

Experiments towards resolving the proton charge radius puzzle

A. Antognini^{1,2,a}, K. Schuhmann^{1,2}, F. D. Amaro³, P. Amaro⁴, M. Abdou-Ahmed⁵, F. Biraben⁶, T.-L. Chen⁷, D. S. Covita⁸, A. J. Dax², M. Diepold⁹, L. M. P. Fernandes³, B. Franke⁹, S. Galtier⁶, A. L. Gouvea³, J. Götzfried⁹, T. Graf⁵, T. W. Hänsch^{9,b}, M. Hildebrandt², P. Indelicato⁶, L. Julien⁶, K. Kirch^{1,2}, A. Knecht², F. Kottmann¹, J. J. Krauth⁹, Y.-W. Liu⁷, J. Machado^{4,6}, C. M. B. Monteiro³, F. Mulhauser⁹, F. Nez⁶, J. P. Santos⁴, J. M. F. dos Santos³, C. I. Szabo⁶, D. Taqqu¹, J. F. C. A. Veloso⁸, A. Voss⁵, B. Weichelt⁵, and R. Pohl⁹

¹Institute for Particle Physics, ETH, 8093 Zurich, Switzerland

²Paul Scherrer Institute, 5232 Villigen-PSI, Switzerland

³LIBPhys, Physics Department, Universidade de Coimbra, 3004-516 Coimbra, Portugal

⁴Laboratório de Instrumentação, Engenharia Biomédica e Física da Radiação (LIBPhys-UNL) e Departamento de Física da Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa, Monte da Caparica, 2892-516 Caparica, Portugal

⁵Institut für Strahlwerkzeuge, Universität Stuttgart, 70569 Stuttgart, Germany

⁶Laboratoire Kastler Brossel, UPMC and CNRS, 75005 Paris, France

⁷Physics Department, National Tsing Hua University, Hsincho 300, Taiwan

⁸i3N, Universidade de Aveiro, Campus de Santiago, 3810-193 Aveiro, Portugal

⁹Max Planck Institute of Quantum Optics, 85748 Garching, Germany

Abstract. We review the status of the proton charge radius puzzle. Emphasis is given to the various experiments initiated to resolve the conflict between the muonic hydrogen results and the results from scattering and regular hydrogen spectroscopy.

1 The proton charge radius puzzle

The historical route to the proton charge radius (r_p) is from elastic electron-proton scattering. In a completely complementary fashion, it has been obtained also from “high-precision” laser spectroscopy of hydrogen (H). Since a few years, “high-sensitivity” laser spectroscopy of muonic hydrogen (μp) offers a third way. The value extracted from μp with a relative accuracy of 5×10^{-4} is an order of magnitude more accurate than obtained from the other methods. Yet the value is 4% smaller than derived from electron-proton scattering and H spectroscopy with a disagreement at the 7σ level [1–5].

In the last five years as summarized in [6, 7] various cross checks and refinements of bound-state QED calculations needed for the extraction of r_p from μp have been performed, together with investigations of the proton structure. Several suggestions in the field of “beyond standard model” BSM physics have been articulated, re-analysis of scattering data have been carried out and new experiments have been initiated. Despite this, presently the discrepancy still persists and the resolution

^ae-mail: aldo@phys.ethz.ch

^bAlso at: Ludwig-Maximilians-Universität, 80539 Munich, Germany.

of the proton radius puzzle remains unknown. In this article, we summarize mainly the ongoing experimental activities which hold the potential to unravel the proton radius puzzle.

As the atomic energy level are slightly modified by the nuclear finite size, it is possible to deduce the nuclear charge radius by performing spectroscopy of the atomic energy levels. In leading order, the energy shift caused by the nuclear finite size is

$$\Delta E_{\text{finite size}} = \frac{2\pi\alpha}{3} |\phi^2(0)|^2 R_E^2 = \frac{2m_r^3 \alpha^4}{3n^3} R_E^2 \quad (1)$$

where $\phi(0)$ is the wavefunction at the origin in coordinate space, m_r the reduced mass of the atomic system, α the fine structure constant and n the principal quantum number. R_E is the charge radius of the nucleus defined in a covariant way as the slope of the electric form factor (G_E) at zero momentum exchange Q^2

$$R_E = -6 \frac{dG_E}{dQ^2} \Big|_{Q^2=0}. \quad (2)$$

Non-relativistically, R_E is the second moment of the electric charge distribution ρ_E of the nucleus $R_E^2 \approx \int d\vec{r} \rho_E(\vec{r}) r^2$.

The m_r^3 dependence of Eq. (1) reveals the advantages related with muonic atoms. As the muon mass is 200 times larger than the electron mass, the muonic wavefunction strongly overlaps with the nucleus ensuing a large shift of the energy levels due to the nuclear finite size. Thus, the muonic bound-states represent ideal systems for the precise determination of nuclear charge radii.

The proton form factors can be obtained from unpolarized differential cross section measurements of electron scattering off protons. In the one-photon approximation the elastic cross section is

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega} \Big|_{\text{Mott}} \times \frac{1}{1+\tau} \left(G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right) \quad (3)$$

where the Mott cross section applies for point-like particles, G_E and G_M are the electric and magnetic form factors of the proton, $\tau = Q^2/4M^2$ and $\epsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta/2)$ are kinematical variables. Commonly, the Rosenbluth separation is applied to disentangle the charge from the magnetic contributions by using the angle-dependence at fixed Q^2 . Therefore, by measuring the differential cross section at various Q^2 and angles θ , one obtains $G_E(Q^2)$ and $G_M(Q^2)$ and via Eq. (2) the radius.

2 Muonic hydrogen and possible new physics explanations

The CREMA collaboration has measured two transition in μp : from the triplet ($2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$) [1] and the singlet ($2S_{1/2}^{F=0} - 2P_{3/2}^{F=1}$) [2] 2S-states yielding a radius of $r_p = 0.84087(39)$ fm. More specifically the two measured energy splittings, from the triplet $h\nu_t$ and from the singlet $h\nu_s$ states, can be combined to obtain both the 2S Lamb shift $E_L = \Delta E(2S - 2P_{1/2})$ and the hyperfine splitting E_{HFS} :

$$E_L = \frac{1}{4} h\nu_s + \frac{3}{4} h\nu_t - 8.8123(2) \text{ meV}, \quad (4)$$

$$E_{\text{HFS}} = h\nu_s - h\nu_t + 3.2480(2) \text{ meV}, \quad (5)$$

where the numerical values follow from reliable proton-independent corrections of the 2P states. Experimentally [2]

$$E_L^{\text{exp}} = 202.3706(23) \text{ meV} \quad (6)$$

equivalent to a frequency 48932.99(55) GHz, limited by statistics while the systematics effects is at the 300 MHz level. From theory [8]

$$E_L^{\text{th}} = 206.0336(15) [\text{meV}] - 5.2275(10) \left[\frac{\text{meV}}{\text{fm}^2} \right] r_p^2 + 0.0332(20) [\text{meV}]. \quad (7)$$

The first term accounts for QED contributions, the second for finite size effects, and the third for the two-photon exchange (TPE) contribution. From the Lamb shift an improved r_p value free from uncertainties related with the HFS splitting has been determined, and from the HFS the Zemach radius, albeit not with the same accuracy as the charge radius.

The consistency of the two μp measurements, represents an important cross check of the muonic results. The typical systematic effects affecting the atomic energy levels are substantially suppressed in μp due to the stronger binding. The internal fields and the level separation of the muonic atoms are greatly enhanced compared to regular atoms making them insensitive to external fields (AC and DC Stark, Zeeman, black-body radiation and pressure shifts). Thus μp turns out to be very sensitive to the proton charge radius (m_r^3 -dependence) and insensitive to systematics which typically scales as $\sim 1/m_r$. The possible involvement of weakly bound three-body systems in the muonic hydrogen spectroscopy experiment [9] has been ruled out by three-body calculations [10], and by the experimental non-observation of sizable additional line broadening, line splitting and event rate decrease.

The bound-state QED corrections which give rise to Eq. (7) have been computed by several groups [11, 12] as summarized in [8] and updated recently in [13, 14]. Particular attention has been devoted to the TPE contribution which has been computed in two frameworks: one making use of dispersion relations and measured inelastic structure functions of the proton [15], the second from chiral perturbation theories [16]. Both ways provide consistent predictions. In the dispersion-based approach a subtraction term in form of an integral from zero to infinite Q^2 is necessary which can not be fully constrained by data. At intermediate Q^2 indeed a modeling of the proton is necessary [17]. The large majority of the community, agrees with a value for this subtraction term which is two order of magnitude smaller than the measured discrepancy of 0.3 meV [18–20]. Even if improbable, in principle still a very un-smooth and physically unmotivated proton structure could be constructed which could shift the μp transition to explain the measured discrepancy [21]. However, the published subtraction functions proposed to solve the proton radius puzzle would affect through the Cottingham formula the proton and neutron masses by 600 MeV [22] which is quite implausible when compared with the measured and computed neutron-proton mass difference of 1.29 MeV [23].

Explaining the discrepancy by this subtraction term can be interpreted as an exotic hadronic effect. Other effects occurring in the vicinity of the nucleus, as the breakdown of the perturbative approach in the electron-proton interaction [24, 25], the interaction with sea $\mu^+\mu^-$ and e^+e^- pairs [26] etc., have been suggested and need to be further investigated but at moment are not yet conclusive. Several BSM extensions have been proposed but the vast majority of them have difficulties to resolve the measured discrepancy without conflicting with other low energy constraints. Still some BSM theories can be formulated but they require fine-tuning (e.g. cancellation between axial and vector components), targeted coupling (e.g. preferentially to the muon or to muon proton) and are problematic to be merged in a gauge invariant way into the standard model [27–29].

3 Hydrogen experiments

In a simplified way, the hydrogen S-state energy levels can be described by

$$E(nS) = \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3} \quad (8)$$

where $R_\infty = 3.289\,841\,960\,355(19) \times 10^{15}$ Hz is the Rydberg constant and approximately

$$L_{1S} \simeq 8171.636(4) \text{ [MHz]} + 1.5645 \left[\frac{\text{MHz}}{\text{fm}^2} \right] r_p^2 \quad (9)$$

the Lamb shift of the ground state given by bound-state QED calculations. The different n -dependence of the two terms in Eq. (8) permits the determination of both R_∞ and r_p , from at least two transition frequencies in H, assuming Eq. (9).

Being the most precisely measured transition ($u_r = 4 \times 10^{-15}$) [30, 31] and because it is showing the largest sensitivity to the Lamb shift contributions usually the 1S-2S transition is used. By combining it with a second transition measurement, it is possible to determine the proton charge radius. When taken individually, the various r_p values extracted from H spectroscopy by combining two frequency measurements (2S-4S, 2S-12D, 2S-6S, 2S-6D, 2S-8S, 1S-3S as “second” transition) are statistically compatible with the value from μp . Only the value extracted by pairing the 1S-2S and the 2S-8D transitions is showing a 3σ deviation while all the others differ only by $\leq 1.5\sigma$.

So the 4.4σ discrepancy between the proton charge radius from μp and H spectroscopy emerges only after an averaging process (mean square adjustments of all measured transitions) of the various “individual” determinations and consequently is less startling than it looks at first glance. A small systematic effect common to the H measurements could be sufficient to explain the deviation between μp and H results. This fact becomes even more evident if we consider the frequency shifts (absolute and normalized to the linewidth) necessary to match the r_p values from μp and H as summarized for selected transitions in Table 1. Obviously the discrepancy can not be solved by slightly tuning

Table 1. Hypothetical shift of the transition frequency value needed to match the r_p from H and μp , expressed also relative to the stated experimental accuracy σ , and to the transition effective linewidths Γ_{eff} .

Transition	Shift rel. to uncertainty	Absolute shift	Shift rel. to effective linewidth
$\mu p(2S-2P)$	100σ	75 GHz	$4 \Gamma_{\text{eff}}$
H(1S-2S)	$4'000 \sigma$	40 kHz	$40 \Gamma_{\text{eff}}$
H(2S-4P)	1.5σ	9 kHz	$7 \times 10^{-4} \Gamma_{\text{eff}}$
H(2S-2P)	1.5σ	5 kHz	$7 \times 10^{-4} \Gamma_{\text{eff}}$
H(2S-8D)	3σ	20 kHz	$2 \times 10^{-2} \Gamma_{\text{eff}}$
H(2S-12D)	1σ	8 kHz	$5 \times 10^{-3} \Gamma_{\text{eff}}$
H(1S-3S)	1σ	13 kHz	$5 \times 10^{-3} \Gamma_{\text{eff}}$

(shifting) the measured values of the 1S-2S transition in H and the 2S-2P transitions in μp because it would require displacements corresponding to 4000σ and 100σ , respectively. Expressing the required frequency shift relative to the linewidth as in the last column allows to better recognize some aspects of the experimental challenges. For example a shift of only $7 \times 10^{-4} \Gamma$ of the $2S - 4P$ transition would be sufficient to explain the discrepancy. A control of the systematics which could distort and shift the line shape on this level of accuracy is far from being a trivial task. Well investigated are the large line broadenings owing to inhomogeneous light shifts which results in profiles with effective experimental widths much larger than the natural linewidths [3].

Another exemplary correction relevant in this context, named quantum interference, has been brought recently back to attention [32] and has been applied also to muonic atoms [33]. An atomic transition can be shifted by the presence of a neighboring line, and this energy shift δE , as a rule of thumb, maximally amounts to [32] $\frac{\delta E}{\Gamma} \approx \frac{\Gamma}{D}$ where D is the energy difference between the two resonances and Γ the transition linewidth. Thus, if a transition frequency is aimed with an absolute accuracy of Γ/x , then the influence of the neighboring lines with $D \leq x\Gamma$ has to be considered.

The precise evaluation of these quantum interference effects are challenging because they require solving numerous differential equations describing the amplitude of the total excitation and detection processes from initial to final state distributions and because it depends on experimental parameters such as the angular position and polarization sensitivity of the detectors, the laser intensity, direction and polarization, the initial population distribution among states, etc.

Generally speaking, transition frequencies involving states with large n are more sensitive to systematic effects caused by external fields. Emblematic is for example the n^7 -dependence of the DC Stark effect. Motivated by the possibility that minor effects in H could be responsible for the observed discrepancy, various activities have been initiated in this field:

- *2S – 4P_{1/2} and 2S – 4P_{3/2} at MPQ Garching [34]*: They are aiming at improving previous measurements by a factor of 5 down to an accuracy of few kHz which would yield a r_p with less than 2% accuracy when paired with the 1S-2S transition. Preparation of the 2S state by means of optical excitation and an almost 4π Lyman-alpha detection system are key elements to control the line pulling due to quantum interference on the $1 \times 10^{-4}\Gamma$ level of accuracy required.
- *1S – 3S transition at LKB and MPQ [35, 36]*: The 2010 results from the LKB group delivered the second most precise transition frequency measurement in H with a total uncertainty of 13 kHz corresponding to a relative accuracy of 4.5×10^{-12} . The error budget was dominated by statistics (12 kHz) and uncertainties in the velocity distribution of the atomic beam (3 kHz). A 1% accuracy of the proton radius will require a measurement of the 1S-3S transition with accuracy of about 2 kHz. To reach this goal, the Paris group is presently pursuing the measurement of the 1S-3S transition at 205 nm wavelength using cw spectroscopy along the same line of investigations as in previous experiment. Special emphasis is devoted to the velocity dependent systematic effects. Oppositely, the MPQ group, to circumvent the difficulties related with the generation of the 205 nm light, has devised an experiment which uses pico-second frequency comb pulses.
- *2S – 2P classical Lamb shift in Toronto [37]*: The measurement of the 2S-2P energy splitting alone can lead to r_p . Indeed, as this transition does not depend on the Rydberg structure there is no need to combine it with a second transition frequency measurement¹. Microwave spectroscopy based on the Ramsey method of separated oscillatory field is used for this purpose. A factor of 5 improvement is anticipated, which implies a determination of r_p to the 0.6% level. To reach this ambitious goal the position of the line has to be determined with 1 part in 10^4 .

The “second” (beside the 1S-2S transition) transition frequency measurement in H can be interpreted as a measurement of the Rydberg constant. An alternative way to an independent determination of the Rydberg constant, is to perform optical spectroscopy of H-like ions between circular Rydberg states where the nuclear size corrections are basically absent, the QED contributions small, and the linewidths narrow [38], or via spectroscopy of positronium and muonium [39].

4 Scattering experiments

The Mainz A1 collaboration at MAMI has measured in 2010 1422 precise relative e-p cross sections in the low- Q^2 regime (0.0038 GeV² to 0.98 GeV²) and a wide range of beam energy and scattering angles [5]. Two spectrometers were moved with overlapping angle settings while a third spectrometer was kept fixed and used as a luminosity monitor.

¹Note that all atomic transition frequencies expressed in Hz depend on R_∞ , thus indirectly via the SI units system also the 2S-2P splitting depends on R_∞ . Yet the to date accuracy of the 2S-2P measurements are several order of magnitude worse than the R_∞ accuracy.

As data are available only down to a minimal Q^2 , extrapolation to $Q^2 = 0$ is required to determine the proton charge radius. Numerous works have been concerned with the issues related with this extrapolation procedure. The Mainz group found a satisfactory goodness of fit ($\chi^2 = 1.14$ for 1422 points) through the use of flexible fitting functions (splines and polynomials) and the resulting radius reads [5] $r_p = 0.879(8)$ fm in agreement with the CODATA06 value of 0.8768(69) fm based mostly on atomic measurements. It is important that the fit function be flexible enough to adequately reproduce the data, without being so flexible that over-fitting occurs. One solution to this problem is given in [4, 41] where the low- Q^2 behavior of the form factor is constrained by using large- r assumption of the charge distribution. Reanalysis of the world data using these constraints yields $r_p = 0.879(11)$ fm [42]. Another approach to address issues of over- or under-fitting data is the use of bounded polynomial z expansion (after conformal mapping) and constraining the expansion coefficients to decrease “perturbatively” with increasing order [40].

Conformal fits to the form factors were performed in [43] yielding $r_p = 0.870(23)(12)$ fm in agreement with the CODATA value. Another group fitting the 1422 Mainz data points found $r_p = 0.840(15)$ fm [44] in agreement with the PSI result but with a $\chi^2 = 1.4$ (using more flexible functions the same group found $\chi^2 = 1.1$). Noteworthy, the value of $r_p \approx 0.84$ fm from analysis of scattering data using dispersion relations and vector-mesons dominance models was obtained by the same group prior to the publication of the muonic result. The use of a form factor model (for all Q^2 -range) based on dispersion relation and vector-meson dominance introduce rigidity in the model which results in the larger χ^2 . So tension exists between the use of a physically motivated model giving a poorer fit or very flexible fit functions without physical constraints yielding better χ^2 . This tension could arise by inappropriateness of the theoretical model, insufficient treatment of experimental details, or due to an underestimation of the scattering cross section uncertainties.

A wide-ranging study of possible systematic effects of the 2010 Mainz data has been recently reported in [40]. Special attention was devoted to the extrapolation procedure, to normalization factors needed to smoothly combine the various spectrometer settings and to refinement of the radiative corrections. This reanalysis yields $r_p = 0.895(20)$ fm. When applied to the world data (excluding Mainz 2010) $r_p = 0.918(24)$ fm is found [40]. Even though some inconsistencies between data sets were found, which have led to the increased uncertainty of the r_p extracted from scattering, it remains difficult to reconcile the scattering results with the muonic results. The only sure conclusion is that analysis of low- Q^2 scattering data is not simple and remains a matter of discussion. Because data at still lower Q^2 would be beneficial, two electron-proton experiments have been initiated:

- *P_{Rad} experiment at Hall B in JLAB [45]*: This experiment planned to operate at Q^2 down to 2×10^{-4} GeV² aims to obtain r_p with sub-percent accuracy. The experiment is based on a windowless target and a downstream calorimeter which allows to extend the cross sections measurements to smaller scattering angles. The need to measure relative cross sections at about 0.2% level requires knowledge of the angle to 10 μ rad accuracy which makes this experiment very challenging.
- *Initial state radiation at MAMI, A1 collaboration [46]*: Making use of the initial state radiation techniques, where the initial electron momentum is degraded by photon emission, the momentum transfer to the proton is reduced. The scattered electron is measured with the usual spectrometers but no information on the photon is observed. However, by comparing measurements with Monte Carlo simulations accurate form factors can be determined down to Q^2 of 2×10^{-4} GeV².

Other scattering experiments can provide very important information:

- *MUSE: muon scattering experiment at PSI [47]*: The MUSE experiment plans to measure $\mu^+ - p$ and $\mu^- - p$ as well as $e^+ - p$ and $e^- - p$ scattering down to 0.002 GeV². By comparing negative with positive charges they will have an handle on the insidious TPE contribution. Comparison between

electron and muon cross sections allows the elimination of common systematical effects, including some extrapolation uncertainties. Thus, this experiment has not only the potential to measure the proton charge radius absolute value to 2% accuracy, but also a possible difference between radii extracted from electron and muon scattering down to a relative accuracy of 1%. In this way possible muon-specific interactions can be disclosed.

- *Deuteron scattering at Mainz [48]*: New e-d scattering data have been collected in Mainz, with the aims to extract a new value of the deuteron charge radius and break-up information which may be used to compute the deuteron polarizability contribution in muonic deuterium.

Of relevance in this context is also the planned measurement at JLAB [49] aiming at the electric form factor of the mirror nuclei ${}^3\text{He}$ and ${}^3\text{H}$ to extract their charge radii difference, and the TREK program at J-PARC that scrutinizes K -decays to search for BSM physics motivated by the r_p puzzle.

5 Muonic deuterium and muonic helium ions

In 2009 the CREMA collaboration measured two 2S-2P transitions in muonic deuterium (μd). The 2S-2P energy splitting in μd was determined with about 1 GHz accuracy, which corresponds to a relative accuracy of 20 ppm and 5% of the linewidth.

Evaluation of the most challenging systematic effect, the quantum interference effect has been recently completed [33]. Due to the proximity of two 2P states (4 Γ apart), the quantum interference effects might be considerable. However, a quantitative evaluation of this effect, when accounting for the used excitation and detection schemes, the detector geometry, the laser direction and polarization etc., yields a line shift $\delta E \leq 0.001\Gamma$, thus far below the statistical accuracy of our experiment.

Moreover, the theory in muonic deuterium has only recently converged [52] to a state which allows a precise determination of the deuteron charge radius from the Lamb shift measured in μd . An impressive progress as been achieved in recent years both on the “purely” QED sector [12, 50, 51], as well as in the computation of the TPE contribution [53–55], yielding [52]:

$$\Delta E_{\mu d}(2S - 2P_{1/2}) = 228.7766(10) [\text{meV}] - 6.1102(3) \left[\frac{\text{meV}}{\text{fm}^2} \right] r_d^2 + 1.7091(200) [\text{meV}] \quad (10)$$

where the first term represent basically the “pure” bound-state QED contributions, the second term with r_d in fm the leading finite size contribution (including mixed radiative-finite-size corrections) and the third the TPE contribution.

Combining our measurements with the prediction of Eq. (10) we will obtain r_d with a relative accuracy of $u_r = 4 \times 10^{-4}$ limited by the TPE contribution. A second route to a precise r_d value is to combine the r_p extracted from μp with the H-D isotopic measurements of the 1S-2S transition [30]. A comparison of these two numbers will check the consistency of the muonic results and will give new constraints to BSM theories, e.g. if and how the “new force carrier” can couple to the neutron.

In 2013 and 2014 we have measured for the first time 2 transitions frequencies in $\mu^4\text{He}^+$ and 3 in $\mu^3\text{He}^+$ with relative accuracy of about 40 ppm. The uncertainty of the transition frequency measurement is entirely given by statistical uncertainty, since systematics effects or uncertainty related with the laser frequency calibration are < 2 ppm. To extract the nuclear charge radii from these measurements the corresponding theoretical predictions have to be known. Preliminary values read

$$\Delta E_{\mu^4\text{He}}(2S - 2P_{1/2}) = 1668.669(20) [\text{meV}] - 106.340 \left[\frac{\text{meV}}{\text{fm}^2} \right] r_{\mu^4\text{He}}^2 + 9.52(30) [\text{meV}] \quad (11)$$

$$\Delta E_{\mu^3\text{He}}(2S - 2P_{1/2}) = 1644.658(20) [\text{meV}] - 103.508 \left[\frac{\text{meV}}{\text{fm}^2} \right] r_{\mu^3\text{He}}^2 + 14.66(40) [\text{meV}]. \quad (12)$$

Final numbers can not be given at this stage because it requires sorting out of the several contributions calculated by various groups using different frameworks [12, 50, 51, 56]. Similar to μd , in recent years there has been an impressive progression of the TPE predictions using state-of-the-art nuclear potentials, rendering the inelastic nuclear contribution with 5% accuracy [57]. Such an accuracy opens the way to alpha-particle and helion radii determination with a relative accuracy better than 1×10^{-3} which will be compared with the very precise value available from scattering [58]. Still, an important contribution of the TPE in $\mu^4\text{He}^+$ and in $\mu^3\text{He}^+$, related with the intrinsic nucleon polarizabilities, has not yet been addressed by the community. “Simple” scaling as used in μd [55] probably does not apply in this situation because of the smaller separation between nuclear and nucleon energies.

Besides providing insights into the r_p puzzle these nuclear radii represent benchmarks to check few-nucleon ab-initio calculations, or vice versa to fix low-energy coefficients (e.g. c_D or c_E of the three-nuclei-interaction) describing the nuclear potential in effective field theories [59]. Moreover they can be used as anchor point for the ^6He - ^4He and ^8He - ^4He isotopic shift measurements [60]. The radii extracted from μHe^+ measurements will be used to disentangle the 4σ discrepancy between two ^3He - ^4He isotopic shifts measurements [61, 62], and their knowledge opens the way to enhanced bound-state QED tests for one- and two-electrons systems in “regular” He^+ [63] and He [64].

6 Conclusions

Various attempts have been made to find a solution of the proton charge radius puzzle which has been exposed by the Lamb shift measurement in muonic hydrogen. A plethora of theoretical works has been devoted in refining and rechecking the underlying theory necessary to extract the charge radius from the muonic measurements, in proposing theories beyond the standard model, in reanalyzing scattering data, and investigating proton structure. After all, the puzzle still persist. As a next step, new experimental inputs are required to provide guidance. Understanding of nuclear effects will be of primary importance for the interpretation of the next muonic measurements.

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