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How to handle the design preferences with Axiomatic Design

António Gonçalves-Coelho[1,2], **João Fradinho**[1,2], **António Gabriel-Santos**[1,2], **Miguel Cavique**[1,3] **and António Mourão**[1,2]

[1] R&D Unit for Mechanical and Industrial Engineering - UNIDEMI, NOVA School of Science and Technology, NOVA University Lisbon, Campus de Caparica, 2829-516 Caparica, PORTUGAL
[2] Department of Mechanical and Industrial Engineering - DEMI, NOVA School of Science and Technology, NOVA University Lisbon, Campus de Caparica, 2829-516 Caparica, PORTUGAL
[3] Naval Academy, Base Naval de Lisboa, Alfeite, 2810-001 Almada, PORTUGAL

gonçalves.coelho@fct.unl.pt

Abstract. Picking the best design solution among a small collection of viable alternatives is often a part of engineering design. The traditional method to do it is the weighted multicriteria decision procedure. However, the Axiomatic Design (AD) framework does not employ weighting factors to deal with design preferences. Instead, AD uses the careful placing of each design range. This paper stresses the reasons AD does not include weighting factors—an imaginary flaw that teachers and designers should not attempt to change because that would break the rational soundness of AD teaching and practice. A numerical example supports our rationale.

1. Introduction

The wording "engineering design" refers to a form of creative, multifaceted, and iterative enterprise that promotes the evolution of humanity. There are many pictorial approaches to explain the expression, and one of the best ones is that of figure 1.

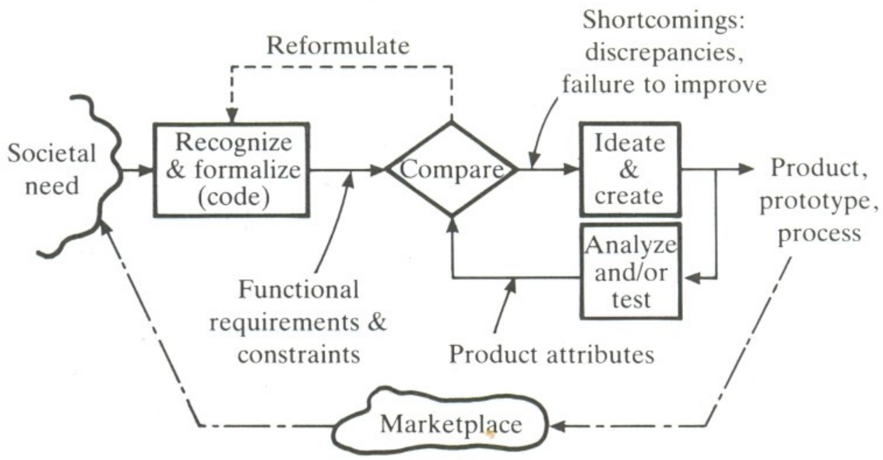

Figure 1. The engineering design after Wilson [1].

The closed-loop paths within figure 1 disclose the iterative nature of the process. Figure 2 depicts engineering design in a much more abridged way.

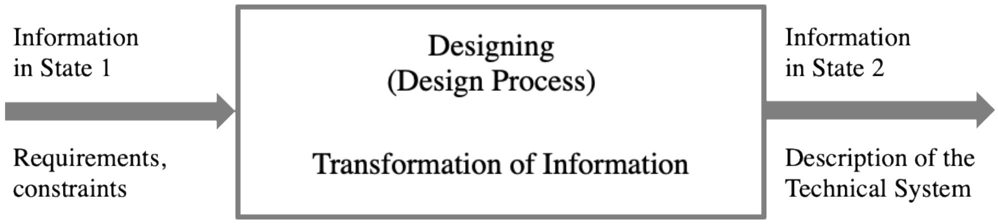

Figure 2. The engineering design after Hubka & Eder [2].

Designers of engineering systems must always consider the different preferences of the multiple stakeholders in their design decisions. For example, a hardware vendor might wish to provide a compact device, and the end-users might prefer easy-to-use systems. In addition, the soon-to-be manufacturer might favor producing an object mainly made of die-stamped parts.

Different methods for decision-making are part of the modern design theory and methodology. This paper aims to show how to deal with the design preferences in Axiomatic Design. Section 2 introduces the basics of the Axiomatic Design (AD). For comparison, section 3 briefly describes the weighted multicriteria decision method (MCDM). Section 4 contains a numerical example, and section 5 is devoted to examining the results attained with both methods. At last, section 6 presents the conclusions.

2. The ABC of Axiomatic Design

Nam P. Suh created the Axiomatic Design (AD) theory in the late 1970s [3] and widely spread it in 1990 through his first textbook on the matter [4]. In the words of Suh, “The ultimate goal of axiomatic design is to establish a scientific basis for design and to improve design activities by providing the designer with a theoretical foundation based on logical and rational thought processes and tools” [5]. In an AD framework, a vector denotes the design object in each of four design domains, as shown in figure 3, and mappings between contiguous design domains depict the design process.

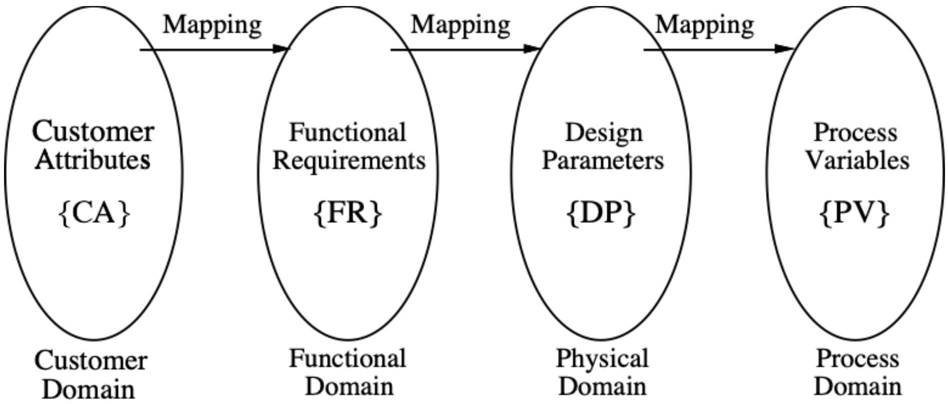

Figure 3. The design world after Suh [5].

The customer attributes vector, {CA}, describes the benefits the customer is expecting. The functional requirements vector, {FR}, arises from the customer attributes and contains the minimum set of requirements that can satisfy the design objectives, stated in precise technical terminology. The functional requirements must be solution-neutral (not biased by solutions imagined by stakeholders). Usually, their phrasing contains action words, such as provide, increase, ensure, etc. [6]. The design parameters vector {DP} specifies the set of elements of the design object meant to satisfy {FR}. It describes the physical features of the design object (aka the embodiment, *e.g.*, parts arrangement, materials, etc.). The process variables vector {PV}, which belongs to the process domain, contains the particularities of the process employed to manufacture the design object. The existing design constraints, (C), do not incorporate figure 1 but represent the economic, social, environmental, political, and technical limits imposed on the design solutions—the input constraints, determined by the stakeholders at the outset of the design process, and the system constraints revealed across the design development. In other words, constraints are limitations to the designers' freedom to search for suitable solutions in the physical and process domains.

Each mapping in figure 3 corresponds to an array of design decisions. For example, the mapping from the functional to the physical domain is denoted by equation (1), where [A] is the design matrix.

$$\{FR\} = [A] \cdot \{DP\},\ A_{ij} = \frac{\partial FR_i}{\partial DR_j}, i = 1..n, j = 1..m \tag{1}$$

In the Axiomatic Design viewpoint, good design decisions conform to the following axioms [7]:

- Axiom 1: Maintain the independence of the functional requirements (FRs).
- Axiom 2: Minimize the information content of the design.

2.1. *The independence axiom*

Concerning axiom 1, the designer must find at least one set of design parameters that fulfill the given functional requirements, which means that the design matrix of equation (1) should be diagonal or at least triangular (m=n in both cases). The diagonal matrix corresponds to an uncoupled design and the triangular to a decoupled design. If $m<n$, then the design matrix is rectangular, and either a coupled design solution results or the FRs cannot be satisfied, as per theorem 1 [7]. The design matrix is also rectangular if $m>n$, but we have a redundant design that is either uncoupled, decoupled, or else coupled, as per theorem 3 [7]. The independence condition is crucial, and more about redundant, independent design solutions can be found in the literature [8].

2.2. *The minimum information axiom*

As for axiom 2, the computation of the information content follows Shannon's definition [9], for whom the probability of success of a one-FR, one-DP design solution is given by

$$I = \log\frac{1}{p} = -\log p\ , \tag{2}$$

where p is the probability of success of the related event. For a one-FR, one-DP design, the probability of success is the ratio of the areas of common range and the system range in figure 4 and is given by

$$p = \frac{\text{Area of the common range}}{\text{Area of the system range}} \tag{3}$$

Figure 4 fully clarifies equation (3). The system probability density function (pdf) of a design solution, p, denotes its probability of success. Vertical straight lines represent the bounds of the design range, and the grey zone is the area within the common range, *i.e.*, the area under the section of the $p(FR)$ curve lying inside the design range. The information content dimensionless unit is the *bit* if base 2 logarithms are used, or the *nat* for natural logarithms.

The information content is given by

$$I = -log \int_{cr} p(FR)dFR \tag{4}$$

and for the whole system range it will be

$$\int_{-\infty}^{+\infty} p(FR)dFR = 1 \tag{5}$$

For a n-FR uncoupled design we have

$$I = -\log\left(\prod_{i=1}^{n} p_i\right) = -\sum_{i=1}^{n} \log p_i \tag{6}$$

The computation of the information content of decoupled and coupled designs is path-dependent, so that "The information contents of coupled and decoupled designs depend on the sequence by which the DPs are changed to satisfy the given set of FRs" as per theorem 7 [7].

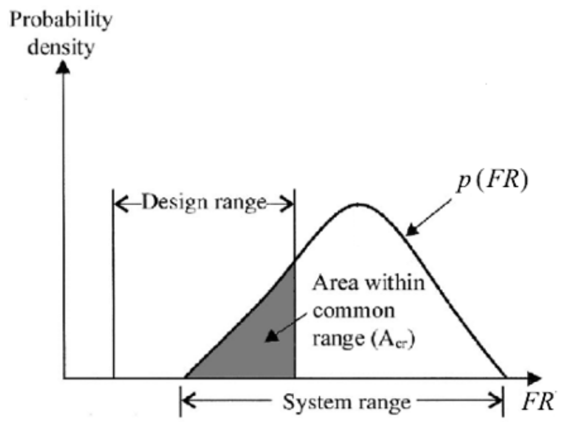

Figure 4. Figuring the information content from the probability density function (adapted from [5]).

Since we know the path for determining the values of the FRs in decoupled design solutions, then employing the fundamental knowledge of conditional probability allows computing their information content. Theorem 17 (Design in the absence of complete information)—which states, "Design can proceed even in the absence of complete information only in the case of a decoupled design if the missing information is related to the off-diagonal elements" [7]—may help in this demanding task. The abovesaid path is undefined in coupled designs so that we cannot compute their information content.

In the AD framework, the minimum information content axiom is the key to select the best of a set of uncoupled and (or) decoupled alternative design solutions. It is worth noting that, by definition, alternative solutions are the ones that share the same FR vector.

2.3. The zigzag decomposition

The design process follows a top-down hierarchical zigzagging pattern between each pair of adjacent design domains, from the most abstract level to a more detailed one, as shown in figure 5. The partly pictured design process in the figure, which top FR is “source of mechanical energy”, yields to a two-stroke internal combustion engine.

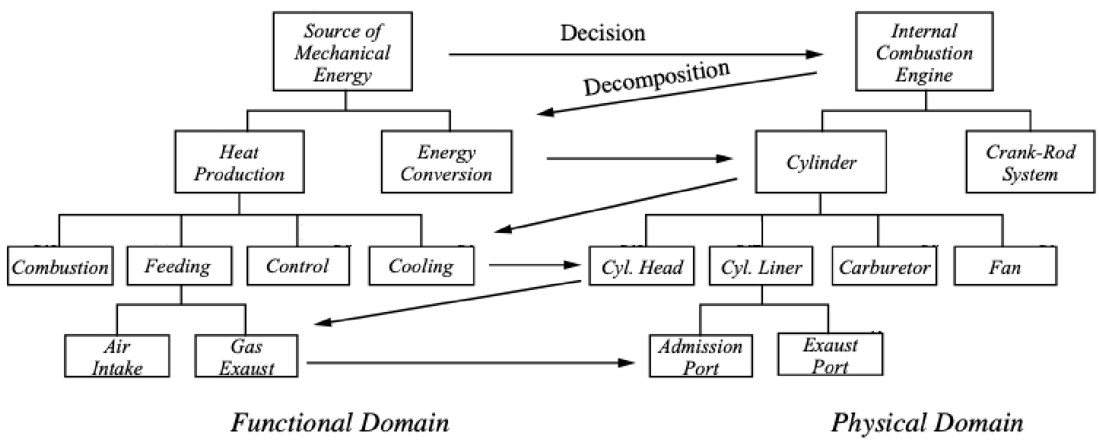

Figure 5. The zigzag decomposition procedure (adapted from [8]).

The design must comply with axiom 1 at each hierarchical level of the decomposition. Zigzag decomposition is unique to AD and should end after attaining the detail required to describe the design object.

3. The weighted multicriteria decision method

Out of the AD context, weighted multicriteria decision method (MCDM) is often the means to choose the best alternative solution.

It is worth noting that the simplest method to deal with such a problem is to sequentially apply the chosen criteria, one at a time, to all the alternative solutions. This procedure allows immediately rejecting the alternatives not complying with at least one of the applicable criteria. However, it has an evident shortcoming: the method does not allow to order the surviving solutions.

The simplest way to determine that ranking is by getting the overall scores for the alternative solutions, C_T, adding their scores associated to each specific criterion, c_i, corrected via weighting factors, w_i, according to the equation:

$$C_T = \sum_{i=1}^{n} w_i c_i, 0 < w_i < 1, \sum_{i=1}^{n} w_i = 1 \quad (7)$$

The criteria to use are indicative values for the functional requirements, FR_i. All the specific criteria are expressed in the same dimensionless scale, for example, 0-10, of type the higher, the better (or inversely, the lower, the better) so that equation (7) yields the overall score for each solution, C_T, denoted in the same dimensionless scale.

Everybody got in touch with some variants of the MCDM method at school since they are used everywhere to grade tests and compare pupils' performance. Employing preferences in the form of weighting factors for the computation of the information content is most probably inspired by this method. But that idea is wrong, and the weighted multicriteria decision method was included here for a better understanding of our claim.

4. A numerical example

The benchmarking of rapid prototyping (RP) machines done almost twenty years ago by Ryder *et al.* [10] suggested this numerical problem. Although its results are outdated, Ryder's work provided the data that inspired this example. The problem here simulated is choosing the best polymer-based additive manufacturing machines to a workshop for mechanical parts with prescribed engineering characteristics. The preselected machines use the following technologies:

- Stereolithography (SLA)
- Fused Deposition Modelling (FDM)
- Laminated Object Modelling (LOM)
- 3-Dimensional Printing (3DP)
- Multi Jet Modelling (MJM)
- Object QUADRA (Obj)
- Droplet on Demand (DoD)

The key factors, or criteria, for selecting the manufacturing technologies were:

i) **Building accuracy.** The exactness of the manufactured parts
ii) **Robustness.** The ability to resist change without modifying its initial configuration
iii) **Building speed.** The speed of the additive manufacturing
iv) **Capital costs.** The cost of machinery, tooling, and workshop infrastructure.
v) **Running costs.** The cost of raw materials, electrical power, labor, etc.

Table 1 lists the capabilities of the machines, with those capabilities assumed as independent in the light of axiom 1. The used values are dimensionless, with the higher, the better.

The scales of items in table 1 have different amplitudes: 0-10 for building accuracy and speed, 0-9 for capital and operating costs, and 0-8 for robustness. All the scales are monotonically increasing, but this characteristic is not mandatory. The different scales' maximum values occur because the original benchmarking included other machines not considered in this work. Besides the different amplitudes, it is worth mentioning that FRs with diverse dimensions are allowed altogether, *e.g.*, speed in ft/s, tensile strength in MPa, production rate in parts per hour, and cost in Euro—because information content depends on a ratio of areas with same dimensions, as shown by the association of equations (2) and (3) with figure 4. The bounds we considered for the system ranges of the studied machines derive from table 1 and are displayed in table 2.

Table 1. The capability of the rapid prototyping machines (built from [10]).

	Building accuracy	Robustness	Building speed	Capital costs	Running costs
SLA	8	5	6	2	1
FDM	6	8	5	5	4
LOM	6	6	5	5	3
3DP	4	6	9	8	9
MJM	6	2	8	9	7
Obj	8	7	10	8	7
DoD	10	2	2	9	3

Table 2. The bounds of the machines' system ranges.

	Building accuracy		Robustness		Building speed		Capital costs		Running costs	
	min.	max.	min.	max.	min.	max.	min.	max.	min.	max.
SLA	0.0	8.0	0.0	5.0	0.0	6.0	1.5	2.5	0.5	1.5
FDM	0.0	6.0	0.0	8.0	0.0	5.0	4.5	5.5	3.5	4.5
LOM	0.0	6.0	0.0	6.0	0.0	5.0	4.5	5.5	2.5	3.5
3DP	0.0	4.0	0.0	6.0	0.0	9.9	7.5	8.5	8.5	9.5
MJM	0.0	6.0	0.0	2.0	0.0	8.8	8.5	9.5	6.5	7.5
Obj	0.0	8.0	0.0	7.0	0.0	10.0	7.5	8.5	6.5	7.5
DoD	0.0	10.0	0.0	2.0	0.0	2.0	8.5	9.5	2.5	3.5

Table 3 contains the limits of the considered design ranges for the machines under analysis. In terms of duty, four different cases were considered.

Table 3. The design ranges of the studied cases.

		Building accuracy	Robustness	Building speed	Capital costs	Running costs
Case A	min	3.5	3.5	3.5	0.0	0.0
	max	6.5	6.5	6.5	10.0	10.0
Case B	min	7.0	3.5	3.5	0.0	0.0
	max	10.0	6.5	6.5	10.0	10.0
Case C	min	3.5	3.5	7.0	0.0	0.0
	max	6.5	6.5	10.0	10.0	10.0
Case D	min	3.5	3.5	0.0	0.0	0.0
	max	6.5	6.5	10.0	3.0	3.0

Case A depicts general-purpose operation: the design ranges for accuracy, robustness, and building speed are centered close to the midpoint of the system ranges, with an extent (or tolerance) of around ±15% the corresponding scale. The costs are not critical.

Case B differs from case A in that it requires higher accuracy. It reflects the need for a system to produce high-accuracy, one-off parts.

Case C and A are alike, but a higher building speed is required. The associated capital and running costs are not critical. Case C serves the production of small batches.

Lastly, Case D also arises from case A, but low costs are required, and the building speed is not critical. It portrays the regular manufacture of prototypes.

In the real world, the systems' probability distribution functions are often bell-shaped, but uniform functions are used here due to the lack of accurate data, a usual simplification that originates an error in the computed values.

Table 4 displays the information content for the four studied cases, as computed by equation (3) for the hypothesis of uniform distribution. The values used in the computation were the bounds of the system and design ranges from tables 2 and 3.

Table 4. The total information content (bit) of the studied cases.

	Case A	Case B	Case C	Case D
SLA	4.4	6.0	∞	3.2
FDM	4.2	∞	∞	∞
LOM	5.9	∞	∞	∞
3DP	∞	∞	6.4	∞
MJM	∞	∞	∞	∞
Obj	∞	5.7	4.4	∞
DoD	∞	∞	∞	∞

Table 4 is easy to read: The probability of success increases as the information content decreases, and success is fully guaranteed when the information content is zero. Machines with infinite information content are to avoid because they cannot accomplish all the FRs.

The use of the MCDM for the same RP systems is presented next for a matter of comparison. But one has to normalize the data about the robustness, capital costs, and running costs given by table 1 so that the dimensionless scales of all the systems' capabilities are the same. Table 5 includes the results of such conversion for robustness, capital costs and running costs.

Table 5. The capability of the rapid prototyping machines (normalized).

	Building accuracy	Robustness	Building speed	Capital costs	Running costs
SLA	8.0	6.3	6.0	2.2	1.1
FDM	6.0	10.0	5.0	5.6	4.4
LOM	6.0	7.5	5.0	5.6	3.3
3DP	4.0	7.5	9.0	8.9	10.0
MJM	6.0	2.5	8.0	10.0	7.8
Obj	8.0	8.8	10.0	8.9	7.8
DoD	10.0	2.5	2.0	10.0	3.3

Employing equation (7), we used the data of table 5 to evaluate the different RP machines for the equally weighted preferences hypothesis. No more transformations of scale are required since they are all monotonically ascending. Monotonicity of the same sense for all criteria is mandatory in MCDM.

Table 6 displays the attained results, C_T, were c_i and w_i have the same meaning as in equation (7). These results entail the following remark: just by coincidence, the achieved scores (the higher, the better) are not very different from those achieved for Case A via the AD's procedure (*cf.* Table 4, where the lower, the better). The misleading MCDM scores related to the machines discarded through AD are highlighted in table 6 by a light grey background and may look better than the valid ones. They correspond to systems unable to fulfill all the case A requirements, and traditional single-criterion tests would promptly reject the same machines. The above-said coincidence is due to design ranges spanning in the neighborhood of the system range midpoints. Nonetheless, there is no actual similarity between both approaches because they address different issues. The AD procedure deals with relatively well-defined functional contexts, expressed by system and design ranges, while the MCDM tackles vague problems described by indicative values.

Table 6. The computed machine scores for equally weighted preferences.

		Building accuracy	Robustness	Building speed	Capital costs	Running costs	C_T
	w_i	0.2	0.2	0.2	0.2	0.2	
SLA	c_i	8.0	6.3	6.0	2.2	1.1	
	w_ic_i	1.6	1.3	1.2	0.4	0.2	4.7
FDM	c_i	6.0	10.0	5.0	5.6	4.4	
	w_ic_i	1.2	2.0	1.0	1.1	0.9	6.2
LOM	c_i	6.0	7.5	5.0	5.6	3.3	
	w_ic_i	1.2	1.5	1.0	1.1	0.7	5.5
3DP	c_i	4.0	7.5	9.0	8.9	10.0	
	w_ic_i	0.8	1.5	1.8	1.8	2.0	7.9
MJM	c_i	6.0	2.5	8.0	10.0	7.8	
	w_ic_i	1.2	0.5	1.6	2.0	1.6	6.9
Obj	c_i	8.0	8.8	10.0	8.9	7.8	
	w_ic_i	1.6	1.8	2.0	1.8	1.6	8.7
DoD	c_i	10.0	2.0	2.5	10.0	3.3	
	w_ic_i	2.0	0.4	0.5	2.0	0.7	5.6

5. Discussion

The AD's information-based decision-making procedure requires accurate quantitative data about the limits of the design ranges. It also needs data about the boundaries of the system ranges and the corresponding probability density functions. In the lack of reliable data about the pdfs, simplified surrogates may be used, including uniform pdfs. The relative error induced by the substitute pdfs used in this paper is likely similar for the different systems under assessment because they are all of electro-mechanical nature and pursue the same goals. However, the designers can adopt other types of simple pdf, *e.g.*, polygonal, if they find too large the error caused by uniform functions. The fuzzy set theory could help to find proxy distribution functions [11]. One should stress that the AD method's ability to detect unacceptable solutions does not depend on the shape of the probability distribution functions but only on the occurrence of one or more empty intersections of systems and design ranges.

Besides, "All information contents that are relevant to the design task are equally important regardless of their physical origin, and no weighting factor should be applied to them" as per theorem 16 [7]. Therefore, the AD framework excludes weighing factors. Theorem 16 is not surprising since the AD's concept of information content came from Shannon's information theory, and using weighting factors to determine the information contents associated with simultaneous random events is nonsense.

The careful regard on the design ranges' location and span is the right way to reflect preferences in the design decision-making. After all, one cannot sustain that a specific criterion is critical and, at the same time, prescribe for it a wide design range. For example, the design range for the maximum acceleration of a racing car should be small so that the information content of all the alternative solutions is large, which might ease finding the best one. Additionally, it could facilitate rejecting some alternatives if their information content becomes infinite.

The plain MCDM presented here highlights the traditional viewpoint on the link between weighting factors (a concept that is alien to AD) and design preferences. Note that the weighting factors are not independent of each other, as required by axiom 1, making it impossible to change the weighting factors one at a time. However, this method deserves a word of appreciation because it is the *de facto* technique when the decision criteria are qualitative, as it often happens in artificial intelligence and social sciences applications. And yes! It is also helpful in engineering design when design decisions rely on qualitative data, such as expert opinions, in case of which the AD decision technique based on the information content becomes useless.

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6. Conclusion

Selecting rapid prototyping machines helped us to remember a procedure alien to the Axiomatic Design framework, which is the use of weighting factors to express design preferences in decision-making processes—as in the case of the weighted multicriteria decision method.

From the practical point of view, the weighted multicriteria decision method might look simpler. But it does not allow a reliable analysis because it is impossible to find out a methodologically correct matching between the weighting factors and the service conditions under study.

The numerical example emphasized that the AD decision method entirely relies on quantitative data about the wanted performance for the design object in the previously defined service conditions.

The chief conclusion is that appropriate position and tolerance of design ranges concerning the corresponding system ranges is the right way to express so-called design preferences since no weighting factors should intervene in an Axiomatic Design framework.

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ORCID IDs
A Gonçalves-Coelho https://orcid.org/0000-0001-9428-0936
J Fradinho https://orcid.org/0000-0001-5889-771X
A Gabriel-Santos https://orcid.org/0000-0002-1797-2020
M Cavique https://orcid.org/0000-0002-9204-7521
A Mourão https://orcid.org/0000-0002-9470-8477

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