



The 8th International Conference on Information Technology and Quantitative Management
(ITQM 2020 & 2021)

Assessing Normalization Techniques for Simple Additive Weighting Method

Nazanin Vafaei*, Rita A. Ribeiro, Luis M. Camarinha-Matos

CTS-UNINOVA and School of Science and Technology, NOVA University of Lisbon
2829-516 Caparica, Portugal

Abstract

One of the current topics of attention in data analysis is the selection of the best normalization technique in the aggregation process when using Multi-Criteria Decision Making (MCDM) methods for solving decision problems. This is particularly critical in complex collaborative decision-making systems dealing with a large variety of heterogeneous data sources. Using different normalization techniques may result in different rankings of alternatives. So, enhancing the accuracy of the final ranking of alternatives could be achieved by selecting the most proper normalization techniques for each MCDM decision problem. In this direction, several attempts have been carried out, however, the lack of coherence and lack of a robust assessment framework persist. This situation encouraged the authors to propose an assessment framework that is enriched with several metrics for the evaluation of different normalization techniques in MCDM problems with the focus on partner/supplier selection in collaborative networks. As an illustration of the approach, in this work we assess different normalization techniques with the Simple Additive Weighting (SAW) method using metrics from the proposed assessment framework and select the most adequate technique for a small case study that is borrowed from literature. The suggested approach contributes to increasing the accuracy of final results for MCDM methods.

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Peer-review under responsibility of the scientific committee of the The 8th International Conference on Information Technology and Quantitative Management (ITQM 2020 & 2021)

Keywords: Normalization; MCDM; SAW; Decision making; data fusion; aggregation; Collaborative network

* Corresponding author.

E-mail address: nazanin.vafaei@uninova.pt.

1. Introduction

Over the last decade, decision makers deal with several challenges for finding optimal solutions for complex decision problems in the context of new developments in the data science. Multi-Criteria Decision Making (MCDM) is one of the desirable methods that is often adopted and for which there are well-known methods. Each MCDM method is defined by a decision matrix that consists of a set of alternatives A_i ($i=1, \dots, m$), a set of criteria C_j ($j=1, \dots, n$), the relative importance of the criteria (or weights) W_j , and r_{ij} , corresponding the ranking of alternative i with respect to criterion j [1]. In most MCDM problems criteria are defined with different units of measure. For instance, in a car selection decision problem, criteria such as fuel consumption, price, and speed are measured in Liter, Euro, and Km/h respectively. To adequately rank different cars, criteria values should be defined in the same (“neutral” or dimensionless) units. Thus, decision makers need a preprocessing that is called normalization for transforming all criteria into the same unit and producing comparable data from heterogeneous input data sets. Normalization techniques scale (transform) all criteria into the interval [0-1] and enable decision makers to compare criteria with different original units. But the way such transformation is done can have an impact on the solution. Thus, the normalization process has an essential role in most MCDM problems and selecting the most proper normalization techniques among the vast number of existing techniques is a major task for decision makers.

Due to rapid changes and advances in information and communication systems, Small and Medium sized Enterprises (SME) cannot respond to turbulent market individually. So, they are motivated to use collaborative networks (CN) mean by sharing skills and resources to deal with their limitations. One of the major challenges in CN is the selection of more suitable partners through several criteria (quantitative or qualitative criteria) that is the MCDM problem [2]. In the other word, fast-growing globalization scenarios and hyper-connected society emphasize the role of collaborative decision making when selecting business partners in their joint efforts to overcome the difficulties [2]. Therefore, partner selection has an important role in formation of any collaboration networks that is consider as a MCDM problem. So, the usage of suitable normalization techniques in collaborative decision models can contribute to effective partner/resource selection and lead to an increase in the creation of value and reduction of risks [2], [3]. In this paper we will focus on collaborative networks with a decision making perspective, for selection problems – i.e., where MCDM methods can be applied - such as selection of supplier, partner, resources, etc.

Several normalization techniques are proposed in the literature [1], hence some of them are used for the specific MCDM methods. For example, the Vector normalization technique is implemented with TOPSIS method and Sum is utilized with AHP. But there is no consensus among researchers about the conditions for utilizing each specific normalization technique. A number of papers have paid attention to the role and importance of normalization techniques in MCDM methods [4]–[10]. They pointed that using different normalization techniques may lead to different rankings of alternatives and cause a degradation in the accuracy of the final solution [4]–[10]. This situation motivated us to explore the suitability of normalization techniques to be used with MCDM methods.

In this paper, we evaluate the suitability of four normalization techniques using the assessment framework proposed in [9] and do a benchmarking analysis with a borrowed case study from [11]. We assess the effects of normalization techniques such as Max, Max-Min, Sum, and Vector using the SAW that is one of the popular MCDM methods. This study aims to show the robustness of the assessment framework and recommend a more proper technique for the given case study.

2. Assessment Framework for Evaluation of Normalization Techniques

Normalization techniques are used in the aggregation process to produce dimensionless data from heterogonous input data sets. Several normalization techniques are introduced in the literature that could be used

in different MCDM problems. For instance, Jahan and Edwards [1] collected 31 techniques from literature and discussed the advantages and disadvantages of them. They also addressed the interesting point of view about producing different rankings of alternatives in MCDM decision problems using different normalization techniques [1]. However, they did not provide an answer to the question “which normalization techniques are more appropriate for a given MCDM method?”.

There are some other research works on evaluating different normalization techniques to be used in MCDM problems. For example, Mathew et al. [7] used Spearman correlation to analyze the effect of six normalization techniques (Vector, Max, Max-Min, Sum, Logarithmic, and Enhanced accuracy) using WASPAS method and recommended Max-Min as the best technique. Also, Lakshmi and Venkatesan [12] calculated time and space complexity in MATLAB software to assess five normalization techniques, namely Max, Max-Min, Sum, Vector, Fuzzification (Gaussian membership function) with Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method. The obtained results introduced Sum normalization technique as the most proper technique for the related case study [12]. Furthermore, Aytekin [13] analyzed and tested the effects of 23 normalization techniques on 14 different scenarios (decision matrices) and used the SAW method as an aggregation technique. The authors pointed out that several features have effects on the selection of normalization techniques such as rank reversal, the range of normalized values, obtaining the same optimization aspect for all criteria, and the validity of results [13]. Moreover, Charaborty and Yeh [5], [6] proposed and implemented a Ranking Consistency Index (RCI) to assess four normalization techniques namely Max, Max-Min, Sum, and Vector for the TOPSIS [5] and SAW [6] methods. In another study, Charaborty and Yeh [11] compared the suitability of four normalization techniques for the SAW and TOPSIS methods and compared their results with Weighted Product (WP) method's results.

Recently, Vafaei et al. [8]–[10], [14], [15] proposed a general assessment framework that contains several metrics for evaluating different normalization techniques using MCDM methods. Figure 1 depicts such assessment framework.

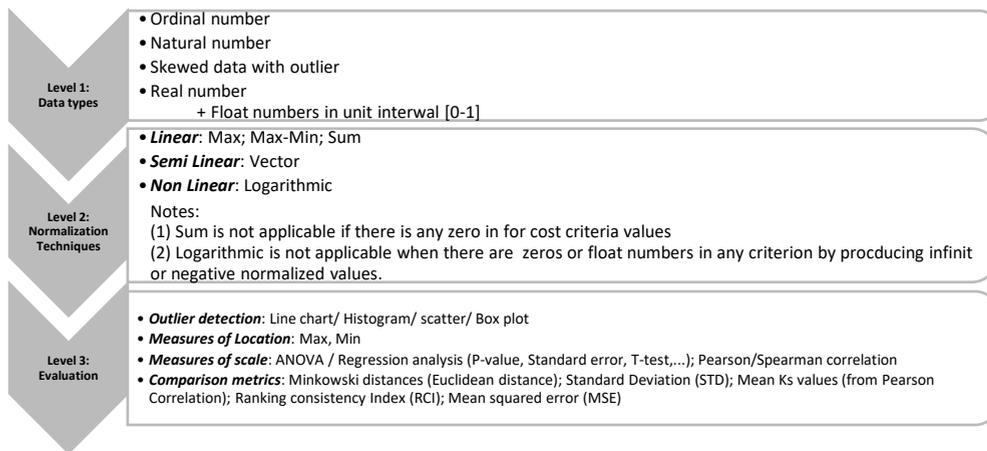


Figure 1: Three level of the evaluation framework (adapted from [9])

Figure 1 shows that the assessment framework includes three levels, namely (1) data types, (2) normalization techniques, and (3) evaluation. In the first level distinguishing the data types of decision matrices is necessary because proceeding to the second level needs the elimination of the normalization that produces infinite or negative normalized values in the presence of zero or decimal numbers in decision matrices. For instance, Sum normalization technique produces infinite normalized values when decision matrices contain zero for cost criteria using the cost formula. The third level of the framework consists of several metrics that enable us to evaluate the

selected normalization techniques (for more detail about the calculation process of these metric please see [9]). In this paper because of space limitation, we only implement comparison metrics of the third level (Euclidean distance; Standard Deviation (STD); Mean Ks values (from Pearson Correlation); Ranking consistency Index (RCI); Mean squared error (MSE)).

It should be noticed that in this study we show the applicability of the assessment framework using a small case study borrowed from [11] for the selection of the graduate fellowship applicants using SAW method. The authors of [11] assessed four normalization techniques namely Max, Max-Min, Sum, and Vector. In order to provide the possibility of benchmarking analysis between our results and the initial results from [11] we use the same normalization techniques. Table 1 shows the formulas of the chosen normalization techniques for benefit and cost criteria. The higher values for benefit criteria are preferable and the lower values for cost criteria are desirable.

Table 1: Selected normalization techniques (Adopted from [11])

Normalization technique	Condition of use	Formula
Linear: Max	Benefit criteria	$n_{ij}^+ = \frac{r_{ij}}{r_{max}}$
	Cost criteria	$n_{ij}^- = 1 - \frac{r_{ij}}{r_{max}}$
Linear: Max-Min	Benefit criteria	$n_{ij}^+ = \frac{r_{ij} - r_{min}}{r_{max} - r_{min}}$
	Cost criteria	$n_{ij}^- = \frac{r_{max} - r_{ij}}{r_{max} - r_{min}}$
Linear: Sum	Benefit criteria	$n_{ij}^+ = \frac{r_{ij}}{\sum_{i=1}^m r_{ij}}$
	Cost criteria	$n_{ij}^- = \frac{1/r_{ij}}{\sum_{i=1}^m 1/r_{ij}}$
Semi-Linear: Vector	Benefit criteria	$n_{ij}^+ = \frac{r_{ij}}{\sqrt{\sum_{i=1}^m r_{ij}^2}}$
	Cost criteria	$n_{ij}^- = 1 - \frac{r_{ij}}{\sqrt{\sum_{i=1}^m r_{ij}^2}}$

3. Comparison of Normalization Techniques with an Illustrative Example for SAW Method

In this part, we test the proposed assessment framework using benchmarking with the borrowed case study from [11]. This case study analyzes the effect of four normalization techniques and recommends the most proper one for the selection of the graduate fellowship applicants using the SAW, TOPSIS, and WP methods. In this study, to test the framework, we apply a benchmark regarding part of the paper [11] that has used the SAW method.

The SAW method was first defined by Churchman and Ackoff (1945) for the portfolio selection problem. The related formula to rank alternative is determined with the following equation:

$$R_i = \sum_{j=1}^n n_{ij} w_j \tag{1}$$

Where R_i is the ranking of i^{th} alternative, n_{ij} is the normalized values of r_{ij} , and w_j is the weight of j^{th} criterion. The case study consists of 5 criteria (C1, ..., C5) and 6 alternatives (A1, ..., A6) with the assigned weights (Table 2). In this case, C2 is the cost criteria (the lower values the better) and the others are benefit criteria (the higher values the better).

Table 2. Decision matrix input data and assigned weights for the case study borrowed from Charaborty and Yeh [11]

	C1	C2	C3	C4	C5
Weights	0.03	0.1	0.3	0.15	0.15
A1	690	3.1	9	7	4
A2	590	3.9	7	6	10
A3	600	3.6	8	8	7
A4	620	3.8	7	10	6
A5	700	2.8	10	4	6
A6	650	4	6	9	8

The authors of the borrowed case selected four normalization techniques as Max, Max-Min, Sum, and Vector for the related case study [11] to analyze the effect of the mentioned techniques on the ranking of alternatives. Table 3 shows the alternative values and ranking of alternatives for the SAW method using the four selected normalization techniques.

Table 3. Results with SAW method (adapted from [11])

	Max		Max-Min		Sum		Vector	
	Alt. Value	Rank						
A1	0.5421	5	0.3527	2	0.1159	6	0.3987	6
A2	0.5404	6	0.2664	6	0.1179	5	0.4041	5
A3	0.5541	3	0.3000	5	0.1209	2	0.4190	1
A4	0.5528	4	0.3082	4	0.1195	4	0.4070	4
A5	0.5682	1	0.4050	1	0.1202	3	0.4106	3
A6	0.5575	2	0.3523	3	0.1232	1	0.4137	2

As Table 3 shows, different normalization techniques produced different rankings of alternatives. The authors of this case study used Spearman correlation and calculated Mean ks values for each normalization techniques. In the end they recommend the Max normalization technique because of having the highest Mean ks value among the selected techniques [11].

We now apply our assessment framework to recommend the most suitable normalization techniques to this case study. Regarding the first level, the decision matrix does not contain zero and decimal numbers and proceed to the second level to choose candidate normalization techniques. As mentioned before to guarantee the benchmarking condition we used the same chosen normalization techniques by the authors of [11] as Max, Max-Min, Sum and Vector. Proceeding to the third level enables us to assess the selected normalization techniques using various comparison metrics, namely the Minkowski distances (Euclidean distance), Standard Deviation (STD), Mean Ks values (from Pearson Correlation), Ranking consistency Index (RCI), and Mean squared error (MSE). For details about calculating the Euclidean distance and STD please see [2] and for calculation of Mean Ks value and RCI please see [8]. Furthermore, MSE is the average of mean squared error for each normalization technique comparing with the other techniques using the ranking of alternatives [16], [17]. For the interpretation of the results, the lower value of MSE is preferred in order to reach less error, while for the Euclidean distance, STD, RCI, and Mean Ks the higher values are more desirable [9]. The obtained results for applied metrics are depicted in Table 4.

Ordering of the normalization techniques using the results of applied metrics (Table 4) are presented in Table 5. By observing these results, still we are not being able to select a more suitable normalization technique because each metric provided different ordering for the considered techniques. So, in order to summarize the results, plurality voting (PV) is then applied [9]. Plurality voting determines the number of times that each normalization technique be in the first rank regarding the different used metrics [9]. PV votes to the technique that has the largest number of times being the first rank and selects it as a more proper technique.

Table 4. Results of different metrics of assessment framework using SAW method for the borrowed case study

	Euclidean↑	STD↑	RCI↑	MSE↓	Mean ks↑
Max	0.2172	0.0397	3	1.7778	0.4640
Max-Min	0.5912	0.1079	2.6667	4.7778	0.1500
Sum	0.1790	0.0327	4	2.4444	0.3210
Vector	0.1850	0.0338	3.6667	2.7778	0.393

Table 5: Ordering of normalization techniques with respect to the metrics and using plurality voting

	Euclidean↑	STD↑	RCI↑	MSE↓	Mean ks↑	PV
Max	2	2	3	1	1	2
Max-Min	1	1	4	4	4	2
Sum	4	4	1	2	3	1
Vector	3	3	2	3	2	0

By implementing PV, we conclude that our framework selects the Max and Max-Min normalization as the best techniques for this case because it has the highest PV, while the approach by Chakraborty and Yeh [11], recommended the Max normalization technique. However, our framework provides more confidence and consistency because it uses metrics from different categories (STD from measures of data dispersion; Euclidean distance from the measure of proximity; Mean Ks, RCI, and MSE from comparison metrics) and could guarantee the robustness of the comparison between different normalization techniques. On the other hand, implementing PV worked as a good aggregation process to summarize results from different metrics. Concluding, results from our framework (Table 5) prove, with more certainty and consistency, the results from [11], where they just applied Spearman correlation and calculated Mean ks values for the four chosen normalization techniques.

Regarding the obtained results, using the assessment framework for selecting the best normalization techniques in MCDM decision problems with the focus of partner/supplier selection enhances the accuracy of the final result/ranking.

The practical design model of the assessment framework is visually described in Figure 2. These kinds of designing models demonstrate the process of an information system that includes several decisions and parts at different stages [18]. The bellow practical design consists of the main part and three phases as phase1-MCDM, phase1-normalization techniques (NT), and phase3-metrics. The decision maker can start the decision model from the main part of the practical design by defining the MCDM problem using the phase1 (MCDM steps). In this phase, determination of the decision matrix, assigning of the weights and type (cost/benefit) of criteria, and the desired MCDM method (TOPSIS, SAW, etc.) will be described. Then by proceeding in the main part, decision maker should choose the candidate normalization techniques (Max, Max-Min, Sum, Vector, Logarithmic) from the second phase (normalization techniques). Implementing different normalization techniques leads to the different ranking of alternatives that is the major challenge for the decision maker, so, proceeding to the last phase of the conceptual model is necessary. So, implementation of the different metrics as RCI, STD, MSE, Mean ks and Euclidean is suggested in the third phase. Then, in order to obtain a single result from the used metrics, applying plurality voting is vital. So, the practical model recommends the normalization techniques that are more fit to the related decision problem.

Finally, as Figure 2 shows, decision makers can apply different normalization techniques and use the proposed assessment framework to improve the accuracy of the final ranking results by following the practical design model that is presented in Figure 2.

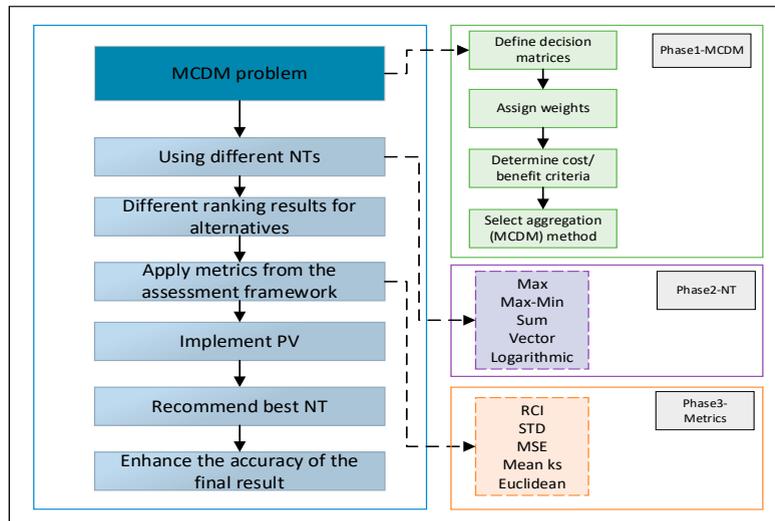


Figure 2: Practical design model for MCDM problems using the proposed assessment framework (Legend: NT= Normalization Technique, PV=Plurality Voting)

4. Conclusion

The results of this work aim to help decision makers to select the most proper normalization technique for their decision problems. This research contributes using different metrics from the proposed assessment framework to recommend the best technique among the six selected normalization techniques (Vector, Max, Max-Min, Sum, Logarithmic, and Enhanced accuracy) using SAW method for the borrowed case study from [7]. The obtained results recommended Max-Min normalization as the best technique for the related case study which is consistent with the initial results by the authors of [7]. Nevertheless, our results demonstrated more coherence of results than the initial study from [7] because of its privilege for implementing several metrics from different categories (data dispersion, data proximity, and etc.) and the usage of plurality voting to obtain single results from the different results of applied metrics. Furthermore, implementation practical design of the conceptual model for the proposed assessment framework (Figure 2) helps decision makers to reach more accurate ranking results. Besides, enterprises can apply assessment framework and practical design for partner selection in CN that leads to select more suitable partner/resource in the current fast changing environment.

As part of ongoing research, we address the implementation of the proposed assessment framework for other normalization technique that are introduced in the literature [1] as well as different MCDM methods (PROMETHEE, MOORA, etc.) which have not been evaluated yet. Moreover, implementation of the proposed assessment framework on real-world case studies, namely in the context of collaborative networks is considered as an open issue for further research.

Acknowledgements

This work was funded in part by the Center of Technology and Systems (CTS) and the Portuguese Foundation for Science and Technology (FCT) through the Strategic Program UIDB/00066/2020.

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