Real Transfers and the Friedman Rule*

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Abstract

We find that the Friedman rule is not optimal with real government transfers and distortionary taxation. As transfers cannot be taxed, a positive nominal net interest rate is the indirect way to tax the additional income derived from transfers. This result holds for heterogeneous agents, standard homogeneous preferences, and constant returns to scale production functions. The presence of real transfers changes the standard optimal taxation result of uniform taxation. Higher transfers imply higher optimal inflation rates. We calibrate a model with transfers to the US economy and obtain optimal values for inflation substantially above the Friedman rule.

Keywords: Friedman rule, fiscal policy, monetary policy, taxes, transfers, inflation.

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1 Introduction

Friedman (1969) shows that a policy rule of zero nominal interest rate maximizes welfare in an economy without distortions and where lump sum taxes are available. This policy rule, which is known as the Friedman rule, corresponds to a negative rate of inflation. Phelps (1973) challenged the relevance of the Friedman rule, as lump sum taxes are usually not available. Without lump sum taxes, the Friedman rule might not be optimal, as seigniorage revenue would have to be replaced by revenue from other taxes, which could be more distortionary than inflation.

However, the fact that governments must choose among distortionary taxes does not necessarily invalidate the Friedman rule. Contrary to Phelps's conjecture, the inflation tax usually adds more distortions than any other tax. Atkinson and Stiglitz (1972) established that it is optimal not to distort the relative prices of different consumption goods when preferences are separable in leisure and homothetic in consumption goods. Taxes should be uniform. This result was applied to economies with cash and credit goods to study the optimal inflation tax. In this context, the inflation tax becomes an effective tax on cash goods. Following the result on uniform taxation, the Friedman rule is optimal, as it does not distort the relative price between cash and credit goods.[1] Moreover, consumption and labor income taxes are perfect substitutes. Optimal taxation is obtained by with zero seigniorage and by taxing either labor income or consumption expenditures.

Nevertheless, the Friedman rule might not be optimal when the tax system is incomplete. That is, when the government is unable to tax all sources of income (Chari and Kehoe 1999). In this case, positive inflation may be optimal to tax the source of income that cannot be freely taxed. As all types of private income are devoted to consumption at some point, and because inflation acts as a tax on consumption, a positive nominal interest rate constitutes an indirect way to tax income. Missing tax instruments have been considered in the literature. Schmitt-Grohé and Uribe (2004), for example, consider the case in which profits cannot be taxed. Nicolini (1998), Cavalcanti and Villamil (2003), and Arbex (2013) consider the presence of an informal sector where agents can evade taxes.

[1] Among many others, Lucas and Stokey (1983), Kimbrough (1986), Chari et al. (1996), Correia and Teles (1999), Cunha (2008), and Schmitt-Grohé and Uribe (2010) have shown that the Friedman rule holds if the social planner has access to a sufficiently large number of tax instruments.

Typically, missing tax instruments do not imply large deviations from the Friedman rule. Schmitt-Grohé and Uribe (2010) calibrate three different models to the US economy, each with a source of income that cannot be taxed. They consider a model with untaxed profits from decreasing returns to scale, another with monopolistic competition, and an underground economy in which firms can evade income taxes. They find that neither of these models can explain why central banks follow an inflation target of two percent per year. They obtain optimal rates of inflation insignificantly above zero.

Here, we study optimal monetary policy in the presence of another type of untaxed income: real government transfers. Transfers are payments to economic agents for which no current goods or services are received in return. Transfers can be payments such as social security, pensions, scholarships, financial aid, medicare, and subsidies.[2] They are substantial in all developed countries. In the US, government transfer payments increased from 4.6 percent of GDP in 1947 to 15 percent in 2016. Considering only federal transfers of social benefits to persons (the main component of transfers), the increase in the same period was from 3.2 percent of GDP to 10.8 percent.[3]

We show that the optimal tax policy changes significantly in the presence of transfers. Uniform taxation is not optimal anymore. And the efficient inflation tax is positive. In our calibration of the model to the US, we obtain optimal inflation rates significantly above zero. When transfers as a percentage of GDP are 10 percent, the optimal inflation rate is about 6 percent. Contrary to the results with the untaxed sources of income considered in Schmitt-Grohé and Uribe (2010), an inflation target higher than two percent per year might be warranted in the presence of transfers.

Additionally, we find that the equivalence between the tax instruments depends on the way in which transfers are introduced. The tax on consumption and the tax on labor income are perfect substitutes if transfers, adjusted for the price gross of all taxes (including the nominal interest rate), are constant. Instead, if the path of transfers is constant, adjusted only for the price, then the optimal labor income tax is zero.

[2] Governments usually provide transfers as in-kind transactions or as specific services, such as scholarships or medical procedures, or periodically adjust nominal transfers to maintain their real value. The definition of transfer in the Bureau of Economic Analysis is "a transaction in which one party provides a good, service, or asset to another party without receiving anything directly in return."

[3] One of the objectives of these transfers is to redistribute income from the rich to the poor. As such, they are usually not taxed. We discuss this point in more detail in sections 2 and 4.

The paper is organized as follows. In section 2 we show that uniform taxation is not optimal in an economy with a cash good and a credit good. In section 3, we calibrate a simplified version of the model to the US and obtain an estimate of the optimal inflation rate. In section 4, we consider a version of the model with cash goods only and show that the optimal tax policy changes with the particular way in which transfers are introduced. Section 5 states the main conclusions.

2 The Model

Consider an economy with heterogeneous households and two types of consumption goods: a cash good and a credit good. Each household makes decisions on consumption and labor, and cash is required to purchase the cash good. Households have measure 1, are uniformly distributed over $[0, 1]$, and are indexed by $i \in [0, 1]$. Households have different preferences, levels of efficiency and initial wealth. The efficiency level is denoted by e_i and initial wealth by W_{i0}. The efficiency level affects the result of labor in the following way: l_{it} hours of work imply $e_i l_{it}$ units of efficiency.[4] Labor income depends on the efficiency units. The productivity, or real wage, for each unit of $e_i l_{it}$ is normalized to one. Time is discrete, $t = 0, 1, 2, \ldots$ There is a constant returns to scale technology that transforms units of efficiency into output. Output can be used for private consumption of cash goods c_{1it}, credit goods c_{2it}, and public consumption g_t. The resource constraint is

$$\int_0^1 (c_{1it} + c_{2it})\, di + g_t = \int_0^1 e_i l_{it} di. \quad (1)$$

Households are infinitely lived and have an endowment of time, which is normalized to one. Household i has preferences over streams of consumption goods and leisure given by

$$\sum_{t=0}^{\infty} \beta^t U_i(c_{1it}, c_{2it}, 1 - l_{it}), \quad (2)$$

[4]Different levels of efficiency and of initial wealth are two ways of introducing inequality in the model. See Castañeda et al. (2003), Correia (2010), and Diaz-Gimenez et al. (2011) for a discussion on inequality and the distribution of wealth, earnings, and income.

where $\beta \in (0,1)$, for $i \in [0,1]$. The current-period utility function U_i is strictly concave, satisfies the Inada conditions, is additively separable in leisure and homogeneous in consumption.[5]

Households trade money, bonds, and goods in markets that obey the Lucas and Stokey (1983) timing. At the beginning of period t household i has wealth W_{it}. Household i uses this wealth to obtain a portfolio of money, M_{it}, and bonds, B_{it}, by trading in a centralized market,

$$M_{it} + B_{it} \leq W_{it}, \tag{3}$$

for $t \geq 0$. After this trade, the household splits into a shopper and a worker. The shopper uses money to buy the cash good. To purchase the credit good, the shopper issues nominal claims, which are settled in the assets markets at the beginning of period $t+1$. The worker is paid in cash at the end of period t and receives a real transfer z_{it} from the government. Therefore, household i brings to period $t+1$ wealth

$$\begin{aligned} W_{it+1} \leq p_t(1-\tau_t)e_i l_{it} - p_t\left(1+\tau_{1t}\right)c_{1it} - p_t\left(1+\tau_{2t}\right)c_{2it} \\ + R_t B_{it} + M_{it} + p_t\left(1+\tau_{2t}\right)z_{it}, \end{aligned} \tag{4}$$

for $t \geq 0$, where p_t denotes the price level, R_t denotes the gross nominal interest rate, τ_{1t} and τ_{2t} denote the consumption tax rate on the cash good and on the credit good, respectively, and τ_t denotes the tax rate on the labor income.[6]

It simplifies the analysis to rewrite the household budget constraints. Instead of two constraints per period, we consider one budget constraint per period for each household by substituting the variable W_{it} for $t \geq 1$. In this manner, the budget constraint of household i for the asset market at the beginning of period 0 is given by

$$M_{i0} + B_{i0} \leq W_{i0}, \tag{5}$$

[5]A function $U(c_1, c_2, 1-l)$ is additively separable in leisure if it can be written as $U(c_1, c_2, 1-l) = F(c_1, c_2) + G(1-l)$. A function $F(x)$, $x \in \mathbb{R}^n$, is homogeneous of degree r, where $r \in \mathbb{Z}$, if $F(\lambda x) = \lambda^r F(x)$ for all $\lambda > 0$.

[6]We consider here that the transfer z_{it} implies the additional income of $p_t(1+\tau_{2t})z_{it}$ in (4). The interpretation is that z_{it} has a market value of $p_t(1+\tau_{2t})z_{it}$. However, instead of being indexed to the price gross of taxes, another possibility is that transfers are indexed only to the price level, that is, $p_t z_{it}$. We discuss the effects of valuing transfers at $p_t(1+\tau_{2t})z_{it}$ or $p_t z_{it}$ in section 4.

where the initial wealth W_{i0} is exogenous. By combining (3) and (4), the budget constraints of household i for $t \geq 1$ are

$$M_{it} + B_{it} \leq p_{t-1}(1-\tau_{t-1})e_i l_{it-1} - p_{t-1}\left(1+\tau_{1t-1}\right)c_{1it-1} \\ - p_{t-1}\left(1+\tau_{2t-1}\right)c_{2it-1} + R_{t-1}B_{it-1} + M_{it-1} + p_{t-1}\left(1+\tau_{2t-1}\right)z_{it-1}. \tag{6}$$

There is also a no-Ponzi condition

$$\lim_{t\to\infty} Q_t\left(M_{it}+B_{it}\right) \geq 0, \tag{7}$$

where $Q_t \equiv \prod_{k=0}^{t-1} R_k^{-1}$, $Q_0 \equiv 1$, is the price at 0 of a bond that pays 1 dollar at t.

The cash-in-advance constraint is given by

$$p_t\left(1+\tau_{1t}\right)c_{1it} \leq M_{it}, \tag{8}$$

for $t \geq 0$. Without loss of generality, we assume that the cash in advance restriction holds with equality.[7]

The period t government budget constraint is

$$R_t \int_0^1 B_{it}di + p_t\left(1+\tau_{2t}\right)\int_0^1 z_{it}di + p_t g_t = p_t\tau_{1t}\int_0^1 c_{1it}di + \\ p_t\tau_{2t}\int_0^1 c_{2it}di + p_t\tau_t\int_0^1 e_i l_{it}di + \int_0^1 B_{it+1}di + \int_0^1 M_{it+1}di - \int_0^1 M_{it}di, \tag{9}$$

for $t \geq 0$. The terms on the left hand side of (9) are outflows and the terms on the right hand side of (9) are inflows. The government date-t policy on taxes is given by $u_t \equiv (\tau_t, \tau_{1t}, \tau_{2t})$. It is assumed that the government must charge the same tax rate to each household.

Let $x_{it} \equiv (c_{1it}, c_{2it}, l_{it}, M_{it}, B_{it})$denote the date-$t$ bundle and portfolio for agent $i \in [0,1]$, and let $v_t \equiv (p_t, R_t)$ denote the date-t vector of prices for the economy. The problem of household i is to choose a sequence $\{x_{it}\}_{t=0}^{\infty}$ that maximizes (2) given $\{v_t\}_{t=0}^{\infty}$, $\{u_t\}_{t=0}^{\infty}$, $\{z_{it}\}_{t=0}^{\infty}$, W_{i0}, and the constraints (5), (6) and (8). A competitive equilibrium is an array $\{(x_{it})_{i\in[0,1]}\}_{t=0}^{\infty}$, a price system $\{v_t\}_{t=0}^{\infty}$ and a policy $\{u_t\}_{t=0}^{\infty}$ such that: (i) for each $i \in [0,1]$,

[7] A sufficient condition for that to happen is $R > 1$.

the sequence $\{x_{it}\}_{t=0}^{\infty}$ solves the problem of household i given the price system $\{v_t\}_{t=0}^{\infty}$, the government policy $\{u_t\}_{t=0}^{\infty}$, and the transfers $\{z_{it}\}_{t=0}^{\infty}$; and (ii) the resource constraint (1) is satisfied.

A Ramsey (1927) equilibrium is as an allocation, a set of prices and policy variables such that the households' welfare is maximized and the allocation can be decentralized as a competitive equilibrium with those prices and policy variables. In the context of our model, the Ramsey problem consists in choosing the paths of the nominal interest rate, the consumption tax rates on cash and credit goods, and the labor income tax rate, that implement the competitive equilibrium allocation and that yield the highest level of welfare to households. We say that the government follows an optimal policy if the policy solves the Ramsey problem.

We follow the primal approach to obtain the Ramsey allocation and policy variables. According to this approach, the government maximizes welfare by choosing directly the allocations of households, taking into account the resource constraint of the economy, and the fact that households react to the tax rates. As in Lucas and Stokey (1983), we solve the problem in two steps. We first use the first order conditions of the maximization problem of the households to write the tax rates as functions of the allocations. Then, we solve for the optimal allocations after replacing the tax rates by these functions.

The first order conditions of the maximization problem of household i imply

$$\frac{U_{i1}(t)}{U_{i2}(t)} = \frac{R_t(1+\tau_{1t})}{1+\tau_{2t}}, \text{ for } i \in [0,1], \tag{10}$$

$$\frac{U_{i2}\left(t\right)}{U_{i3}(t)} = \frac{1+\tau_{2t}}{\left(1-\tau_t\right)e_i}, \text{ for } i \in [0,1], \tag{11}$$

$$\frac{U_{i1}(t)}{p_t\left(1+\tau_{1t}\right)} = \beta R_t \frac{U_{i1}(t+1)}{p_{t+1}\left(1+\tau_{1t+1}\right)}, \text{ for } i \in [0,1]. \tag{12}$$

The notation $U_{ij}(t)$, $j = 1,2,3$, denotes the first derivative of $U_i(c_{1it}, c_{2it}, 1-l_{it})$, for household i at time t, with respect to the argument j. Equations (10)-(12) impose restrictions on the tax rates, prices and allocations.

Next, we rewrite the intertemporal budget constraint of household i as a function of the allocations. We first multiply condition (5) by Q_0 and condition (6) for time t by Q_t,

$t = 1, \ldots, T$, with $T > 1$. Adding the resulting inequalities implies

$$\sum_{t=0}^{T-1} Q_{t+1} p_t \left(1 + \tau_{1t}\right) c_{1it} + \sum_{t=0}^{T-1} Q_{t+1} p_t \left(1 + \tau_{2t}\right) c_{2it} + \sum_{t=0}^{T-1} Q_{t+1} \left(R_t - 1\right) M_{it}$$
$$- \sum_{t=0}^{T-1} Q_{t+1} p_t (1 - \tau_t) e_i l_{it} - \sum_{t=0}^{T-1} Q_{t+1} p_t \left(1 + \tau_{2t}\right) z_{it} + Q_t \left(M_{it} + B_{it}\right) \leq W_{i0}. \tag{13}$$

Using the no-Ponzi condition (7), this inequality implies

$$\sum_{t=0}^{\infty} Q_{t+1} p_t \left(1 + \tau_{1t}\right) c_{1it} + \sum_{t=0}^{\infty} Q_{t+1} p_t \left(1 + \tau_{2t}\right) c_{2it} + \sum_{t=0}^{\infty} Q_{t+1} \left(R_t - 1\right) M_{it}$$
$$- \sum_{t=0}^{\infty} Q_{t+1} p_t (1 - \tau_t) e_i l_{it} - \sum_{t=0}^{\infty} Q_{t+1} p_t \left(1 + \tau_{2t}\right) z_{it} \leq W_{i0}. \tag{14}$$

At the optimum for household i, the intertemporal budget constraint holds with equality. With the cash-in-advance constraint holding with equality, the intertemporal budget constraint can then be rewritten as

$$\sum_{t=0}^{\infty} q_{t+1} \left(1 + \tau_{1t}\right) R_t c_{1it} + \sum_{t=0}^{\infty} q_{t+1} \left(1 + \tau_{2t}\right) c_{2it}$$
$$- \sum_{t=0}^{\infty} q_{t+1} (1 - \tau_t) e_i l_{it} - \sum_{t=0}^{\infty} q_{t+1} \left(1 + \tau_{2t}\right) z_{it} = \frac{W_{i0}}{p_0}, \tag{15}$$

where $q_t \equiv Q_t \frac{p_{t-1}}{p_0}$. As it is standard in the literature, we assume that the government is able to fully tax the initial wealth of the households W_{i0}, but that this revenue is not enough to pay for the present value of public expenditures.[8] As a result, it is necessary to raise additional government revenues by resorting to distortionary taxation. Without loss of generality, therefore, we set $\frac{W_{i0}}{p_0} = 0$.

Using the first order conditions (10), (11) and (12) in (15), we obtain

$$\sum_{t=0}^{\infty} \beta^t U_{i1}(t) c_{1it} + \sum_{t=0}^{\infty} \beta^t U_{i2}(t) c_{2it} = \sum_{t=0}^{\infty} \beta^t U_{i3}(t) e_i l_{it} + \sum_{t=0}^{\infty} \beta^t U_{i2}(t) z_{it}, \tag{16}$$

for $i \in [0, 1]$. This equation is known as the implementability condition.

For all $i \in [0, 1]$, and for each sequence of allocations $\{\tilde{x}_{it}\}_{t=0}^{\infty} \equiv \{c_{1it}, c_{2it},\, l_{it}\}_{t=0}^{\infty}$ that

[8] This can be done either with a tax over the initial wealth or by making the initial price level approach infinity (when $W_{i0} > 0$).

satisfies (1) and (16), it is always possible to find a policy $\{u_t\}_{t=0}^{\infty}$ that satisfies conditions (10), (11), and a price system $\{v_t\}_{t=0}^{\infty}$ that satisfies (12). This policy $\{u_t\}_{t=0}^{\infty}$ and this price system $\{v_t\}_{t=0}^{\infty}$, together with the set of bundle and portfolio sequences, $\{x_{it}\}_{t=0}^{\infty} \equiv \{\tilde{x}_{it}, M_{it}, B_{it}\}_{t=0}^{\infty}$, for agent $i \in [0,1]$, where M_{it} satisfies (8) with equality and B_{it} satisfies (6), constitute a competitive equilibrium.

The Ramsey problem is defined as the problem of choosing an array $\{(\tilde{x}_{it})_{i\in[0,1]}\}_{t=0}^{\infty}$ that maximizes

$$\int_0^1 \omega_i \sum_{t=0}^{\infty} \beta^t U_i(c_{1it}, c_{2it}, 1 - l_{it}) di, \tag{17}$$

with $\omega_i > 0$, for $i \in [0,1]$, and satisfies the restrictions (16), one for each household $i \in [0,1]$, and the resource constraint (1). Let $\beta^t \alpha_t$ and λ_i be the Lagrange multipliers associated with the restrictions (1) and (16) respectively. The first order conditions of this problem are

$$(\omega_i + \lambda_i) U_{i1}(t) + \lambda_i (U_{i11}(t) c_{1it} + U_{i21}(t) c_{2it} - U_{i21}(t) z_{it}) = \alpha_t, \tag{18}$$

$$(\omega_i + \lambda) U_{i2}(t) + \lambda_i (U_{i12}(t) c_{1it} + U_{22}(t) c_{2it} - U_{i22}(t) z_{it}) = \alpha_t, \tag{19}$$

and

$$-(\omega_i + \lambda_i e_i) U_{i3}(t) + \lambda_i U_{i33}(t) e_i l_{it} = -e_i \alpha_t. \tag{20}$$

The first order conditions do not have cross partial derivatives between consumption goods and leisure, as the utility function is additively separable in leisure.

If the utility function is homogeneous of degree r in the consumption goods, then the partial derivatives with respect to the consumption goods of the utility function are homogeneous of degree $r-1$. Moreover, according to Euler's theorem, the marginal utility function $U_{ij}(t)$, for all $i \in [0,1]$, $j = 1, 2$ and $t \geq 0$, satisfies

$$U_{ij1}(t) c_{1it} + U_{ij2}(t) c_{2it} = (r-1) U_{ij}(t), \text{ for } j = 1, 2. \tag{21}$$

We can then prove the following proposition.

Proposition 1. *If the utility function is additively separable in leisure and homogeneous in consumption, and if transfers are positive, then the Ramsey allocation is such that* $\frac{U_{i1}(t)}{U_{i2}(t)} \neq 1$.

As the optimal effective tax over the cash good, $R_t(1+\tau_{1t})$, is different from the tax on the credit good, $1+\tau_{2t}$, then optimal commodity taxation is not uniform.

Proof. As U is homogeneous in consumption then equation (21) holds. As $U_{i12}(t) = U_{i21}(t)$, we obtain

$$\frac{U_{i11}(t)c_{1it} + U_{i21}(t)c_{2it}}{U_{i1}(t)} = \frac{U_{i12}(t)c_{1it} + U_{i22}(t)c_{2i,t}}{U_{i2}(t)} \equiv \mu, \tag{22}$$

where μ is a constant (equal to $r-1$). Then, the first order conditions for the optimum (18) and (19) together with (22) imply

$$\frac{U_{i1}(t)}{U_{i2}(t)} = \frac{\omega_i + \lambda_i\left(1+\mu-\frac{U_{i22}(t)}{U_{i2}(t)}z_{it}\right)}{\omega_i + \lambda_i\left(1+\mu-\frac{U_{i21}(t)}{U_{i1}(t)}z_{it}\right)}, \tag{23}$$

where $\lambda_i \neq 0$. In general, $\frac{U_{i22}(t)}{U_{i2}(t)} \neq \frac{U_{i21}(t)}{U_{i1}(t)}$. Therefore, if $z_i > 0$ then $\frac{U_{i1}(t)}{U_{i2}(t)} \neq 1$. □

If $z_{it} = 0$, for all $i \in [0,1]$ and $t \geq 0$, then the optimal allocation must be such that $U_{i1}(t)/U_{i2}(t) = 1$. This is the standard result. The optimal commodity taxation must be uniform. The effective consumption tax on the two goods must be the same. The policy that implements this allocation must satisfy condition (10), which implies $R_t(1+\tau_{1t}) = 1+\tau_{2t}$. There are many combinations of taxes and nominal interest rates that satisfy this condition. However, if the tax on the cash good cannot be different from the tax on the credit good, that is $\tau_{1t} = \tau_{2t}$, then the Friedman rule, $R_t = 1$, is the only efficient policy.

If $z_{it} > 0$, for some $i \in [0,1]$ and $t \geq 0$, then $U_{i1}(t)/U_{i2}(t) \neq 1$. From (10), $R_t(1+\tau_{1t}) \neq 1+\tau_{2t}$. The optimal taxes are not uniform anymore as the effective taxes on the consumption goods are different. The reason for this result is the extra term $\sum_{t=0}^{\infty}\beta^t U_{i2}(t)z_{it}$ in the implementability condition (16). This term appears because we assumed that transfers are positive, exogenous, and because there is not a specific tax on transfers.[9]

We assume that transfers are exogenous because it simplifies the analysis. In reality, transfers are the result of government choices. In an extended Ramsey problem that considers optimal transfers, the planner should have the freedom to choose the net transfers to each household to maximize the society welfare (17) subject to implementability conditions similar to (16) and the resource constraint (1). The objective function of this problem would be

[9] Proposition 1 only requires $z_{it} \neq 0$, but we focus on the case $z_{it} > 0$. As we do not allow for lump sum taxes, transfers cannot be negative to any household.

maximized if marginal utilities of consumption, weighted by the ω_i's, were equalized across households. If households were homogeneous, and the ω_i's equal across households, then optimal transfers would be zero. However, if households are heterogeneous, then transfers have the potential to reduce inequality across households and improve social welfare.[10]

In the extended Ramsey problem, the tax over government transfers does not constitute an additional policy instrument. What matters is the net transfers to households, $\{z_{it}\}_{t=0}^{\infty} \equiv \{(1-\tau_{zt})\,\tilde{z}_{it}\}_{t=0}^{\infty}$, where τ_{zt} is the tax on transfers and $\tilde{z}_{it}$ denotes the gross transfers. We assume implicitly, and without loss of generality, that the sequence of net transfers $\{z_{it}\}_{t=0}^{\infty}$, for $i \in [0,1]$, taken as exogenous, solves the extended Ramsey problem when $\tau_{zt} = 0$. This assumption also simplifies notation.

3 Quantitative Results and Discussion

To calculate the quantitative implications of the model and discuss its results, we simplify some aspects of the general economy above. Consider an economy with homogeneous households, where $e_i = 1$, and $W_{i0} = W_0$. All households have standard current-period utility function given by

$$U(c_{1t}, c_{2t}, 1-l_t) = \frac{c_{1t}^{1-\theta}}{1-\theta} + \gamma \frac{c_{2t}^{1-\theta}}{1-\theta} + \eta \frac{(1-l_t)^{1-\theta}}{1-\theta}, \tag{24}$$

where θ, γ and η are positive constants.[11] We follow the same notation. That is, c_{1t} and c_{2t} denote the cash and credit goods at time t, respectively, and l_t denotes hours of work at time t. The parameter θ is the coefficient of relative risk aversion. The parameters γ and η determine the relative weight on credit goods and leisure $1-l_t$, respectively.

The first order conditions of the households' problem imply equations (10)-(12), which yield

$$R_t = \frac{1+\tau_{2t}}{1+\tau_{1t}} \frac{1}{\gamma} \left(\frac{c_{2t}}{c_{1t}}\right)^{\theta}, \tag{25}$$

$$\frac{1+\tau_{2t}}{1-\tau_t} = \frac{\gamma}{\eta} \left(\frac{1-l_t}{c_{2t}}\right)^{\theta}, \tag{26}$$

[10] For example, in Werning (2007), positive transfers are optimal if the weights on poor households are high enough or if there is enough inequality.

[11] Tiago Cavalcanti suggested this functional form for the utility function.

$$1 + \pi_{t+1} = R_t \frac{1 + \tau_{1t}}{1 + \tau_{1t+1}} \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\theta}, \tag{27}$$

where $\pi_{t+1} \equiv p_{t+1}/p_t - 1$ is the inflation rate from period t to period $t+1$.

The resource constraint is now given by

$$c_{1t} + c_{2t} + g = l_t, \tag{28}$$

where we let $g_t = g$ and $z_t = z$ to focus on a stationary equilibrium.

As we did in the previous section, we now solve the Ramsey problem. The solution that solves the Ramsey problem is the allocation that maximizes (24) subject to the resource constraint (28) and the implementability condition below, which is analogous to (16), with $e_i = 1$ and without the subscript i in the variables,

$$\sum_{t=0}^{\infty} \beta^t U_1(t) c_{1t} + \sum_{t=0}^{\infty} \beta^t U_2(t) c_{2t} = \sum_{t=0}^{\infty} \beta^t U_3(t) l_t + \sum_{t=0}^{\infty} \beta^t U_2(t) z. \tag{29}$$

Let $\beta^t \alpha_t$ and λ be the Lagrange multipliers respectively associated with (28) and (29). As the objective function of this problem is concave, the Ramsey allocation must be stationary. That is, in equilibrium, $c_{1t} = c_1$, $c_{2t} = c_2$, $l_t = l$ and $\alpha_t = \alpha$.[12] The first order conditions (18)-(20), (28) and (29) imply the system of equations

$$1 + \lambda (1 - \theta) = \alpha c_1^{\theta}, \tag{30}$$

$$1 + \lambda (1 - \theta) + \lambda \theta \frac{z}{c_2} = \frac{1}{\gamma} \alpha c_2^{\theta}, \tag{31}$$

$$1 + \lambda + \lambda \theta \frac{l}{1 - l} = \frac{1}{\eta} \alpha_t (1 - l)^{\theta}, \tag{32}$$

$$c_1^{1-\theta} + \gamma c_2^{1-\theta} = \eta (1 - l)^{-\theta} l + \gamma c_2^{-\theta} z, \tag{33}$$

$$c_1 + c_2 + g = l. \tag{34}$$

The system (30)-(34) is a set of five equations with five endogenous variables, c_1, c_2, l, α and λ. The solution to this problem is the Ramsey allocation for this economy. We retrieve

[12] This result is also a consequence of the fact that, in this formulation, $z_t = z$ and $g_t = g$ in the household budget constraint.

the optimal tax rates and interest rates using the first order conditions of the households' problem, constraints (25)-(27).

This problem is useful to understand that $z = 0$ implies uniform taxation, and the Friedman rule, and that $z > 0$ implies a departure from the Friedman rule whenever it is not possible to set different consumption taxes on the cash and credit goods. To obtain analytical expressions, let $\gamma = 1$ and $\theta = 1$.[13] From equations (30) and (31), we obtain

$$\frac{c_2}{c_1} = 1 + \lambda \frac{z}{c_2}. \tag{35}$$

Moreover, the tax rates and the gross interest rate must satisfy

$$\frac{R\,(1+\tau_1)}{1+\tau_2} = \frac{c_2}{c_1}, \tag{36}$$

$$\frac{1+\tau_2}{1-\tau} = \frac{1}{\eta}\left(\frac{1-l}{c_2}\right), \tag{37}$$

$$1+\pi = \beta R. \tag{38}$$

Without loss of generality, equations (36)-(38) focus on the case with constant interest rate and taxes.

Let $z = 0$. This is the case in which we have uniform taxation. From equation (35), we obtain $c_2 = c_1$. Moreover,

$$R = \frac{1+\tau_2}{1+\tau_1}, \tag{39}$$

$$\frac{1+\tau_2}{1-\tau} = \frac{1}{\eta}\left(\frac{1-l}{c_2}\right), \tag{40}$$

$$1+\pi = \beta R. \tag{41}$$

The system (39)-(41) is indeterminate as there are five endogenous variables (R, τ_2, τ_1, τ and π), and three equations. However, if the tax rate on the cash good cannot be different from the tax rate on the credit good, $\tau_1 = \tau_2$, then the Friedman rule, $R = 1$, is the unique solution to the system.

[13] We interpret the case $\theta = 1$ as the logarithmic utility. The logarithmic utility function is not homogeneous, but there exists a monotonic transformation to the logarithmic utility that describes the same preferences and implies a homogeneous utility function.

Let now $z > 0$. Equation (35) then implies $c_2/c_1 > 1$, as $\lambda > 0$. We still have an indeterminacy of the optimal taxes and interest rate from (36)-(38). However, from equation (36), optimality now requires non-uniform taxation, as $R(1+\tau_1) > 1+\tau_2$. Uniform taxation is not optimal if transfers are positive.

Transfers are a pure rent and efficiency requires that they should be completely taxed. That is, if it were possible, they should have had a tax rate of 100 percent. Since transfers cannot be taxed directly, optimality requires that they should be taxed indirectly. This is achieved by taxing more the cash good than the credit good.

The Ramsey planner has an incentive to inflate above the level implied by the Friedman rule as a way to levy an indirect tax on transfers. As Schmitt-Grohé and Uribe (2010) put it, if we add a source of income, then the government is likely to depart from the Friedman rule, if the instrument to tax that income is not available or if there is an upper limit on that instrument tax rate.

Suppose, for example, that the government cannot set different taxes for cash and credit goods. In this case, $\tau_1 = \tau_2$.[14] The government, for example, might not be able to distinguish cash and credit goods. One of the reasons for the difficulty to distinguish cash and credit goods is that the same good can be a cash good for some households and a credit good for others. Households are heterogeneous and this implies different consumption choices. Different consumption choices have implications with respect to the instruments used to make transactions. Avery et al. (1987), Kennickell et al. (1997), Mulligan and Sala-i-Martin (2000) and Attanasio et al. (2002), among others, point out that high-income households use a smaller fraction of cash on their transactions than low-income households. The poorest households do not own a checking account.[15] Therefore, the same good can be a cash good for a poor household and a credit good for a rich household.

If the government is constrained to set $\tau_1 = \tau_2$, equations (35) and (36) imply

$$R > 1. \tag{42}$$

[14]It is common to assume the same tax rate for cash and credit goods. This is done, for example, in Cooley and Hansen (1992).

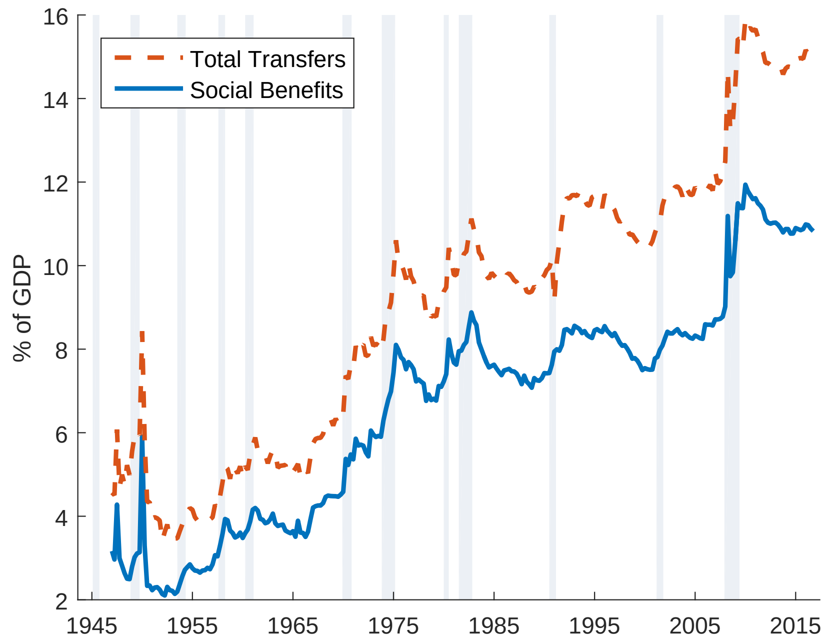
[15]Erosa and Ventura (2002) find that inflation acts as a regressive consumption tax, increasing inequality, as lower-income households tend to use more cash as a percentage of their total expenditures.

The Friedman rule does not hold. As the government cannot set higher taxes for cash goods, as it would be implied by equations (35)-(36), then the government needs to set $R > 1$ to obtain the Ramsey policy. The cash good is taxed more than the credit good, but now this is done through the inflation tax. This policy reaches the cash good because this good is subject to the cash-in-advance constraint. From equation (41), we have $\pi > \beta - 1$. The inflation rate is higher than $\beta - 1$, which is the inflation rate implied by the Friedman rule. Although $R > 1$ when $\tau_1 = \tau_2$, the values of the consumption and labor taxes are still indeterminate. Given the optimal allocation, equation (40) determines the labor tax once a value for the consumption tax is chosen.

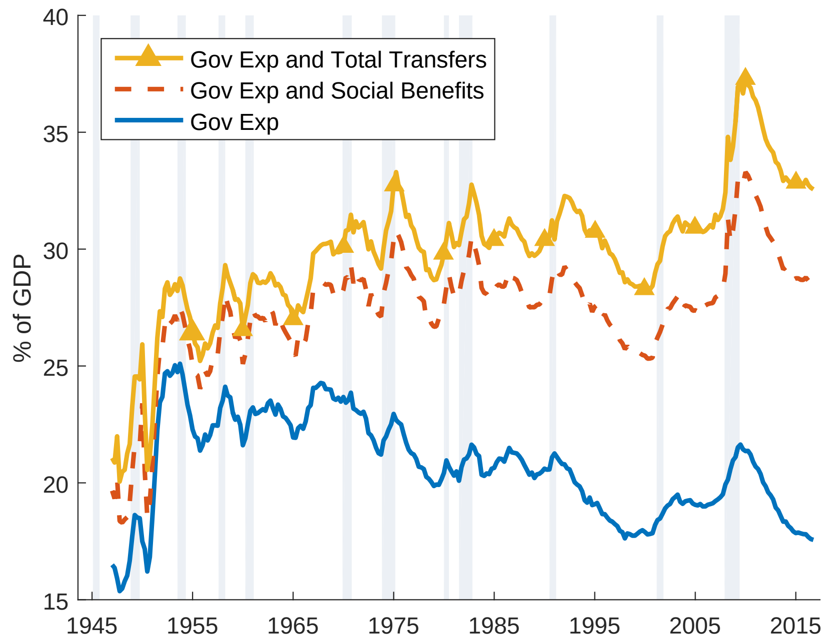
To determine the quantitative implications of our findings, we parameterize the model based on US data and solve the system of equations (30)-(34) together with (36)-(38). Figure 1 shows the evolution of transfers and government expenditures as a percentage of GDP over time. Transfers increased substantially from 1947 to 2016. As stated in the introduction, total government transfer payments during the period increased from 4.6 percent of GDP to 15 percent of GDP. Federal government transfers as social benefits to persons increased from 3.2 percent of GDP to 10.8 percent of GDP. In contrast, government expenditures have shown a more stable behavior. Government expenditures changed from 16.5 percent of GDP in 1947 to 17.6 percent to 2016; the average for the whole period is 20.7 percent of GDP. As we show below, this change in the composition of transfers and government expenditures has important consequences for optimal taxation.[16]

We need to set values for the preferences parameters, government expenditures g, and transfers z. We set $\beta = 0.98$ for the intertemporal discount, which implies a real interest rate of 2 percent per year. The value of the weight on leisure η is determined so that hours of work are equal to 0.3 when the ratio of government expenditures to GDP is equal to 20 percent and the ratio of transfers to GDP is equal to 8 percent. The ratio of g over GDP reflects the mean of this variable since 1947. The value for transfers replicates the mean of the ratio

[16] Federal transfers of social benefits to persons include social security, medicare, veterans' benefits, unemployment insurance, and other transfers. Social security and medicare are about 70 percent of social benefits since the mid 1960s. Veterans' benefits decreased from 70 percent of social benefits in 1947 to 5 percent in 2016. Unemployment insurance from 2000 to 2016 is on average 4 percent of social benefits. Total government transfers include federal social benefits, medicaid, state and local transfers, and transfers to the rest of the world. Medicare and medicaid together comprise about 40 percent of total government transfers. Government expenditures include consumption expenditures and gross investment. For these computations, we use data from the Federal Reserve Bank of St. Louis FRED dataset.



(a) Transfers



(b) Government Expenditures and Transfers

Figure 1: Transfers and government expenditures over time. Social benefits are total federal transfers of social benefits to persons (social security, medicare, veterans' benefits and other transfers). Total transfers include medicaid, state and local transfers, and transfers to the rest of the world. Shaded areas indicate NBER U.S. recessions. Source: Federal Reserve Bank of St. Louis.

of federal transfers in the form of social benefits to persons over GDP since 1970. We set $\gamma = 1$ so that cash and credit goods have an equal weight.[17] We follow the same procedure to obtain the parameters for $\theta = 0.5$, 1, and 2. The value $\theta = 1$ implies logarithmic utility. Once we set the parameters, we calculate the optimal allocations and taxes for different values of z. We change z so that the ratio of transfers to GDP varies from zero to 15 percent. Table 1 shows the parameters used in the simulations.

Table 1: Parameters

	Parameter	Value
Intertemporal discount factor	β	0.98
Coefficient of relative risk aversion (CRRA)	θ	1
Preference parameter on credit goods	γ	1
Preference parameter on leisure	η	4.213

The values of η, g and z are found simultaneously such that hours of work are equal to 0.3 when the transfers-to-GDP ratio z/y is equal to 8 percent and the government-to-GDP ratio g/y is equal to 20 percent. For $\theta = 0.5$, $\eta = 1.743$; and for $\theta = 2$, $\eta = 24.58$. $\theta = 1$ in figures 2 and 3. The value of g is maintained constant during the simulations while the value of z increases from $z = 0$ to a value such that $z/y = 15$ percent.

Table 2 and figures 2 and 3 show the main results. Table 2 shows results for different values of θ. Figures 2 and 3 show the results for $\theta = 1$. For $z = 0$, the Ramsey policy implies the Friedman rule, with $R = 1$ and inflation of -2 percent per year. The consideration of positive transfers implies a substantial departure from the Friedman rule. As transfers increase to 5 percent of GDP, the optimal policy for $\theta = 1$ implies $R = 1.029$ and $\pi = 0.8$ percent per year. With transfers of 10 percent of GDP, the optimal policy implies inflation of 6.15 percent per year. The values are robust to changes in θ.

Table 2: Interest rates, inflation and labor taxes for different values for transfers

	$\theta = 0.5$			$\theta = 1$			$\theta = 2$		
z (% of GDP)	R	π (% p.a.)	τ_l (%)	R	π (% p.a.)	τ_l (%)	R	π (% p.a.)	τ_l (%)
0	1.000	−2.00	11.6	1.000	−2.00	13.4	1.000	−2.00	14.1
5	1.027	0.61	17.6	1.029	0.84	18.3	1.030	0.93	18.6
10	1.084	6.27	23.2	1.083	6.15	22.5	1.083	6.15	22.2
15	1.199	17.5	28.6	1.172	14.9	25.6	1.167	14.38	24.6

Gross interest rate R, inflation π, and labor tax τ_l for different coefficients of relative risk aversion θ and different levels of transfers z. Inflation in percent per annum. To determine τ_l, the consumption tax is set to 6.5% ($\tau_1 = \tau_2$). Parameters in table 1. Results obtained from equations (30)-(34) and (36)-(38).

[17] Cooley and Hansen (1991, 1992) use $\gamma = 1$ and smaller values such as $\gamma = 0.2$. We also used these values in the simulations and obtained results qualitatively similar.

The increase in transfers requires higher inflation and higher labor taxes. As leisure is not taxed, hours of work decrease when transfers increase. To determine the optimal value for the labor tax τ_l, we use equation (40) together with a value for the consumption tax, τ_c. To obtain a value for the consumption tax, we use estimates for the effective tax rates on consumption, as described by Mendoza et al. (1994).

The calculations of the effective tax rates take into account aggregate tax revenues from consumption taxes and aggregate sales. Mendoza et al. (1994) find τ_c between 6.4 and 5.1 percent for the US from 1965 to 1988, with smaller rates for the most recent periods. Silva (2008), using a similar procedure, finds values of τ_c for the US between 5 percent and 7.1 percent for 1970-2001. Carey and Rabesona (2002), with a revised methodology, find values between 6.4 percent and 6.7 percent. The values, therefore, are largely compatible across estimates.

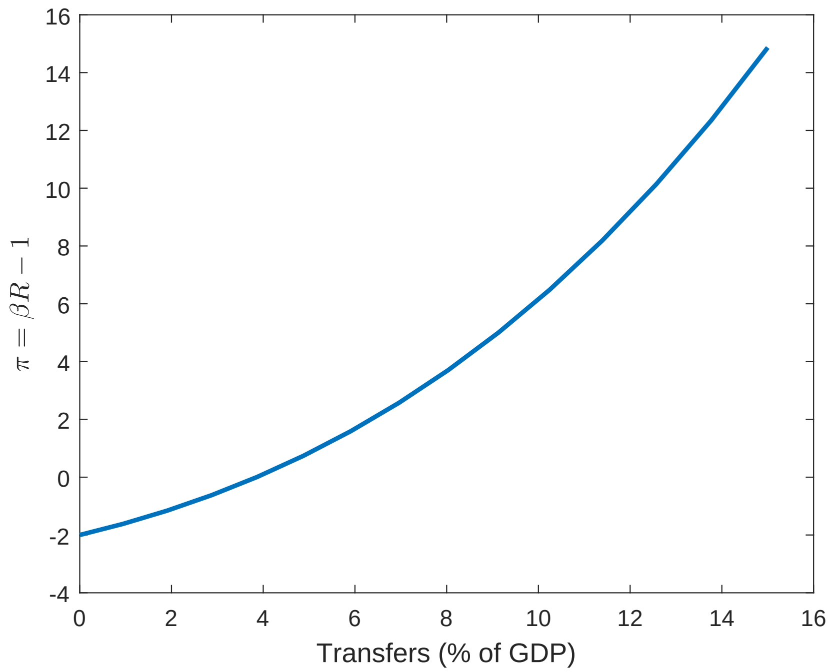
For table 2, we set $\tau_c = 6.5$ percent and find $\tau_l = 22.5$ percent, for $\theta = 1$, when transfers are 10 percent of GDP. The values for the labor tax are similar for different values of θ. In figure 2, we calculate the optimal labor tax for $\tau_c = 5$, 7.5 and 10 percent. For higher consumption taxes, the required labor taxes are smaller. When transfers are 10 percent of GDP, the optimal labor tax varies between 19.9 percent and 23.6 percent.

The main result of this section is that transfers have a significant impact on the estimates of optimal inflation in the standard cash-in-advance model with a credit good. The optimal inflation reacts strongly to changes in the level of transfers. When transfers are zero the optimal inflation rate is -2 percent. When transfers are 10 percent of GDP, which according to the data is a conservative value, the optimal inflation is around 6 percent.

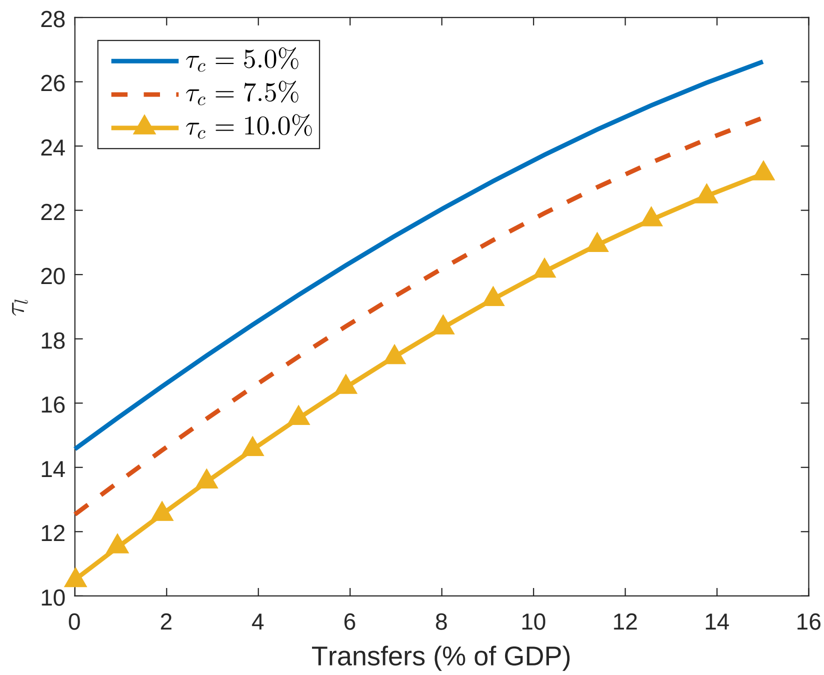
Our results were obtained assuming that the government maintains the real value of transfers every period, taking into account the full tax on the consumption goods. This assumption was made to simplify the analysis. In the next section, we investigate whether the results are robust to changes in this assumption.

4 Alternative Paths for Real Transfers

The optimal tax policy depends on the way real transfers are distributed. Let $\{Z_{it}\}_{t=0}^{\infty}$, for $i \in [0, 1]$, be nominal transfers. There are two interesting possibilities for the path for transfers,

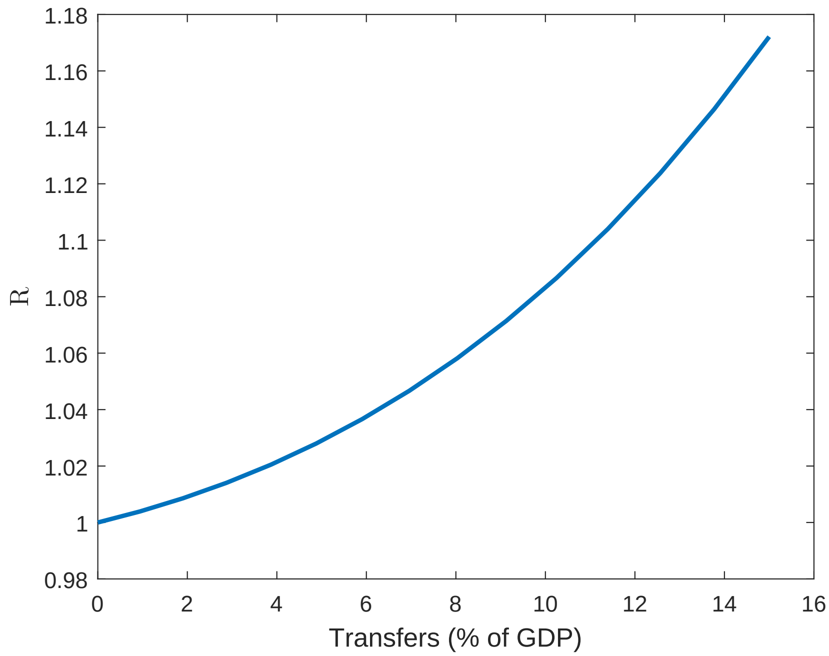


(a) Inflation (% per year)

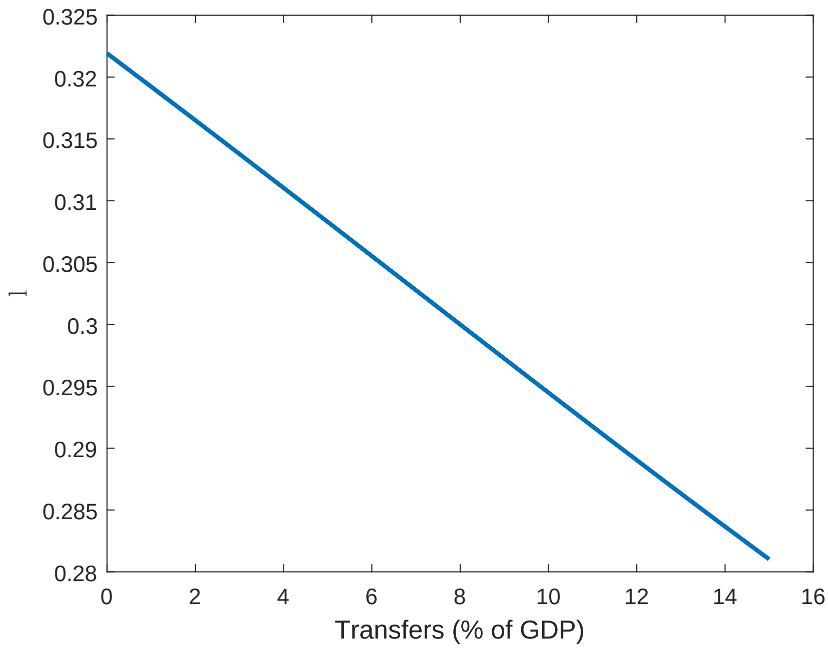


(b) Labor tax (%)

Figure 2: The Friedman rule holds for $R = 1$ or inflation equal to -2 percent per year. It holds when transfers are equal to zero. Results from simulations. See table 1 for parameters. Transfers z increase for a constant given value of government expenditures g.



(a) Gross interest rate



(b) Hours of work

Figure 3: The Friedman rule ($R = 1$) holds when transfers are equal to zero. Total hours of work are normalized to 1. Results from simulations. See table 1 for the parameters.

either real transfers adjusted for taxes are time invariant, that is, $Z_{it} = p_t (1 + \tau_2) z_i$, or real transfers adjusted only for the price level are constant, that is, $Z_{it} = p_t z_i$. We compare in this section these two alternative assumptions. It turns out that the labor income tax and the consumption tax are equivalent instruments under the first assumption, but not under the second assumption. As we show below, an implication is that, when transfers follow the second path, the inflation tax is a more efficient instrument than the labor income tax.

We consider a simpler version of the economy in section 2 with one consumption good instead of two. There is a constant returns to scale technology that transforms units of efficiency into output. Output can be used for private consumption of cash goods and public consumption. The resource constraint is

$$\int_0^1 c_{it} di + g_t = \int_0^1 e_i l_{it} di. \tag{43}$$

The private consumption good must be bought with money according to the standard cash-in-advance constraint

$$p_t \left(1 + \tau_{ct}\right) c_{it} \leq M_{it}, \tag{44}$$

where τ_{ct} is the tax rate on the consumption good. The preferences of household i are given by

$$\sum_{t=0}^{\infty} \beta^t U_i(c_{it}, 1 - l_{it}), \tag{45}$$

with $0 < \beta < 1$. As before, we normalize total time to one; and $1 - l_{it}$ denotes leisure. Utility function U is strictly concave and satisfies the Inada conditions.

The budget constraint of each household for the asset market at the beginning of time t is given by

$$M_{it} + B_{it} \leq R_{t-1} B_{it-1} + M_{it-1} - p_{t-1}(1 + \tau_{ct-1}) c_{it-1} + p_{t-1}(1 - \tau_{t-1}) e_i l_{it-1} + Z_{it}. \tag{46}$$

Households are subject to the no-Ponzi condition (7).

The household i's problem is to choose the sequence $\{(M_{it}, B_{it}, c_{it}, l_{it})\}_{t=0}^{\infty}$ that maximizes (45) subject to (46), (44), the no-Ponzi condition, and the initial condition W_{i0}. The first-order conditions of household i's problem include

$$\frac{U_{i1}(t)}{U_{i2}(t)} = \frac{R_t \left(1 + \tau_{ct}\right)}{\left(1 - \tau_t\right) e_i}, \tag{47}$$

and

$$\frac{U_{i1}(t)}{p_t\,(1+\tau_{ct})} = \beta R_t \frac{U_{i1}(t+1)}{p_{t+1}\,(1+\tau_{ct+1})}. \tag{48}$$

The notation $U_{ij}(t)$, $j = 1, 2$ denotes the first derivative of $U_i(c_{it}, 1-l_{it})$ with respect to the argument j.

The intertemporal budget constraint for the household i is

$$\sum_{t=0}^{\infty} q_{t+1}\,(1+\tau_{ct})\,R_t c_{it} = \frac{W_{i0}}{p_0} + \sum_{t=0}^{\infty} q_{t+1}\left(\frac{Z_{it}}{p_t} + (1-\tau)e_i l_{it}\right). \tag{49}$$

An efficient way of raising government revenue is to tax the initial real wealth, as it does not introduce any distortion in the economy. Therefore, we assume that the initial real nominal wealth of the household is fully taxed.

As the utility function U_i is strictly concave and transfers and public consumption are time invariant, then consumption and leisure are also time invariant. It follows from (47) that time invariant consumption and leisure can be achieved with constant tax rates and a constant nominal interest rate. Thus, from now on, without loss of generality, we assume that τ_{ct}, τ_t, R_t are time invariant.[18]

Let the path of transfers $Z_{it} = p_t\,(1+\tau_c)\,Rz_i$, $t \geq 0$, be path 1, and the path $Z_{it} = p_t z_i$, $t \geq 0$, be path 2. Define a policy vector by the vector (τ_c, τ, R). We say that two policy vectors are equivalent when they decentralize the same equilibrium allocation. The components of the policy vectors are then equivalent policy instruments. The following lemma states that, in a cash in advance economy, the consumption tax, the labor income tax and the nominal interest rate are equivalent instruments when the path for transfers follows path 1.

Lemma 1. *Assume that transfers follow path 1. The policy vector* (τ_c^a, τ^a, R^a) *is equivalent to the policy vector* $\left(\tau_c^b, \tau^b, R^b\right)$ *where* $\frac{(1+\tau_c^a)R^a}{(1-\tau^a)} = \frac{\left(1+\tau_c^b\right)R^b}{(1-\tau^b)}$.

Proof. When transfers follow path 1, the budget constraint for each household i can be written as

$$\sum_{t=0}^{\infty} q_{t+1} \frac{(1+\tau_c)\,R}{(1-\tau)} c_{it} = \sum_{t=0}^{\infty} q_{t+1}\left(\frac{(1+\tau_c)\,R}{(1-\tau)} z_i + e_i l_{it}\right), \tag{50}$$

as initial wealth is fully taxed. This constraint for the vector (τ_c^a, τ^a, R^a) is identical to the

[18] To avoid confusion, we do not suppress the subscript t in c_{it} and l_{it}.

one for the vector $\left(\tau_c^b, \tau^b, R^b\right)$. In the same way, the first order conditions are identical under the two alternative policies. Given the values for government consumption and transfers, the equilibrium prices gross of taxes are the same under the two policy vectors. Moreover, aggregate and individual allocations are also the same under the two policy vectors. Therefore, the two policy vectors are equivalent. □

However, when transfers follow path 2, there is no equivalence between the nominal interest rate and the labor tax rate. Moreover, the optimal labor income tax is zero.[19] To prove these results, it is convenient to write the budget constraint (49) of individual i with $Z_{it} = p_t z_i$ and replace q_{t+1} using (48). We obtain

$$\sum_{t=0}^{\infty} \beta^t (1+\tau_c) R c_{it} = \Psi_i + \sum_{t=0}^{\infty} \beta^t (1-\tau) e_i l_{it}, \tag{51}$$

where $\Psi_i = \sum_{t=0}^{\infty} \beta^t z_i$. The variable Ψ_i is the present value of the transfers to household i. Consider fiscal policies of the type $f = (\tau_c, \tau, R, L)$, where L is a virtual levy on Ψ_i. This levy is virtual because we assume that the government cannot use it. The virtual levy on the present value of transfers is equivalent to a lump-sum tax. The next Lemma implies that the policy $f^a = (\tau_c^a, \tau^a, R^a, 0)$ is equivalent to the virtual policy $f^{av} = (\tau_c^{av}, \tau^{av}, R^{av}, L)$, where $(1+\tau_c^{av}) R^{av} = (1+\tau_c^a) R^a/\phi$, $(1-\tau^{av}) = (1-\tau^a)/\phi$, and $1 - L = 1/\phi$, with $\phi > 1$.

Lemma 2. *When transfers follow path 2, the policy $f^a = (\tau_c^a, \tau^a, R^a, 0)$ is equivalent to the virtual policy $f^{av} = (\tau_c^{av}, \tau^{av}, R^{av}, L)$, where $R^{av} = R^a/\phi$, $(1-\tau^{av}) = (1-\tau^a)/\phi$, and $1 - L = 1/\phi$.*

Proof. When transfers follow path 2 the budget constraint (51) of household i, under policy f^a, is

$$\sum_{t=0}^{\infty} \beta^t \left(1+\tau_c^a\right) R^a c_{it} = \Psi_i + \sum_{t=0}^{\infty} \beta^t (1-\tau^a) e_i l_{it}. \tag{52}$$

Dividing by ϕ, the budget constraint of household i becomes

$$\sum_{t=0}^{\infty} \beta^t \left(1+\tau_c^{av}\right) R^{av} c_{it} = (1-L)\, \Psi_i + \sum_{t=0}^{\infty} \beta^t (1-\tau^{av}) e_i l_{it}. \tag{53}$$

[19] In the appendix, we provide an alternative proof of this result.

The individual first order conditions are identical under the two alternative policies. Given the same government consumption and the same paths for individual consumption and leisure, the resource constraint will be satisfied and, by Walras law, the government budget constraint will also be satisfied. As a result, the equilibrium prices gross of taxes are identical under f^a or f^{av}. Moreover, aggregate and individual allocations are the same under the two policies. □

Next, we establish that the inflation tax is a better instrument to finance government transfers than the labor income tax. To simplify notation, let $\tau_c = 0$.

Proposition 2. *When transfers follow path 2, the inflation tax is a more efficient instrument than the labor income tax.*

Proof. Suppose that there are two policies $f^a = (R^a, \tau^a, 0)$ and $f^b = \left(R^b, \tau^b, 0\right)$ that generate the same fiscal revenue necessary to finance government transfers, with $R^b > R^a$ and $0 \leq \tau^b < \tau^a$. Using Lemma 2, the policy f^a is equivalent to the virtual policy $f^{av} = (1, \tau^{av}, L^{av})$ and f^b is equivalent to $f^{bv} = \left(1, \tau^{bv}, L^{bv}\right)$. Since $R^b > R^a$, then $L^{bv} > L^{av}$. As the lump-sum tax is larger under the virtual policy f^{bv}, then $\tau^{bv} < \tau^{av}$. Therefore, policy f^b is more efficient than policy f^a as the same path of government transfers is financed with a lower distortionary tax. □

If transfers follow path 2 and there are no constraints on either the consumption tax or on the inflation tax, then the labor income tax should be set to zero. The inflation tax and the consumption tax are indeterminate. If the inflation tax has lower administrative costs, then the Friedman rule is not optimal. On the other hand, if there are political reasons that constraint the inflation rate, then the government should rely more on the consumption tax to raise revenue.

5 Conclusions

We find that the apparently innocuous introduction of real government transfers changes standard results on optimal taxation. The Friedman rule is not optimal anymore. The Friedman rule is not optimal because transfers constitute a source of income that cannot be taxed.

For the same reason, the Friedman rule does not hold for economies that have an underground sector or profits that cannot be taxed. However, Schmitt-Grohé and Uribe (2010) conclude that these frictions imply insignificant levels of inflation and do not justify an inflation target above zero. In contrast, we find optimal values for inflation substantially higher than the ones obtained in the literature when these other frictions are considered.

The calibrated model abstracts from the fact that households are asymmetric with respect to wealth and income. A model and a calibration that allow for these asymmetries could imply different estimates. Nevertheless, the exercise shows that the presence of transfers alone has the potential to justify the targets for inflation followed by most central banks.[20]

A Appendix

We provide an alternative proof to proposition 2 of section 4.

Proposition 2. *When transfers follow path 2, the inflation tax is a more efficient instrument than the labor income tax.*

Proof. Consider the simple economy of section 4 with the path of transfers following path 2, $e_i = 1$, $g = 0$, and $\tau_c = 0$. Define $\Gamma_t \equiv \frac{1}{1-\tau_t}$ and $c_t \equiv f(\Gamma_t R_t)$ as the value of consumption that solves equations (43) and (47). Define the instantaneous indirect utility as $V(\Gamma_t R_t) \equiv U(f(\Gamma_t R_t), 1 - f(\Gamma_t R_t))$, using the fact that $c_t = l_t$. Since U is strictly concave, the optimal allocation is stationary, which implies that Γ_t and R_t should be stationary too. It is trivial to see that V is decreasing in ΓR. Therefore, the optimal tax policy solves the problem $\min_{\Gamma,R} \Gamma R$ subject to the government budget constraint

$$\frac{\Gamma R - 1}{\Gamma} f(\Gamma R) = z. \tag{A.1}$$

Suppose that $0 < \tau < 1$, $\Gamma \equiv \frac{1}{1-\tau} > 1$, and that Γ and R satisfy (A.1). We can show that it is always possible to decrease $\Gamma \geq 1$ and increase R so that ΓR decreases and the constraint (A.1) is still satisfied. As a result, the solution of the problem cannot involve $\Gamma > 1$. First, a change in Γ together with a change in R so that $\frac{d\Gamma}{dR} = -\frac{\Gamma}{R}$ maintains the

[20] We focus here on the effect of real transfers on optimal inflation. However, there are other reasons to keep inflation low. For example, if there are costs of changing the composition of the portfolio of assets (Silva 2012, Adão and Silva 2018).

value of ΓR constant. Consider an increase in R, $dR > 0$. If Γ changes by $d\Gamma = -\frac{\Gamma}{R}dR - \varepsilon$, for $\varepsilon > 0$, then, as $d(\Gamma R) = \Gamma dR + Rd\Gamma$, this change in Γ and R implies a change in ΓR equal to $-R\varepsilon < 0$. On the other hand, $d\Gamma$ and dR change government revenues by $(\frac{1}{\Gamma^2} f(\Gamma R) + \frac{\Gamma R - 1}{\Gamma} f'(\Gamma R) R) d\Gamma + (f(\Gamma R) + (\Gamma R - 1) f'(\Gamma R)) dR$. With $d\Gamma = -\frac{\Gamma}{R}dR - \varepsilon$, this change in government revenues is equal to

$$\left(1 - \frac{1}{\Gamma R}\right) f(\Gamma R)\, dR - \left(\frac{1}{\Gamma^2} f(\Gamma R) + \frac{\Gamma R - 1}{\Gamma} f'(\Gamma R)\, R\right) \varepsilon. \tag{A.2}$$

As $\Gamma R > 1$ and $f(\Gamma R) > 0$, the coefficient on dR is strictly positive. Therefore, for any $dR > 0$, there is a sufficiently small $\varepsilon > 0$ such that the expression in (A.2) is positive. This means that ΓR decreases but government revenues increase. We then have that a pair (Γ, R), with $\Gamma > 1$, cannot be the solution to the Ramsey problem as there is a decrease in the labor tax rate and an increase in the nominal interest rate such that the distortion ΓR decreases and government revenues do not decrease. It follows that the solution to the Ramsey problem requires $\Gamma^* = 1$. Moreover, setting $\Gamma^* = 1$ in (A.1) implies that R^* is equal to the smallest value that satisfies $(R - 1) f(R) = z$. As $\Gamma = \frac{1}{1-\tau}$ and $z > 0$, then $\tau = 0$ and $R > 1$. □

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