implementation of hands-on practical activities, in class, by teachers.

Costa and Domingos (2019) report an empirical study on mathematics education in a STEM-integrated context, in which they conclude that it is crucial to implement a collaborative PDP that supports teachers in developing interdisciplinary tasks in the classroom. In this research, we explore what knowledge is needed for teachers to be able to implement this approach with their students, in particular, what knowledge is needed to develop and implement STEM-integrated mathematics tasks. In the literature review on this topic, no studies were found that presented a theoretical framework for knowledge related to

DOI: 10.20396/zet.v30i00.8661697

STEM integration, which justifies the need to develop research on this topic. Thus, the characterization of the knowledge to teach that is necessary to develop mathematics tasks that integrate STEM can help to understand how to make a PDP more effective, where one of the main objectives is that teachers are able to implement interdisciplinary mathematics teaching practices, namely STEM-related teaching practices.

In this sense, we have tried to incorporate the ideas of the authors cited in the literature review in order to design a theoretical framework that can be used in the context of STEM-related knowledge. The acronym STEM stands for Science (S), Technology (T), Engineering (E) and Mathematics (M) (Figure 4).


Figure 4 - The four areas that make up STEM
Source: Authors' own elaboration

The literature presents some attempts to integrate these, the most common of which are represented in Figure 5.

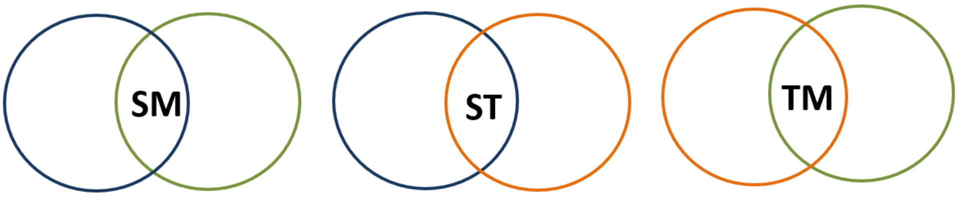


Figure 5 - Some intersections of the areas that integrate STEM.
Source: Authors' own elaboration

In the scope of the work presented here, the aim was to extend this integration to all STEM-related areas. Moreover, there is a particularity to highlight that has to do with the fact that this approach arises in an educational context that involves hands-on STEM-related activities. From this point of view, students are involved in investigative activities, with which they have the opportunity to manipulate materials, question, discuss observations and results with peers with the aim of drawing conclusions.

In some preliminary studies developed by Costa and Domingos (2017; 2018) it was verified the importance of knowledge of the subjects to be taught (SMCK), in particular robust knowledge about science, without which teachers have neither motivation nor confidence to develop the new approaches proposed. Thus, it is to be expected that in order to implement tasks that integrate STEM, knowledge of the subject matter to be taught (SMCK) about Science, Technology, Engineering and Mathematics is required. But since an integrated approach is intended, the tasks should include the various topics, which means that they appear in an integrated way without a boundary between them (Figure 6).

DOI: 10.20396/zet.v30i00.8661697

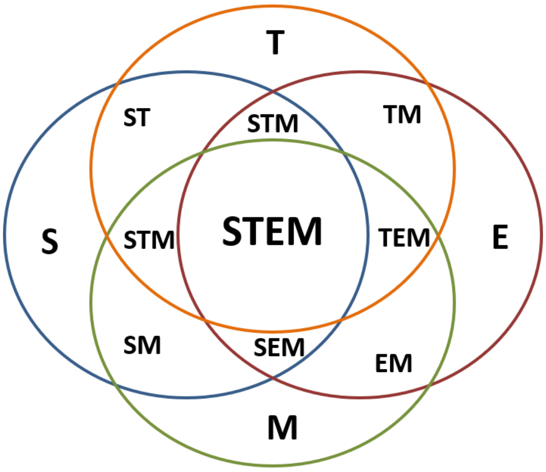


Figure 6 - The integration of STEM.
Source: Authors' own elaboration

However, it is not enough to have STEM-related SMCK to effectively implement the tasks in the classroom. According to the previous sections, teachers must also have the ability to systematize and transform SMCK so that it is understood by their students. This is a unique expertise for teaching that is essentially a combination of content knowledge and pedagogical knowledge (Figure 7), resulting in pedagogical content knowledge (PCK) as introduced by Shulman (1986).

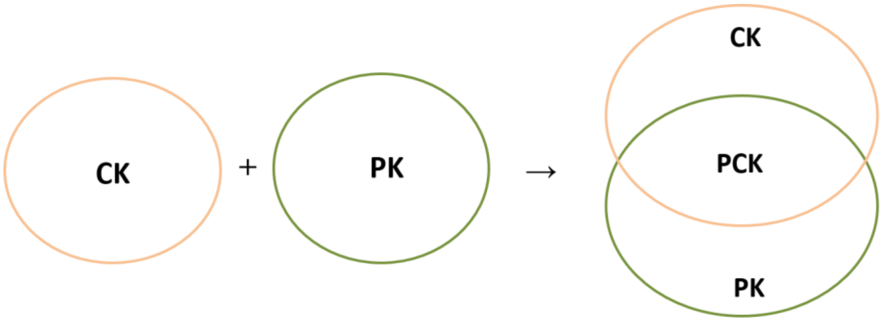


Figure 7 - Pedagogical Content Knowledge (PCK).
Source: Authors' own elaboration

In fact, PCK has been developed by several authors and applied to different curricular areas, such as mathematics (Ball et al., 2008) or science (e.g., Park & Oliver, 2008), or even related to the introduction of technology (Koehler et al., 2013). This is considered to be subject matter knowledge for teaching, which is responsible for transforming subject matter knowledge to be taught (SMCK) so that it becomes accessible to students (Ball et al., 2008; Shulman, 1986).

Taking into account all the categories of knowledge identified in Figures 6 and 7 and the PCK, more categories are obtained (Figure 8).

DOI: 10.20396/zet.v30i00.8661697

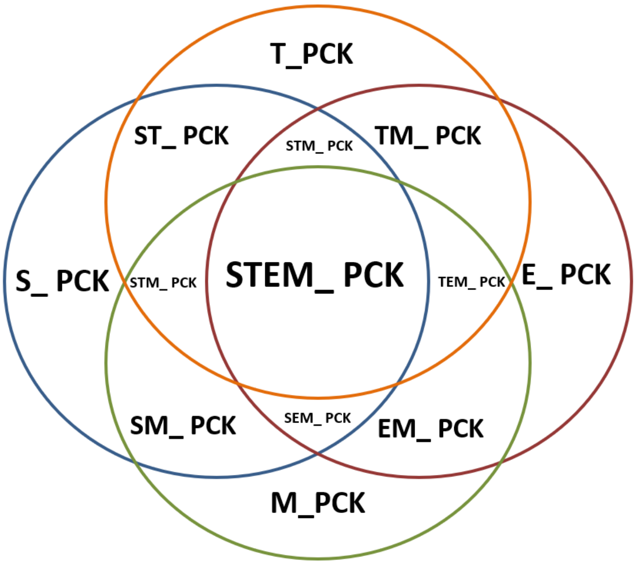


Figure 8 - The various categories of Knowledge needed to teach STEM.
Source: Authors' own elaboration

As mentioned above, one of the main goals of the professional development program is to develop teachers' competencies so that they are able to implement practical, hands-on activities related to STEM. Thus, teachers need not only to acquire theoretical knowledge about the subjects to be taught (SMCK), but also to be able to carry out hands-on practical activities related to these subjects in a way that makes sense to the students. Therefore, the model proposed here needs to be further explored to understand whether it reflects the practical nature of the tasks performed. In this sense, we intend to characterize the teachers' knowledge that is necessary for them to be able to implement the proposed approaches, with a particular focus on mathematics. With this goal in mind, we now proceed to the analysis of the reports of the teachers who participated in the PDP.

*Analysis of the teachers' reports*

Teacher Aúrea participated in the PDP during the 2016/2017 school year. She was 62 years old and was the holder of a 2nd grade class. Her perceptions represent her vision of the training context, the expectations she had of the PDP and its impact. The following excerpt from a teacher's final report shows the initial expectations she had about the PDP she chose to attend, as well as its relevance:

> It was essential to participate in a training course that would contribute to fully configure the way to operationalize contents that may be shrouded in some opacity, reduce or solve some gaps in my theoretical scientific knowledge, and transversally, through the meeting with other teachers and theoretical and practical experts, provide me with a collective reflection on topics of mathematics, science and technology, in order to be able to design a teaching/learning path of my class with more quality and to improve the success of my students. (Aúrea, Final Report, June 2017)

In view of this excerpt from the report, we can see that the teacher recognizes that it is essential to attend a training course of this nature. The justification for recognizing this fact has to do with "contents (...) shrouded in some opacity", which implies the need to update her

DOI: 10.20396/zet.v30i00.8661697

"theoretical scientific knowledge". From this point of view, there is reference to a specialized Theoretical Knowledge related to this theme, which apparently is not clear for teachers and, therefore, the need to acquire it is recognized. On the other hand, it is also necessary to "configure the way to operationalize contents", which justifies the relevance of a theoretical-practical format, as mentioned by the teacher. In this way, Aúrea refers to the trainers as "theoretical-practical experts", which reflects the context of this PDP where theoretical contents are introduced while teachers perform hands-on practical activities. Finally, the teacher mentions that she wants to "design a teaching/learning path for my class, with more quality and enhancing the success of my students." This quote has to do with Pedagogical Knowledge, since its goal is to adapt the theoretical and practical knowledge in order to make it meaningful to students. When we analyze this reflection of the teacher, we can see not only the reference to a specialized Theoretical Knowledge related to the subject to be taught (SMCK), but also Theoretical-Practical Knowledge and, also, Pedagogical Knowledge to transform this knowledge to make it meaningful to students, promoting their learning about the topics covered.

Teacher Aúrea concludes by saying that this PDP helped to reinforce her knowledge and improve her performance in class:

> I am sure that I have strengthened the knowledge and sensitivity necessary to be able to improve my performance in the classroom, using the variations and modeling that were offered to me and that the context of the class requires, in the sense of being able to develop in students the idea that one learns by doing and that mistakes are intermediate skills for success. (Aúrea, Final Report, June 2017)

In the previous excerpt, the acquisition of knowledge to teach is evident, both at the subject and pedagogical level. The students' knowledge is also considered, since there is a concern to consider the class context, as well as to develop the students' ideas.

Like Aúrea, other teachers also mentioned the importance of this professional development program and how it contributed to innovate their teaching practices. For example, teacher Marisa, who participated in the PDP in the school years 2016/2017 and 2017/2018, highlights an "innovative intervention in experimental science teaching" that is related to the integration of "theory and practice during the exploration of activities":

> All of the above justifies the relevance of this action that leads teachers to an innovative intervention in experimental science teaching in the early years of schooling. Very important, since it integrates theory and practice during the exploration of the activities, leading teachers to transfer learning from the training context to the application context (classroom). (Marisa, 2017)

In the above quote, the integration of "theory and practice" is again mentioned, indicating the importance of an approach that includes them. Furthermore, it can be seen that the teacher expresses that this training has given her the ability to apply what she has learned "in the training context to the (...) classroom context". From this point of view, not only were SMCK acquired, but also PCK that allowed her to implement these practices with her

DOI: 10.20396/zet.v30i00.8661697
students. These are characteristics identified in the literature as related to the effectiveness of PDPs (e.g. Desimone, 2009).

Regarding the knowledge to teach pointed out, it should be noted that both teachers (Aúrea and Marisa) referred to the theoretical-practical component and/or the integration of theory and practice, which allows us to identify a pattern that has to do with this specific training context aimed at implementing hands-on practical activities related to STEM. From this perspective, the teachers recognized the need not only for a solid theoretical knowledge of the subjects to be taught, but also for "practical" knowledge in order to carry out the practical tasks related to the topics covered. In addition, pedagogical knowledge is highlighted when they report that they have acquired the ability to innovate their teaching practices in the classroom.

In terms of professional knowledge for teaching, Ivete believes that she lacked some knowledge, for example, related to sound, which is a topic that integrates the curricular content of the 4th grade:

> In the third session, about the mysteries of sound, we were given some knowledge and experiences that we can do in the classroom because, as we know, you learn more by doing than by hearing. Although we have to make some experiences about sound with 4th grade students (the year I teach), I recognize that I had little knowledge. This training allowed me to acquire some important knowledge about sound and awakened in me the curiosity to want to know more about this subject. (Ivete, 2017)

The teacher demonstrates knowledge of the curriculum to be taught, as she knew that sound and experiments with sound should be taught in the 4th grade. However, she recognizes that she did not have enough expertise to implement it, so this training helped to fill that gap, not only by providing her with new knowledge, but also by creating a desire to learn more. Although sound is part of the curriculum of primary school, Ivete recognizes a lack of specialized knowledge to work with this topic in the classroom. Therefore, she lacked theoretical knowledge about sound and also specialized knowledge to implement hands-on practical activities related to sound. Still regarding the specialized knowledge to teach related to electricity, Ivete also reveals that she acquired content knowledge about this topic:

> In the fourth session "Hands on Electricity" we first discussed the history of electrical phenomena before they were understood and the attempts that have been made over the years to explain what electricity is. We did some experiments, in pairs, related to this theme. I used a multimeter for the first time. After sharing the conclusions of the experiments, we also assembled an electric circuit with the association of biological batteries in parallel and experimented with the inclusion of an electric component, provided by the trainer. (Ivete, 2017)

In the excerpt above, there is acquisition not only of specific theoretical knowledge (TheoK) related to electricity, but also of technical knowledge (TechK) related to the hands-on practical activities performed. For example, Ivete mentions that she had never used a multimeter. In fact, the "practical" component of this training model is highlighted, which involves carrying out several hands-on activities performed by the teachers themselves in a collaborative environment, where experiences and conclusions about them are shared.

DOI: 10.20396/zet.v30i00.8661697

Still regarding electricity, Ivete's students received the team of trainers to perform activities:

> On May 11, the trainer (...) came to work with my class, accompanied by two colleagues, physics teachers at IPT. They developed experiments with electricity and the students really enjoyed it. They brought a lot of materials, were very good at talking to the students and keeping them involved in the tasks. It was a morning well spent and that part of the subject was given. Even the biological battery, whose explanation for its construction comes in the students' manual, there was a chance to build it, with oranges. (Ivete, 2017)

Once again, the knowledge of the curriculum (CuK) emerges by recognizing that the experiments were included in the curriculum of environmental study subject matter. Although there are suggestions for experiments in the textbooks, as mentioned by the teacher, she had never done them because she did not have the knowledge to carry them out.

In addition to the aspects mentioned above, the promotion of interdisciplinarity in the context of hands-on science activities is one of the innovations in teaching practices mentioned by the teacher:

> I did not always relate the mathematical content to the experiments that the students had in class, now I have this concern, as was evident in all the sessions of this training course. (Ivete, 2017)

Another important dimension highlighted in the above excerpts has to do with the knowledge of the trainers. Among several aspects, the teachers recognized that the trainers are theoretical-practical experts, which shows that the trainers have not only a theoretical knowledge related to the subject to be taught, but also a practical knowledge related to the hands-on approach of the tasks implemented. Now, this knowledge may fit into the SMCK, but it is different. A theorist may know how to explain why phenomena occur but may not be used to doing practical hands-on activities to demonstrate them. On the other hand, a lab technician may be used to setting up various hands-on activities but may not be in possession of all the theoretical concepts that are usually introduced by the Ph.D. professor in the field. Given this distinction, it is reasonable to consider these two types of knowledge: STEM-related theoretical knowledge (STEM_TheoK) and technical knowledge to perform hands-on practical STEM-related activities (STEM_TechK).

But it is not enough to know the theoretical concepts and how to carry out practical activities. For example, it is undeniable that some scientists have these two types of knowledge to develop sophisticated experiments in the laboratory, but this does not mean that they have the ability to make them accessible in order to promote student learning about them. This is where pedagogical knowledge for teaching comes in. In fact, a teacher must not only have specialized knowledge of the subject to be taught, but also needs to know how to transform it in order to make it understandable for students. But again, in the case under study, there is also specific pedagogical knowledge to implement the hands-on STEM-related practical activities. Within this knowledge, one needs to know how to adapt theoretical knowledge to make it accessible to students. Last, but not least, it is necessary to know how

DOI: 10.20396/zet.v30i00.8661697

to carry out the practical activities so that they are meaningful for the students. In this sense, teachers must be able to carry out the tasks in a way that promotes reflection and learning about them, namely by applying the recommended strategies for their implementation. Thus, it makes sense to introduce specific pedagogical knowledge to make the theoretical concepts about STEM-related fields appropriate and accessible to the students (STEM_TheoPCK - Theoretical Pedagogical Content Knowledge about STEM), as well as specific pedagogical knowledge to introduce hands-on practical activities to be accessible to the students and promote their learning about the topics covered (STEM_TechPCK - Technical Pedagogical Content Knowledge about STEM). Now, these are very specific dimensions of this educational context that are not presented in the works of Lee Shulman or Debora Ball, which justifies the need to introduce them in this research, as shown in the following table.

Table 1: Knowledge dimensions for implementing hands-on science activities.

| SMCK | | PCK | |
|---|---|---|---|
| STEM_TheoCK | STEM_TechCK | STEM_TheoPCK | STEM_TechPCK |

Source: Authors' own elaboration

Table 1 frames the new dimensions related to the specific knowledge to implement hands-on STEM activities in the knowledge proposed by the previously mentioned authors (Ball et al., 2008; Shulman, 1986), respecting the international acronyms proposed by them (SMCK and PCK).

*Teacher Josephine's tasks*

Teacher Josephine participated in the PDP during the school year 2016/2017 and chose electricity to work with her students in the classroom, knowing that she could count on the help of the trainers to support her in the tasks to be implemented. Electricity was introduced by the teacher to draw the students' attention to sustainable development goals, namely, to make them aware of the importance of recycling and preserving the environment. In this sense, she asked the students to bring old batteries and cell phone batteries that they had at home and that they no longer used. In class, after collecting the batteries, the teacher asked the students to sort them by size and model. In this way, the students tried to find patterns in the material they had collected in order to organize it (Figure 9).

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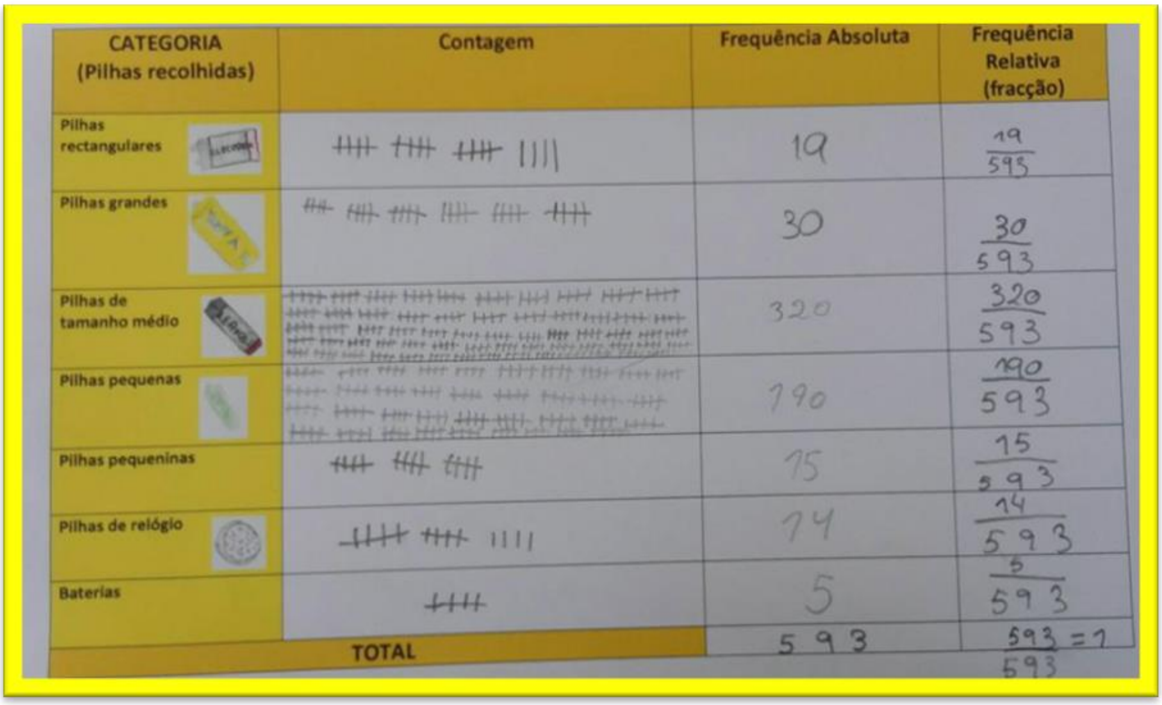
Figure 9 -Organization of the batteries according to their sizes and models.
Source: Authors' own file

After organizing all the batteries, the teacher asked the students to draw the identified models and to do counts to find out how many batteries there were of each type. The class identified 6 models, took note of how many batteries there were of each model, and organized the data in a table (Figure 10).



| CATEGORIA (Pilhas recolhidas) | Contagem | Frequência Absoluta | Frequência Relativa (fracção) |
|---|---|---|---|
| Pilhas rectangulares | 卌 卌 卌 \|\|\|\| | 19 | 19/593 |
| Pilhas grandes | 卌 卌 卌 卌 卌 卌 | 30 | 30/593 |
| Pilhas de tamanho médio | [illegible] | 320 | 320/593 |
| Pilhas pequenas | [illegible] | 190 | 190/593 |
| Pilhas pequeninas | 卌 卌 卌 | 15 | 15/593 |
| Pilhas de relógio | 卌 卌 \|\|\|\| | 14 | 14/593 |
| Baterias | 卌 | 5 | 5/593 |
| TOTAL | | 593 | 593/593 = 1 |

Figure 10 - Organizing and Processing Data from the old batteries and cell phone batteries.
Source: Authors' own file

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With the data recorded in the table, shown in Figure 10, the teacher encouraged the students to work on mathematics, making bar graphs and stem and leaf diagrams, among other tasks. The tasks performed by the students, involved topics from various subject matters such as: Environmental Studies, Artistic and Physical-Motor Expressions, and Mathematics (CuK, STEM_TheoCK, and STEM_TechCK).

In another session, the multimeter was used to measure the current of all the batteries, separating those that had no charge from those that still had. After a brief explanation of how it would be possible to measure and what measurements were used (STEM_TheoPCK) such as volt (V) or ampere (A), the teacher handed out a multimeter to each group of 3 to 4 students. After explaining and exemplifying how it works (STEM_TechPCK), she asked them to measure the potential difference (p.d.) of the batteries, in volts (Figure 11).

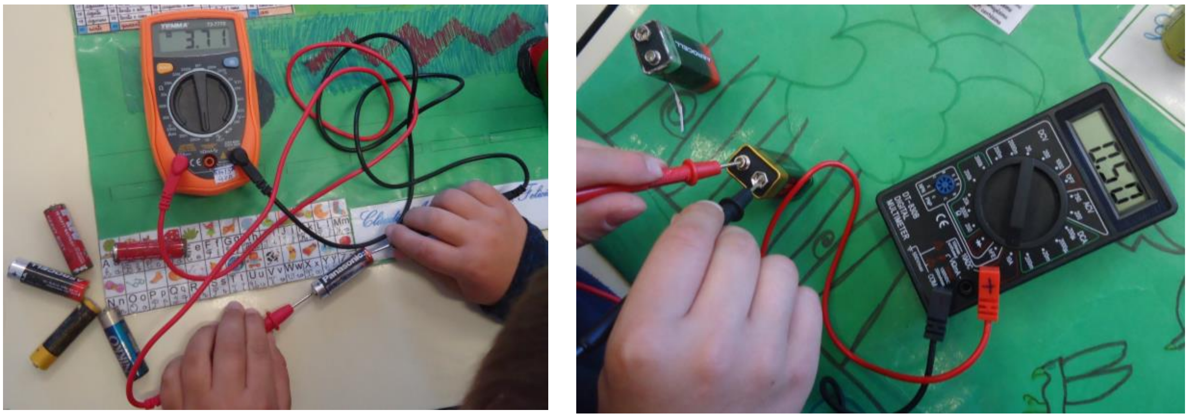

Figure 11 - Measurement of the potential difference of the batteries.
Source: Authors' own file

It was explained that the uncharged batteries went to the pillion rider. The remaining batteries were used to build circuits to light bulbs, run motors, clocks, thermometers, toys, etc. (Figure 12).

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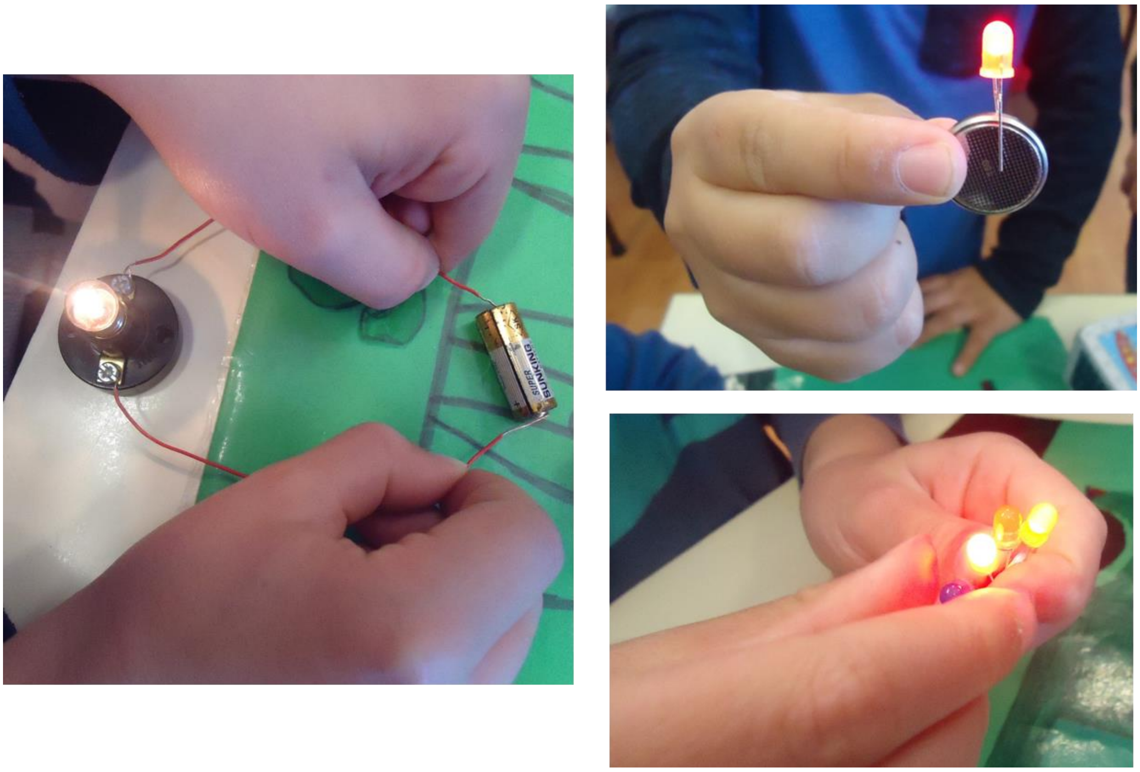

Figure 12 - Lighting lamps with the old batteries.
Source: Authors' own file

In another session the teacher first explained that the batteries used in the previous session were called chemical batteries and that there were other kinds of batteries, such as biological batteries that can be built from fruit or vegetables (electrolytes) and with two different kinds of metals (electrodes), which reveals STEM_TheoCK. She then helped the students build biological batteries (for example with fruit, with a nail and a copper wire) and asked the students to record some measurements of the various batteries they had built (Figure 13 and 14), which reveals STEM_TechCK. The results of these measurements were recorded on the board for all students to observe.

| Pilha Biológica Legumes | Moeda de [illegible] à distância de... | Medida em Volt | Intensidade da corrente (ampere) | Potência máxima Watt |
|---|---|---|---|---|
| cenoura | 2 cm | 0,89 V | 229 Ma | 203,81 W |
| batata | 2 cm | 0,95 V | 559 Ma | 531,05 W |
| batata doce | 2 cm | 0,79 V | 1106 Ma | 873,74 W |
| cheronia | 2 cm | | | |
| cogumelo | 2,5 cm | 0,65 V | 274 Ma | [illegible] W |
| chuchu | 2 cm | 0,72 V | 724 Ma | 527,28 W |
| cebola | 2,5 cm | 0,83 V | 332 Ma | 275,56 W |
| cabeça de nabo | 2 cm | 0,72 V | 192 Ma | 138,24 W |

Figure 13 - Measurement of p.d. and current intensity of biological batteries. Calculation of the maximum power supplied.
Source: Authors' own file

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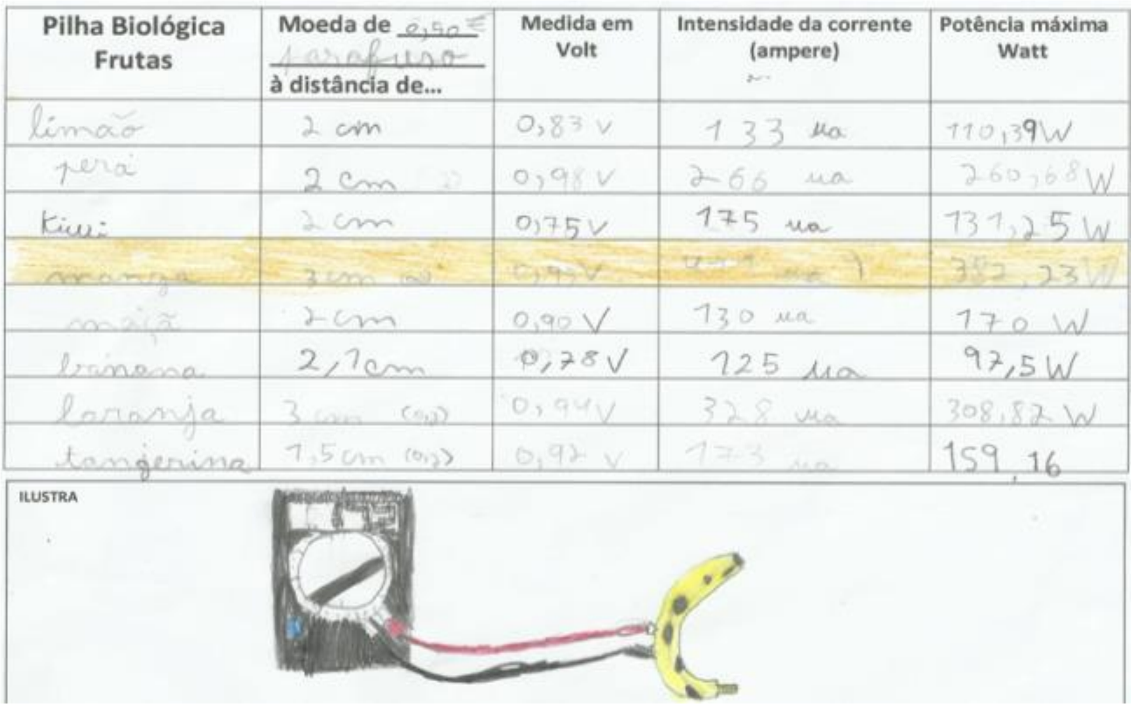

| Pilha Biológica Frutas | Moeda de 0,50 € ~~parafuso~~ à distância de... | Medida em Volt | Intensidade da corrente (ampere) | Potência máxima Watt |
|---|---|---|---|---|
| limão | 2 cm | 0,83 V | 133 ua | 110,39 W |
| pera | 2 cm | 0,98 V | 266 ua | 260,68 W |
| Kiwi | 2 cm | 0,75 V | 175 ua | 131,25 W |
| [illegible] | 3 cm [illegible] | [illegible] | [illegible] | 382,23 W |
| maçã | 2 cm | 0,90 V | 130 ua | 170 W |
| banana | 2,1 cm | 0,78 V | 125 ua | 97,5 W |
| laranja | 3 cm (0,2) | 0,94 V | 328 ua | 308,82 W |
| tangerina | 1,5 cm (0,2) | 0,92 V | 173 ua | 159,16 |



Figure 14 - Measurement of p.d. and current intensity of biological batteries. Calculation of the maximum power
Source: Authors' own file

All of these sessions were accompanied by a lot of questioning in order to drive the tasks towards student learning (STEM_TheoCK and STEM_TechCK), as exemplified in the following excerpt:

> Teacher: What is the potential difference of the orange?
> Student: It is 0.51 volts.
> Teacher: How much does the light bulb need to light up? [Teacher asks them to look up the information on the bulb]
> Student: It needs 1.5 volts.
> Teacher: Do you think it will be possible to light the lamp with an orange?
> Student: No. The lamp needs 1.5 volts. Look, it's almost triple!
> Teacher: So how many oranges do you need to get 1.5 volts?
> Student: It takes three.

The teacher continued the questioning, trying to develop investigative tasks so that they could draw conclusions:

> Teacher: What if you break the orange in half? Do you think the d.p. is the same for each half?
> Student: Of course not! It must be half.
> Teacher: Then cut the orange in half and measure the w.d. of each of the halves!
> Student: Ah! It was almost the same as the whole orange! It can't be ....
> Teacher: Cut the halves in half and measure again? What do you think will happen?
> Student: Maybe it will be the same... that's right! The size of the fruit doesn't count!
> Teacher: Does it really take three oranges to light the lamp?

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Student: No. Three pieces should be enough ... I'll try it! ... It lit!

At the end, from the various records that were on the board, the teacher asked several questions. For example:

Teacher: Which fruit or vegetable has the highest p.d.? And with the smallest p.d.?
Students: It's the tomato. It's the apple.

In addition to working on math, the teacher was able to introduce biological batteries, and to teach her students how to measure potential difference and current intensity, as well as how to build electrical circuits. In fact, in the course of several sessions dedicated to the theme, the teacher was able to work mathematics from hands-on activities related to electricity, in the context of STEM integration. In this way, Josephine developed tasks, based on math and science concepts and procedures, while incorporating engineering design methodology and using appropriate technology (Shaughnessy, 2013). Table 2 presents the STEM-related content that was worked on in class, according to the current mathematics curriculum.

Table 2: Content of STEM-related tasks

| Sciences | Technology | Engineering | Mathematics |
|---|---|---|---|
| Electricity | Cell phones<br>Toys<br>Multimeters<br>Bulbs | Plan, design and build electrical circuits. | Geometry and Measurement in organizing batteries according to their sizes, models and patterns.<br>Measuring potential difference and intensity in volt and ampere.<br>Numbers and Operations when the students organized and counted the batteries.<br>Analysis and Treatment of Data from the counts and measurements taken. |

Source: Authors' own elaboration

In view of the above, Josephine acquired specific Content Knowledge in order to be able to introduce practical hands-on tasks related to electricity and working with mathematics. In fact, the teacher was able to implement tasks that integrated STEM, which means that she acquired interdisciplinary knowledge related to STEM. Thus, it turns out that she put into action the specific Content Knowledge to integrate STEM (STEM_TheoCK and STEM_TechCK). On the other hand, Josephine was able to adapt the tasks to be understandable to her students, which reveals Pedagogical Content Knowledge (STEM_TheoPCK and STEM_TechPCK).

## Final considerations

This article investigated what knowledge is necessary for teachers to be able to innovate their mathematics practices in the context of a PDP that promotes STEM integration. Although the knowledge characterized by other authors, such as knowledge of

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the curriculum, or knowledge of the students, among others, is also present, the aim of this investigation was not to discuss the previous knowledge, but to characterize the knowledge that raised out in this context of STEM education.

Based on an empirical study, which took place over 3 school years, within a PDP that involved the implementation of hands-on, STEM-related practical activities, it was found that there were new categories of knowledge that were not characterized in the literature, such as theoretical knowledge and technical knowledge (Table 1). Thus, Theoretical Knowledge related to STEM (STEM_TheoCK) and Technical Knowledge to perform hands-on, STEM-related practical activities (STEM_TechCK) stand out. But this knowledge needs to be transformed in a way that makes sense to students, which requires pedagogical knowledge that in turn leads to STEM_TheoPCK and STEM_TechPCK.

From testimonies presented in the teachers' reports, it was evident that the PDP training model was adequate and innovative, and it was mentioned that it strengthened their knowledge, both of the subjects to be taught and of pedagogy, which will be reflected in their classes, improving their teaching practices. These are characteristics that point to the effectiveness of the PDP, as suggested by Desimone (2009).

In the particular case of teacher Josephine, it was found that she showed STEM_TheoCK and STEM_TechCK to develop and implement the interdisciplinary mathematics activities presented in the previous section. Moreover, she had the ability to introduce the tasks in such a way that they were meaningful to the students, promoting their learning about the presented contents, which reveals that she also acquired STEM_TheoPCK and STEM_TechPCK.

Thus, in a training context related to STEM education, it is concluded that there is expertise that is essential for teachers to be able to develop and implement interdisciplinary mathematics practices in class, which should be taken into account for the effectiveness of teacher training.

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