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Government Financing, Inflation, and the Financial Sector

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Abstract

We calculate the effects of an increase in government spending financed with labor income taxes or inflation. We consider government spending in the form of government consumption or transfers. We use a model in which agents increase the use of financial services to avoid losses from inflation, as empirically the financial sector increases with inflation. The financial sector size is constant in standard cash-in-advance models, which implies optimal positive inflation. We reverse this result when we take into account the increase in the financial sector. In our framework, it is optimal to use taxes to finance the government. This result is robust to alternative specifications and definitions of seigniorage and government spending.

JEL Codes: E52, E62, E63.

Keywords: fiscal policy, monetary policy, government financing, demand for money, financial sector.

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1 Introduction

We calculate the effects of an increase in government spending in a model in which the changes in the behavior of agents toward inflation are reflected in the size of the financial sector. We consider government spending in the form of government consumption or transfers. It is important to consider transfers as this form of government spending increased substantially in the last decades. Government spending can be financed by an increase in inflation or in distortionary taxes in the form of labor income taxes. Agents choose the frequency of trades of bonds for money, as in Baumol (1952) and Tobin (1956). Standard cash-in-advance models have constant frequency of trades and constant size of the financial sector. We let agents decide the frequency of trades. This change implies different predictions on the effects of financing the government with inflation.

When agents are allowed to change the frequency of trades, the welfare cost of financing the government with inflation is substantially higher. It can be shown that inflation can be optimal with transfers and distortionary taxation in a cash-in-advance model (Adão and Silva forthcoming). However, when inflation increases, agents increase their use of financial services to decrease money balances. When the costs implied by the increase in the use of financial services is taken into account, the welfare cost of inflation increases. As a result, it becomes optimal to avoid inflation and finance the government with taxes. It is important to consider changes in the trading frequency to calculate the effects of policies that involve changes in inflation.

Consider the results from our simulations of an increase in government spending in the form of transfers that corresponds to an increase from 20 to 21 percent of the government-output ratio.[1] If the increase in spending is financed with an increase in labor income taxes then having fixed or endogenous trading frequency implies similar predictions for the welfare cost: 0.95 percent in terms of income. If the increase in spending is financed with an increase in inflation, the predicted welfare cost with fixed trading frequency is 0.49 percent in terms of income. With endogenous periods, the predicted welfare cost is 1.45 percent. A difference of 0.96 percentage points.[2]

One of the reasons for the difference in estimates is the larger decrease in the demand for money with endogenous trading frequency when inflation increases. The decrease in the demand for money implies higher costs for the provision of financial services. Moreover, the decrease in the demand for money implies a decrease in seigniorage as compared with a model with fixed periods for the same rate of inflation. With fixed trading frequency, the inflation

[1]In the simulations, this change in the government-output ratio is obtained with a 5 percent increase in spending.

[2]For an idea of the magnitude, one percent of income is equivalent to more than 130 billion dollars every year or more than one thousand dollars distributed to every household in the United States every year (2010 dollars, data from the BEA and from the US Census Bureau).

rate necessary to cover the increase in government spending is equal to 5.5 percent per year. With endogenous periods, the inflation rate is equal to 12.7 percent per year. A model with fixed trading frequency predicts that a smaller inflation rate generates the same seigniorage as a higher inflation rate in a model with endogenous trading frequency.

The values for inflation and seigniorage implied by this example are within realistic estimates. According to our simulations, seigniorage revenues as a percentage of output in this example are 2.19 percent of output with fixed trading frequency and 1.93 percent of output with endogenous trading frequency. Sargent et al. (2009) estimate that seigniorage exceeded 10 percent of output in Brazil in 1993 and in Argentina several times during 1970-1990. Click (1998) calculates seigniorage to be 2.5 percent of output on average in a database of 90 countries during 1971-1990 and finds that seigniorage exceeded 10 percent of output in some cases.[3] Kimbrough (2006) cites evidence that seigniorage may reach 17 percent of GDP in some countries but that usually it does not exceed 10 percent. Kimbrough considers seigniorage from 5 to 15 percent of output. Our simulations imply an increase in seigniorage revenues of 1 percentage point, and total seigniorage revenues around 2 percent of output, which is much below the admitted maximum value of 10 percent.

Having endogenous or fixed trading frequency is not relevant when the increase in spending is financed by an increase in labor income taxes. When financing is made through labor income taxes, inflation is about the same before and after the increase in spending. Changes in the demand for money are then insignificant. As as result, it is not important to take into account the timing of the trades between bonds and money. The changes in labor taxes are approximately the same with endogenous or fixed intervals; labor taxes increase to about 32 percent. Other effects, such as those on output and consumption, are also similar when financing is made through taxes. Endogenous trading frequency is relevant when the change in policy implies a change in inflation and, consequently, a significant change in the demand for money.[4]

Agents in the model pay a fee in goods to have access to financial markets, where they trade bonds for money. The payment of a fee implies staggered visits to the asset market. As inflation increases, it is optimal to increase the frequency of the trades in the financial markets to decrease real money holdings. The increase in the frequency of trades increases the frequency of fee payments.

[3] Click (1998) calculates seigniorage to be 10.5 percent of government spending on average but to exceed 100 percent of government spending for various countries. In this example, seigniorage is 10.3 percent of government consumption with fixed trading frequency and 9.2 percent with endogenous trading frequency. In the database of Click, U.S. has a seigniorage of 0.43 percent of output. The lowest seigniorage is 0.38 percent of output, for New Zealand. Our model reproduces these values with low inflation. The highest seigniorage in the database is 14.8 percent of output, for Israel.

[4] The area under the demand curve does not approximate the welfare cost of inflation here. The welfare cost is higher than the area under the demand for money with either endogenous or fixed trading frequency. The reason is the existence of distortionary taxation, government spending, and endogenous labor supply.

We interpret the aggregate fee payments in the model as the size of the financial sector. We can in this way use our model to estimate the increase in the financial sector implied by the increase in inflation. In our simulations, an increase in inflation from 0 to 10 percent per year implies an increase in financial services of 1 percentage point. Our estimates are in accordance with the evidence in English (1999). English finds a positive relation between inflation and the size of the financial sector and estimates that a 10 percent increase in inflation in U.S. implies an increase in the financial sector of 1.3 percentage points.

Cooley and Hansen (1991, 1992) study the effects of inflation in a cash-in-advance economy with distortionary taxation. Their economy is similar to the economy that we have here, except that we allow agents to choose optimally the timing of their transactions in the asset market. In Cooley and Hansen (1991), decreasing inflation from 10 to zero percent per year, with seigniorage revenues replaced with labor income taxes, implies a welfare loss of 1.018 percent of output. We emphasize that this exercise generates a welfare loss in Cooley and Hansen (1991) instead of a welfare gain.[5] In Cooley and Hansen (1992), financing the government with distortionary taxes implies a welfare cost of 13.30 percent of GDP when compared with lump sum taxation. If financing is done with inflation, it implies a welfare cost of 12.43 percent. The difference of 0.87 percent between financing with inflation and distortionary taxes is the loss associated with decreasing inflation to zero and increasing taxes to keep government revenues constant. We confirm that replacing inflation with labor income taxes implies a welfare loss when agents cannot choose the timing of transactions in the asset market. However, once we lift the restriction on the timing of asset transactions, the loss of decreasing inflation is reversed to a gain.

Silva (2012) introduces a model to study trading frequency and the welfare cost of inflation, assuming no government spending and that lump sum taxation is available. Here, we introduce distortionary taxation and government spending in the form of government consumption or government transfers. We also consider different forms in which the government can finance itself. We focus here on the implications for government financing under distortionary taxation and inflation in each case. We find that taking into account the decision on the frequency of trades substantially changes the estimates of the welfare cost of inflation. This effect is so large in certain cases that it reverses a prescription of increasing inflation to a prescription of keeping inflation low.

We also find that the response of output depends on the trading frequency being considered. With fixed trading frequency, inflation acts as a labor income tax. An increase in inflation encourages agents to substitute away from labor toward leisure, which decreases output (Cooley and Hansen 1989 and others). This effect is also present in the endogenous trading frequency case. However, to decrease the demand for money, the financial sector increases. As the financial sector requires labor and capital, the increase in the financial sector implies a

[5] About this unexpected result, see also Benabou (1991) and Wright (1991).

positive effect on labor supply. The effect on the labor supply is small as the financial sector as a fraction of output increases only one percentage point when the increase in spending is financed through inflation. However, the effect might be enough to change the sign of the output response. Under certain conditions, there is a positive government multiplier. If the increase in spending is in government consumption, the multiplier is -0.01 with fixed timing and 1.09 with endogenous timing.[6]

The remainder of the paper is organized as follows. Section 2 describes the model. In section 3, we solve the model for the competitive equilibrium steady state. In section 4, we specify the welfare cost criterion, discuss our simulations, and report the results of an increase in government spending financed by distortionary taxes or inflation. Section 5 concludes.

2 The Model

We introduce distortionary taxation, government consumption, and government transfers to a model in which agents exchange bonds for money infrequently and decide their frequency of trading. These modifications allow us to study how the government affects the economy and financial markets if it increases expenditures in the form of transfers or government consumption and finances the increase in expenditures in different ways.

Money must be used to purchase goods, only bonds receive interest payments, and there is a cost to transfer money from bond sales to the goods market. Different from Alvarez et al. (2009) and other models in which there is a separation between the asset market and the goods market (e.g. Grossman and Weiss 1983, Rotemberg 1984), the timing of the infrequent sales of bonds for money is endogenous. Moreover, we have capital and labor decisions, government spending, and distortionary taxation.[7]

Time is continuous and denoted by $t \in [0, +\infty)$. At any moment, there are markets for assets, for the consumption good, and for labor. There are three assets, money, claims to physical capital, and nominal bonds. The markets for assets and the market for the consumption good are physically separated.

There is an unit mass of infinitely lived agents with preferences over consumption and leisure. Agents have two financial accounts: a brokerage account and a bank account. They hold assets in the brokerage account and money in the bank account. We assume that readjustments in the brokerage account have a fixed cost. As only money can be used to buy goods, agents need to maintain an inventory of money in their bank account large enough to pay for their flow of consumption expenditures until the next transfer of funds.

[6]According to Ramey (2011), the evidence points to a government spending multiplier between 0.8 and 1.5. For permanent increases in government spending, the multiplier is closer to the one in our experiments. In this case, Ramey points to a multiplier of 1.2 (Baxter and King 1993). See also Hall (2009) and Woodford (2011).

[7]Apart from the heterogeneity of agents and the decision on the holding periods, the model is similar to standard cash-in-advance models such as Cooley and Hansen (1989).

Let M_0 denote money in the bank account at time zero. Let B_0 denote nominal government bonds and k_0 claims to physical capital, both in the brokerage account at time zero. Index agents by $s = (M_0, B_0, k_0)$.

The agents pay a cost Γ in goods to transfer resources between the brokerage account and the bank account. Γ represents a fixed cost of portfolio adjustment. Let $T_j(s)$, $j = 1, 2, ..$, denote the times of the transfers of agent s. Let $P(t)$ denote the price level. At $t = T_j(s)$, agent s pays $P(T_j(s))\Gamma$ to make a transfer between the brokerage account and the bank account. The agents choose the times $T_j(s)$ of the transfers.

The consumption good is produced by firms. Firms are perfect competitors. They hire labor and rent capital to produce the good. The production function is given by $Y(t) = Y_0 K(t)^\theta H(t)^{1-\theta}$, where $0 < \theta < 1$ and $K(t)$ and $H(t)$ are the aggregate quantities of capital and hours of work at time t. Capital depreciates at the rate δ, $0 < \delta < 1$.

The agent is a composition of a shopper, a trader, and a worker. The shopper uses money in the bank account to buy goods, the trader manages the brokerage account, and the worker supplies labor to the firms. The firms transfer their sales proceeds to their brokerage accounts and convert them into bonds.[8]

The firms pay $w(t)h(t,s)$ and $r^k(t)\,k(t,s)$ to the worker for the hours of work $h(t,s)$ and capital $k(t,s)$; $w(t)$ are real wages and $r^k(t)$ is the real interest rate on capital. The firms make the payments with a transfer from the brokerage account of the firm to the brokerage account of the agent. When the firms make the payments to the agents, the government collects $\tau_L w(t)h(t,s)$ in labor income taxes and sends T in transfers to the agents. As a result. the brokerage account of the worker is credited by $(1-\tau_L)\,w(t)h(t,s) + r^k(t)k(t,s) + T$. These credits can be used at the same date for purchases of bonds.[9]

The government issues bonds that pay a nominal interest rate $r(t)$. Let the price of a bond at time zero be given by $Q(t)$, with $Q(0) = 1$. The nominal interest rate is $r(t) \equiv -d\log Q(t)/dt$. Let inflation be denoted by $\pi(t)$, $\pi(t) = d\log P(t)/dt$. To avoid the opportunity of arbitrage between bonds and capital, the nominal interest rate and the payment of claims to capital satisfies $r(t) - \pi(t) = r^k(t) - \delta$. That is, the rate of return on bonds must be equal to the real return on physical capital discounted by depreciation. With this condition satisfied, the agents are indifferent between converting their income into bonds or capital.

Money holdings at time t of agent s are denoted by $M(t,s)$. Money holdings just after a transfer are denoted by $M^+(T_j(s), s)$ and they are equal to $\lim_{t\to T_j, t>T_j} M(t,s)$. Analogously, $M^-(T_j(s), s) = \lim_{t\to T_j, t<T_j} M(t,s)$ denotes money just before a transfer. The net transfer

[8] In Silva (2012), firms keep a fraction a of the sales proceeds in money and transfer the remaining fraction $1-a$ to their brokerage accounts of the workers, $0 \leq a < 1$. However, the value of a has little impact on the welfare cost, on the demand for money and on other equilibrium values.

[9] We also studied a version in which the deposits could only be used in the following period. The results are not affected by this change.

from the brokerage account to the bank account is given by $M^{+}-M^{-}$. If $M^{+}<M^{-}$, the agent makes a negative net transfer, a transfer from the bank account to the brokerage account, which is immediately converted into bonds. Money holdings in the brokerage account are zero, as bonds receive interest payments and it is not possible to buy goods directly with money in the brokerage account. All money holdings are in the bank account. To have M^{+} just after a transfer at $T_j(s)$, agent s needs to transfer $M^{+}-M^{-}+P(T_j(s))\Gamma$ to the bank account, $P(T_j(s))\Gamma$ is used to buy goods to pay the transfer cost.

Define a holding period as the interval between two consecutive transfer times, that is $[T_j(s), T_{j+1}(s))$. The first time agent s adjusts its portfolio of bonds is $T_1(s)$ and the first holding period of agent s is $[0, T_1(s))$. To simplify the exposition, let $T_0(s) \equiv 0$, but there is not a transfer at $t=0$, unless $T_1(s)=0$.

Denote $B^{-}(T_j(s), s)$, $B^{+}(T_j(s), s)$, $k^{-}(T_j(s), s)$, and $k^{+}(T_j(s), s)$ the quantities of bonds and capital just before and just after a transfer. During a holding period, bond holdings and capital holdings of agent s follow

$$\dot{B}(t,s) = r(t)B(t,s) + P(t)(1-\tau_L)w(t)h(t,s) + T, \tag{1}$$

$$\dot{k}(t,s) = (r^k(t)-\delta)k(t,s). \tag{2}$$

The way in which equations (1) and (2) are written implies that labor income and government transfers are converted into nominal bonds, and that interest payments to capital are converted into new claims to capital. This is done to simplify the expressions of the law of motion of bonds and capital. The agent is indifferent between the asset allocations on bonds and capital, as $r(t)-\pi(t)=r^k(t)-\delta$.

At each date $T_j(s)$, $j=1,2,...$, agent s readjusts its portfolio. At the time of a transfer $T_j(s)$, the quantities of money, bonds, and capital satisfy

$$M^{+}(T_j)+B^{+}(T_j)+P(T_j)k^{+}(T_j)+P(T_j)\Gamma = M^{-}(T_j)+B^{-}(T_j)+P(T_j)k^{-}(T_j), \tag{3}$$

$j=1,2,...$ The portfolio of money, bonds and capital chosen, plus the real cost of readjusting, must be equal to the current wealth. With the evolution of bonds and capital in (1)-(2), we can write $B^{-}(T_j)$ and $k^{-}(T_j)$ as a function of the interest payments accrued during a holding period $[T_{j-1}, T_j)$. Substituting recursively and using the non-Ponzi conditions $\lim_{j\to+\infty} Q(T_j)B^{+}(T_j)=0$ and $\lim_{j\to+\infty} Q(T_j)P(T_j)k^{+}(T_j)=0$, we obtain the present value constraint

$$\sum_{j=1}^{\infty} Q(T_j(s))\left[M^{+}(T_j(s),s)+P(T_j)\Gamma\right] \leq \sum_{j=1}^{\infty} Q(T_j(s))M^{-}(T_j)+W_0(s), \tag{4}$$

where $W_0(s)=B_0+P_0k_0+\int_0^{\infty} Q(t)P(t)(1-\tau_L)w(t)h(t,s)\,dt$. Equation (4) states that the present value of money transfers and transfer fees is equal to the present value of deposits in the brokerage account, plus the initial bond and capital holdings.

In addition to the present value budget constraint (4), the agents face a cash-in-advance constraint

$$\dot{M}(t,s) = -P(t)c(t,s), t \geq 0, t \neq T_1(s), T_2(s), \ldots \tag{5}$$

This constraint emphasizes the transactions role of money, that is, agents need money to buy goods. At $t = T_1(s), T_2(s), ...$, constraint (5) is replaced by $\dot{M}(T_j(s),s)^+ = -P(T_j(s))$ $c^+(T_j(s))$, where $\dot{M}(T_j(s),s)^+$ is the right derivative of $M(t,s)$ with respect to time at $t = T_j(s)$ and $c^+(T_j(s))$ is consumption just after the transfer.

The agents choose consumption $c(t,s)$, hours of work $h(t,s)$, money in the bank account $M(t,s)$, and the transfer times $T_j(s)$, $j = 1, 2, ...$ They make this decision at time zero given the paths of the interest rate and of the price level. The maximization problem of agent $s = (M_0, k_0, B_0)$ is then given by

$$\max_{c,h,T_j,M} \sum_{j=0}^{\infty} \int_{T_j(s)}^{T_{j+1}(s)} e^{-\rho t} u(c(t,s), h(t,s))\, dt \tag{6}$$

subject to (4), (5), $M(t,s) \geq 0$, and $T_{j+1}(s) \geq T_j(s)$, given $M_0 \geq 0$. The parameter $\rho > 0$ is the intertemporal rate of discount. The utility function is $u(c(t,s)) = \log c(t,s) + \alpha \log(1 - h(t,s))$. Preferences are a function of consumption and hours of work only. In particular, the transfer cost does not enter the utility function. These preferences are derived from the King et al. (1988) preferences $u(c,h) = \frac{[c(1-h)^{\alpha}]^{1-1/\eta}}{1-1/\eta}$, with $\eta \to 1$, which are compatible with a balanced growth path.[10]

As bonds receive interest and money does not, the agents transfer the exact amount of money needed to consume until the next transfer. That is, the agents adjust $M^+(T_j)$, T_j, and T_{j+1} to obtain $M^-(T_{j+1}) = 0$, $j \geq 1$. We can still have $M^-(T_1) > 0$ as M_0 is given rather than being a choice. Using (5) and $M^-(T_{j+1}) = 0$ for $j \geq 1$, money just after the transfer at T_j is

$$M^+(T_j(s), s) = \int_{T_j}^{T_{j+1}} P(t)\, c(t,s)\, dt,\ j = 1, 2, ... \tag{7}$$

The government makes consumption expenditures G, distributes transfers T, taxes labor income at the rate τ_L, and issues nominal bonds $B(t)$ and money $M(t)$. Government transfers are distributed to agents in lump sum form. The government controls the aggregate money supply at each time t by making exchanges of bonds and money in the asset markets. The

[10] We also solved a version of the model with $\eta \neq 1$ and a version with Greenwood et al. (1988) preferences. Moreover, we considered cash and credit goods. Silva (2012) considers indivisible labor. These changes in the model do not affect our conclusions.

financial responsibilities of the government at time t satisfy the period budget constraint

$$r(t)B(t) + P(t)G + P(t)T = \dot{B}(t) + \tau_L P(t)w(t)H(t) + \dot{M}(t). \tag{8}$$

That is, the government finances its responsibilities $r(t)B(t) + P(t)G + P(t)T$ by issuing new bonds, using revenues from labor taxes, and by issuing money. With the condition $\lim_{t\to\infty} B(t)e^{-rt} = 0$, the government budget constraint in present value is given by

$$B_0 + \int_0^\infty Q(t)P(t)(G+T)\,dt = \int_0^\infty Q(t)\tau_L w(t)H(t)\,dt + \int_0^\infty Q(t)P(t)\frac{\dot{M}(t)}{P(t)}dt. \tag{9}$$

Seigniorage is equal to the real resources obtained by issuing money, $\dot{M}(t)/P(t)$.

The market clearing conditions for money and bonds are $M(t) = \int M(t,s)\,dF(s)$ and $B_0 = \int B_0(s)\,dF(s)$, where F is a given distribution of s. The market clearing condition for goods takes into account the goods used to pay the transfer cost. Let $A(t,\delta) \equiv \{s : T_j(s) \in [t, t+\delta]\}$ represent the set of agents that make a transfer during $[t, t+\delta]$. The number of goods used on average during $[t, t+\delta]$ to pay the transfer cost is then given by $\int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s)$. Taking the limit to obtain the number of goods used at time t yields the market clearing condition for goods, given by $\int c(t,s)\,dF(s) + \dot{K}(t) + \delta K(t) + G + \lim_{\delta\to 0}\int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s) = Y$. The market clearing conditions for capital and hours of work are $K(t) = \int k(t,s)\,dF(s)$ and $H(t) = \int h(t,s)\,dF(s)$.

An equilibrium is defined as prices $P(t)$, $Q(t)$, allocations $c(t,s)$, $M(t,s)$, $B(t,s)$, $k(t,s)$, transfer times $T_j(s)$, $j = 1, 2, ...$, and a distribution of agents F such that (i) $c(t,s)$, $M(t,s)$, $B(t,s)$, $k(t,s)$, and $T_j(s)$ solve the maximization problem (6) given $P(t)$, $r(t)$, and $r^k(t)$ for all $t \geq 0$ and s in the support of F; (ii) the government budget constraint holds; and (iii) the market clearing conditions for money, bonds, goods, capital, and hours of work hold.

3 Solving the Model

As we study the long run effects of policy toward taxes or inflation, we focus on an equilibrium in the steady state. In this equilibrium, the nominal interest rate is constant at r and the inflation rate is constant at π. The aggregate quantities of capital and labor are constant at K and H, and output is constant.

The transfer cost Γ and the interest payments on capital and bonds induce agents to follow (S,s) policies on consumption, money, capital claims, and bonds. For money, agent s makes a transfer at T_j to obtain money $M^+(T_j, s)$ at the beginning of a holding period. The agent then lets money holdings decrease until $M(t,s) = 0$, just before a new transfer at T_{j+1}. Symmetrically, individual bond holdings $B^+(T_j, s)$ are relatively low at T_j; they increase at the rate r until they reach $B^-(T_{j+1}, s)$, just before T_{j+1}. The same applies to the behavior of $k(t,s)$. We assume that all agents behave in the same way in the steady state, in the sense that they follow the same pattern of consumption along holding periods. With constant inflation and interest rates, this implies that the agents start a holding period with a certain value of consumption, $c^{+}(T_j,s)$, which decreases until the value $c^{-}(T_{j+1},s)$, just before the transfer at T_{j+1}. The agents look the same along holding periods, although in general they are in different positions of the holding period.[11]

In the steady state, agents follow the same pattern of consumption along holding periods and choose the same interval between holding periods N. Let $n \in [0,N)$ denote the position of an agent along a holding period and reindex agents by n. Agent n makes the first transfer at $T_1(n) = n$, and then makes transfers at $n+N$, $n+2N$ and so on. Given that the agents have the same consumption profile across holding periods, the distribution of agents along $[0,N)$ compatible with a steady state equilibrium is a uniform distribution, with density $1/N$. We can then solve backwards to find the initial values of M_0, B_0, and k_0 for each agent $n \in [0,N)$ that implies that the economy is in the steady state since $t=0$.[12]

To characterize the pattern of consumption of each agent, consider the first order conditions of the individual maximization problem (6) with respect to consumption. These first order conditions imply

$$c(t,n) = \frac{e^{-\rho t}}{P(t)\lambda(n)\,Q(T_j)},\ t \in (T_j,T_{j+1})\,,\ j=1,2,\ldots, \tag{10}$$

where $\lambda(n)$ is the Lagrange multiplier associated to the budget constraint (4). Let c_0 denote consumption just after a transfer. In the steady state, $P(t) = P_0 e^{\pi t}$, for a given initial price level P_0, and $Q(T_j) = e^{-rT_j}$. Therefore, rewriting (10), individual consumption along holding periods is given by

$$c(t,n) = c_0 e^{(r-\pi-\rho)t} e^{-r(t-T_j)}, \tag{11}$$

for $t \in [T_j(n), T_{j+1}(n)]$. We find aggregate consumption by integrating (11), using the fact that the distribution of agents along $[0,N)$ is uniform. Aggregate consumption is then

$$C(t) = c_0 e^{(r-\pi-\rho)t} \frac{1-e^{-rN}}{rN}. \tag{12}$$

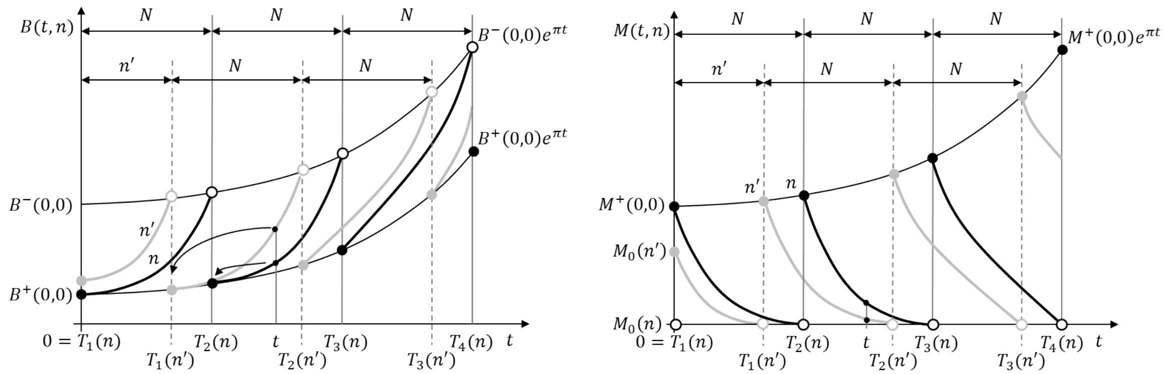
As aggregate consumption is constant in the steady state, the nominal interest rate and the inflation rate that are compatible with the steady state are such that $r = \rho + \pi$.

From (11) and $r = \rho+\pi$, we obtain that individual consumption $c(t,n)$ decreases during

[11] For a description of different applications of (S,s) models in economics, see Caplin and Leahy (2010). See Alvarez et al. (2017) for the relation between inventory theoretical models of the demand for money and the welfare cost of inflation.

[12] See Silva (2011) for additional results on the distribution of agents and for the characterization of M_0 and B_0 for each agent n. The characterization of k_0 is obtained analogously. We obtain government bonds B_0 by the government budget constraint.

the holding period $t \in [T_j, T_{j+1})$ at the rate r. On the other hand, from (12), we obtain that aggregate consumption is constant at $c_0 \frac{1-e^{-rN}}{rN}$. The individual behavior given by $c(t,n)$ is very different from the aggregate behavior, as in other (S,s) models. In particular, the variability of consumption is much larger at the individual level.



(a) Bonds (b) Money

Figure 1: Individual bond and money holdings for two agents, n and n'. Agent n makes the first transfer at time zero, $n = 0$; this agent starts with $M_0(n) = 0$. Agent n' starts with $M_0(n') > 0$ and makes the first transfer at $T_1(n') > 0$. At t, bond and money holdings are a function of the elapsed time since the last time the portfolio was rebalanced. Money at the beginning of holding periods increase at the rate of inflation, π. Individual bond holdings grow at the rate r and aggregate bond holdings grow at the rate π.

A similar situation happens with bonds. During holding periods, individual bond holdings follow equation (1), which implies that $\dot{B}(t,n)/B(t,n) > r$. However, aggregating bond holdings across agents imply that aggregate bond holdings grow at the rate of inflation, $\dot{B}(t)/B(t) = \pi$. As $r = \rho + \pi$, individual bond holdings grow at a higher rate than aggregate bond holdings during a holding period. At the transfer dates T_j, however, bond holdings decrease sharply as the agent sells bonds for money and transfers money to the bank account at these dates. As $B(t)$ grows at the rate of inflation, the value of aggregate bond holdings is constant in real terms. Figure 1 shows the evolution of individual bond holding for two agents, n and n'. Agent n makes the first transfer at time zero and the second transfer at $T_2(n) = N$. Agent n' makes the first transfer at $T_1(n') > 0$ and the second transfer at $T_2(n') = T_1(n') + N$. The fact that $T_1(n) = 0$ implies that agent n starts with zero money holdings and so the agent needs to make a transfer at $t = 0$. Agent n' starts with some money, which delays the first transfer. At time $t > 0$, the agents have different quantities of bonds, $B(t,n)$ and $B(t,n')$.

Given the production function $Y = Y_0 K^\theta H^{1-\theta}$, profit maximization implies prices $w = (1-\theta) Y_0 (K/H)^\theta$ and $r^k = \theta Y_0 (K/H)^{-(1-\theta)}$, which are constant in the steady state. With the non-arbitrage condition $r^k - \delta = r - \pi$ and $r = \rho + \pi$, we have $K/H = [\theta Y_0/(\rho+\delta)]^{1/(1-\theta)}$. Therefore, $K/Y = \theta/(\rho+\delta)$. As K is constant in the steady state, the investment output ratio $(\dot{K}+\delta K)/Y$ is equal to $\delta\theta/(\rho+\delta)$.

The first order conditions for $h(t,n)$ imply

$$1-h(t,n)=\frac{\alpha c_0}{(1-\tau_L)\,w}, \tag{13}$$

which is constant along holding periods, as wages are constant in the steady state. As there is a unit mass of agents, $H=h$. With the expression of wages, we obtain the equilibrium value of the hours of work,

$$h=1-\frac{\alpha c_0}{(1-\tau_L)\,(1-\theta)\,Y_0(K/H)^{\theta}}. \tag{14}$$

As c_0 depends on r and N, equation (14) determines hours of work as a function of r and N.

The market clearing condition for goods in the steady state is $C(t)+\delta K+G+\frac{1}{N}\Gamma=Y$. This equation implies

$$c_0\frac{1-e^{-rN}}{rN}+\delta K+G+\frac{1}{N}\Gamma=Y. \tag{15}$$

Dividing by Y, we obtain an expression for the consumption-income ratio $\hat{c}_0\equiv c_0/Y$ in terms of N and the ratio between government spending and output,

$$\hat{c}_0\frac{1-e^{-rN}}{rN}+\delta\frac{K}{Y}+\frac{G}{Y}+\frac{1}{N}\frac{\Gamma}{Y}=1. \tag{16}$$

The optimal holding period N is obtained with the first order conditions for $T_j(n)$. As derived in the appendix, N must satisfy

$$c_0(r,N)rN\left(1-\frac{1-e^{-\rho N}}{\rho N}\right)=\rho\Gamma. \tag{17}$$

The aggregate demand for money is given by $M(t)=\frac{1}{N}\int M(t,n)\,dn$. Individual money holdings at t, $M(t,n)$, are obtained with the cash-in-advance constraint (5), given individual consumption $c(t,n)$ for an agent that has made a transfer at $T_j(n)$, $M(t,n)=\int_t^{T_{j+1}(n)}P(\tau)\,c(\tau,n)\,d\tau$, $\tau\in(T_j,T_{j+1})$. At any time t, there will be agents in their holding period $j+1$ and others in their holding period j. Taking this fact into account and the behavior of $c(t,n)$ in (11), it is possible to express real money holdings $\tilde{m}\equiv\frac{M}{P}$ in terms of r, N, and c_0. As derived in the appendix,

$$\tilde{m}(r)=\frac{c_0(r,N)\,e^{-rN(r)}}{\rho}\left[\frac{e^{rN(r)}-1}{rN(r)}-\frac{e^{(r-\rho)N(r)}-1}{(r-\rho)\,N(r)}\right]. \tag{18}$$

The values of $\tilde{m}$ and N are written with respect to r to emphasize their dependency on the nominal interest rate r. Dividing (18) by Y yields the money-income ratio $m(r)\equiv M/(PY)$. For a constant value of m in the steady state, as output is constant in the steady state, the growth rate of money must be equal to the inflation rate π.[13]

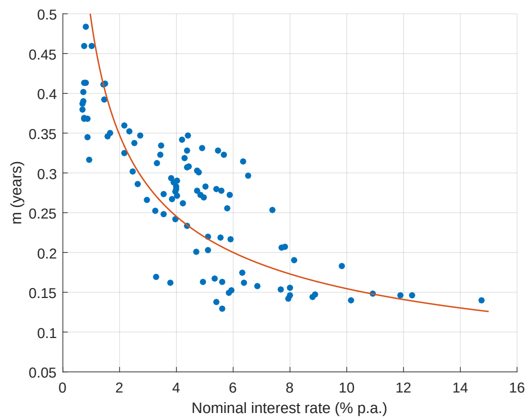


Figure 2: Money-income ratio implied by the model and U.S. annual data. M1 for the monetary aggregate and commercial paper rate for the nominal interest rate.

Figure 2 shows the money-income ratio $m(r) = \tilde{m}(r)/Y$ predicted by the model along with U.S. data.[14] The dataset is similar to the one used in Lucas (2000), Lagos and Wright (2005) and others.[15] The results for $m(r)$ imply an interest-elasticity of $-1/2$, which agrees with the findings in Guerron-Quintana (2009), Alvarez and Lippi (2009) and others.

The equation that closes the model links government spending with tax revenues. The government budget constraint (8) implies

$$rb + G + T = b\frac{\dot{B}}{B} + \tau_L wH + \frac{M}{P}\frac{\dot{M}}{M}, \tag{19}$$

where $b = \frac{B(t)}{P(t)}$. As $\frac{\dot{B}}{B} = \pi$ and $\frac{\dot{M}}{M} = \pi$ in the steady state, we obtain that government

[13]When Y grows at a constant rate, c_0 grows at the same rate of Y and the money-income ratio is constant.

[14]$m(r)$ is calibrated to pass through the geometric mean of the nominal interest rates and the money-income ratio in the data, as described in the next section. In the figure, spending is in the form of transfers and seigniorage is calculated with the nominal interest rate; this implies $\tau_L = 29.8$ percent. The values of τ_L and spending in the figure are then maintained constant for each value of r (the figure changes little if the increase in r is compensated with a decrease in τ_L to maintain constant government revenues). With fixed N, $m(r)$ has elasticity close to zero, which implies a straight line passing through the geometric mean of the data. It is possible to obtain elasticity of the demand for money with N fixed. However, this implies implausible values for parameters such as the elasticity of intertemporal substitution.

[15]The nominal interest rate is given by the commercial paper rate and the monetary aggregate is given by M1. Data are annual from 1900 to 1997 (the last year in which the commercial paper data are available from the same source). M1 and commercial paper rate were also used by Dotsey and Ireland (1996), Lucas (2000), Lagos and Wright (2005), Craig and Rocheteau (2008), among others. We use a similar dataset to facilitate the comparison of results. As it is known, money demand changed substantially at the beginning of 1990s. Teles and Zhou (2005) and Lucas and Nicolini (2015) propose new monetary aggregates to account for the changes in the demand for money. Changing the monetary aggregate does not change our conclusion that the welfare cost of financing the government with inflation with N endogenous increases substantially.

consumption and transfers must satisfy the constraint

$$G + T + (r - \pi)\, b = \tau_L w H + \pi \frac{M}{P}. \tag{20}$$

That is, government spending plus interest payments must be financed through revenues from labor income taxes $\tau_L w H$ or through seigniorage $\pi \frac{M}{P}$. If we consider that $b = 0$, we have

$$G + T = \tau_L w H + \pi \frac{M}{P}. \tag{21}$$

In this formulation, seigniorage is defined as $S = \pi \frac{M}{P}$. In this case, the inflation rate is the analogous to a tax rate on real money holdings.

Alternatively, as in Walsh (2010), add $(r - \pi)\frac{M}{P}$ to both sides of (20). It implies

$$G + T + (r - \pi) d = \tau_L w H + r \frac{M}{P}, \tag{22}$$

where $d = b + \frac{M}{P}$. Seigniorage is now defined as $S = r \frac{M}{P}$, with the nominal interest rate as the analogous to a tax rate on real money holdings. This formulation emphasizes that there are revenues from seigniorage even when inflation is equal to zero. This happens because, if the government finances itself with money, it does not pay interest on the quantity of money issued. Chari et al. (1996), de Fiore and Teles (2003), and others assume that nominal assets $b + \frac{M}{P}$ are equal to zero, that is, $d = 0$. This implies

$$G + T = \tau_L w H + r \frac{M}{P}. \tag{23}$$

Equations (21) or (22) complete the characterization of the equilibrium. As the literature defines seigniorage as $S = \pi \frac{M}{P}$ or $S = r \frac{M}{P}$, we will make separate simulations for both definitions.[16] This formulation implies five equations (equations 14, 15, 17, 18, and either 21 or 23, depending on the definition of seigniorage) and five equilibrium variables (N, h, c_0, $\tilde{m}$, and τ_L or r, depending on the method of financing). Equilibrium output is obtained by $Y = Y_0 (K/H)^{\theta} h$, where Y_0 is normalized to 1. The initial price in equilibrium is obtained by setting an initial value for the money supply. We take as exogenous the values for government consumption G and government transfers T.

4 An Increase in Government Spending

We calculate the effects of an increase in government spending in the form of transfers T or in the form of government consumption expenditures on goods and services G. The economy is

[16] The formulation with π is used in Sargent and Wallace (1981), Cooley and Hansen (1991, 1992), and others. The formulation with r is more common in the optimal taxation literature.

initially in a long run equilibrium, following the equilibrium equations described in section 3. After a change in the value of G or T, we recalculate the equilibrium values of τ_L and r so that the system of equations is satisfied for the new value of government spending.

We change τ_L and r separately. For financing the increase in spending with labor income taxes, we maintain the value of r at its initial value and find τ_L so that the government budget constraint and the remaining equilibrium equations are satisfied. Analogously, for financing the increase in spending with inflation, we maintain τ_L and find the interest rate r such that the equilibrium equations are satisfied. The new inflation rate is given by $r - \rho$.

4.1 Welfare Cost

Let the vector $\Psi_i = (G, T, \tau_L, r)$ denote a fiscal policy i. Let $c_i = c(\Psi_i)$ and $h_i = h(\Psi_i)$ denote the equilibrium profiles of consumption and hours of work for all agents in the support of F under policy $i = A$ or B. The welfare cost of a fiscal policy A with respect to a baseline fiscal policy B is defined as the income compensation w_A that leaves agents indifferent between an economy under fiscal policy A and economy under fiscal policy B. The value of w_A is such that $U^T[(1+w_A)c_A, h_A] = U^T(c_B, h_B)$, where U^T is the aggregate utility with equal weights for all agents.[17] The preferences $u(c,h) = \log c + \alpha \log(1-h)$ imply

$$1 + w_A = \frac{c_0(\Psi_B)}{c_0(\Psi_A)} \left(\frac{1 - h(\Psi_B)}{1 - h(\Psi_A)}\right)^{\alpha} \exp\left(\frac{r(\Psi_A)N(\Psi_A)}{2} - \frac{r(\Psi_B)N(\Psi_B)}{2}\right). \tag{24}$$

The values of $c_0(\Psi_i)$, $h(\Psi_i)$, $N(\Psi_i)$, and $r(\Psi_i)$, $i = A, B$, are given by the equilibrium conditions in section 3. They depend, in particular, on the size of the government spending and on the way in which the government is financed.

Government consumption G does not enter the utility function. Therefore, an increase in G always implies a positive welfare cost with respect to the economy with lower G. Government consumption enters the market clearing condition and decreases the availability of private consumption for the same output. However, we can compare an economy with the same value of government consumption, but in which A denotes financing with inflation and B denotes financing with labor income taxes. If w_A is positive for this case, then the interpretation is that agents would be better off if the government financed government consumption with taxes rather than with inflation.

We also consider an increase in transfers T. Transfers are distributed to agents in lump sum form through the individual budget constraints. On the other hand, the government needs to increase distortionary taxes or inflation to finance the additional transfers. The government taxes the economy in a distortionary way and redistributes the tax revenues in

[17] $U^T(\tilde{c}, \tilde{h}) = \int \int_0^\infty e^{-\rho t} u(c(t,n), h(t,n)) dt dF(n)$, where $\tilde{c}$ and $\tilde{h}$ denote consumption and hours of work for all agents in the support of F and $t \in [0, \infty)$. The definition of w_A uses the fact that consumption is homogeneous of degree one in income.

lump sum form.

As for government consumption, we can calculate the welfare cost of changing the method of financing given the same level of transfers. For a given level of transfers T, let economy A finance the transfers with inflation and economy B with taxes. If $w_A > 0$ then agents would be better off by substituting inflation with taxes.

The two forms of increasing government spending, through government consumption or transfers, yield different results and allow the analysis of different aspects of fiscal policy. When the government increases G, for example, we can study the government consumption multiplier. Also, we can analyze the behavior of the multiplier when the increase in government consumption is financed with inflation or with taxes.

Adão and Silva (forthcoming) show that in a cash-in-advance economy it is optimal to finance transfers with inflation. This case has the counterpart here of an economy with fixed holding periods. As a result, we will see that the welfare cost of substituting inflation with taxes is negative for an economy with fixed holding periods. In contrast, the welfare cost is positive when the holding periods are endogenous.

4.2 Parameters

We set $\theta = 0.36$ for the parameter for capital in the production function and $\delta = 0.10$ for the depreciation (Cooley and Hansen (1989)). We set $\rho = 3$ percent per year (Lucas (2000)). The parameters α and Γ are set so that hours of work are equal to 0.3 and the money-income ratio $m(r)$ passes through the geometric mean of the data; that is, $r_{avg} = 3.64$ percent p.a. and $m_{avg} = 0.257$ year. The data on money-income ratio imply that the average person in the U.S. holds about one quarter of income in money, or that average velocity is about $1/0.25 = 4$ per year).[18] Figure 2 shows the money-income implied by the model together with the data.

We set the initial value of government spending such that the initial ratio of government spending to output is equal to 20 percent.[19] The value of τ_L is obtained so that equations (21) or (23) for the government budget constraint are satisfied. We set the initial value for seigniorage using the mean of the nominal interest rate. This procedure yields an initial value for τ_L equal to 29.79 percent when seigniorage is given by $S = rM/P$ and equal to 30.99 percent when $S = \pi M/P$. Initial seigniorage is equal to 0.94 percent when $S = rM/P$ and to 0.16 percent when $S = \pi M/P$.

[18] Analogously, Alvarez et al. (2009) parameterize the holding period (exogenous in their case) such that the theoretical demand for money approximates the average velocity in the data. Also, Lucas (2000) determines the parameters for the demand for money such that the theoretical demand for money passes through the geometric average of the data.

[19] The average government spending to output ratio for the United States from 1950 to 2016 is 20.9 percent, where government spending is given by government consumption and investment (data from the St. Louis Fed). We also obtained the predictions for different values of the government-output ratio such as zero, 5 percent, and 10 percent. The qualitative results do not change.

We depart from other calibrations in the determination of τ_L as we obtain τ_L such that it satisfies the government budget constraint. We do not take this value from estimates of marginal tax rates. In particular, there are no lump sum taxes to close the government budget constraint. Nevertheless, the value of τ_L is close to the ones in other papers. For example, $\tau_L = 23$ percent in Cooley and Hansen (1992). Carey and Rabesona (2002) estimate an effective labor tax rate of 23.4 percent for the U.S. and a combined consumption and labor tax rate of 28.3 percent.

As hours of work and the money-income ratio are equilibrium variables, the values of α and Γ so that $h = 0.3$ and $m = m_{avg}$ vary if government spending is in the form of transfers or government consumption; and if seigniorage is defined with the inflation rate or with the nominal interest rate. The values, however, do not vary much. The value of α varies from 1.45 for the case of an increase in transfers and $S = rM/P$, the case discussed in the introduction, to 2.00, for the case of an increase in government spending and $S = rM/P$. The value of Γ in these two cases is 36.65 and 51.16 respectively. To have an idea of the meaning of these values, consider the ratio Γ/Y, which yields the cost of a transfer in working days. This ratio is equal to 2.49 with transfers and 3.41 with government consumption.[20] These values imply infrequent transfers from the brokerage account to the bank account, as the initial value of N is equal to 367 days with government consumption and 264 days with transfers. When inflation increases to finance the increase in government spending, the values of N, when endogenous, decrease to 197 days and 127 days.[21]

We can also evaluate the transfer cost by calculating the minutes per week devoted to financial services. As Γ/Y is the cost of transfers in working days and $1/N$ is the number of transfers per year, then $\Gamma/Y \times 1/N$ yields the cost of transfers in working days per year.[22] This measure implies that about 0.95 percent of working days is devoted to financial services in the steady state.[23] To obtain the time devoted to financial transfers implied by the model, we multiply this value by hours of work. The average weekly hours of work in the U.S. from 1957 to 1997 is 36.5 hours per week (data from the OECD). The model then implies 21 minutes per week for the time devoted to financial services.

[20] We also calculated the results using $\Gamma = \gamma Y$ for the cost parameter, which implies a demand for money homogeneous of degree 1 in income. However, as Y is an equilibrium value, γY varies with the policy. In spite of these differences, the values of γ and of Γ/Y are approximately equal. The simulation results are similar with Γ or $\Gamma = \gamma Y$.

[21] For comparison, Alvarez et al. (2009) set N equal to 24 months and 36 months (higher values because Alvarez et al. use M2 instead of M1 and allow for cash inflows). As in other models with market segmentation, the value of N implied by the parameters is large. There is evidence that firms and households in fact rebalance their portfolios infrequently (Christiano et al. 1996, Vissing-Jorgensen 2002, Alvarez et al. 2009). Agents maintain large money holdings even when they pay high interest rates for credit cards (Telyukova 2013).

[22] Γ is in goods per transfer and Y is in goods per working day. So, Γ/Y is in working days per transfer.

[23] Using the equilibrium values of Y and N, we obtain values between 0.94 and 0.95 for the four cases considered in the simulations (the four combinations between government consumption and transfers and seigniorage defined with inflation or nominal interest rates).

4.3 An Increase in Government Spending

We now set the economy with the initial value for the ratio of government spending to output equal to 20 percent and increase the value of government spending by 5 percent. This change increases the ratio of government spending to output by about 1 percentage point, from 20 percent to 21 percent.

We study two ways of financing the increase in spending: by increasing labor income taxes τ_L and by increasing the inflation rate π. We consider seigniorage to be given by $S = rM/P$ or $S = \pi M/P$. The increase in government spending can be in the form of transfers or in the form of government consumption. The initial equilibrium is for the economy under the benchmark parameterization with a balanced government budget constraint.

The results are in tables 1–3 and in figures 3–6. We study the effects of the change in policy using the model with a fixed N and with endogenous N. The value of N when it is fixed is equal to the optimal value of N for the initial situation. All other parameters are the same for the cases with N fixed and N endogenous. The only difference between the two cases is that N is allowed to change optimally in the case of N endogenous.[24] The definition of seigniorage has a small impact. The form of the increase in government spending, either transfers or government consumption, the method of financing, and having N fixed or endogenous are more important for the results.

Table 1: Welfare cost of an increase in government spending

Values in % of income	Model and Method of Financing					
	N Endogenous			N Fixed		
	Inflation	Labor Tax	From Tax to Inflation	Inflation	Labor Tax	From Tax to Inflation
Transfers, Seigniorage $r \times M/P$	1.45	0.95	0.50	0.49	0.96	−0.46
Government Cons, Seigniorage $r \times M/P$	3.03	2.11	0.90	2.01	2.13	−0.11
Transfers, Seigniorage $\pi \times M/P$	0.97	1.01	−0.04	0.51	1.01	−0.50
Government Cons, Seigniorage $\pi \times M/P$	2.41	2.13	0.28	2.00	2.13	−0.12

Government spending increases 5%. The government-output ratio increases from 20% to about 21%. Increase in government spending either in the form of transfers or government consumption. The calculation of the welfare cost follows equation (24). Segniorage is defined with either the nominal interest rate or with the inflation rate. From tax to inflation: welfare cost of changing the method of financing from labor income tax to inflation. A negative sign implies a welfare gain.

The case of fixed N approximates the standard cash-in-advance model with fixed periods.[25]

[24]Figure 6 shows the effects when seigniorage is defined with the interest rate. The qualitative results when seigniorage is defined with inflation are similar.

[25]It is not equal to a standard cash-in-advance model because the agents can still smooth consumption

In particular, the demand for money is approximately constant with N fixed. As it is known, an inflation tax or a labor income tax have similar effects in a cash-in-advance economy. In accordance with this result, the effects across simulations are similar with N fixed. What strongly changes predictions is the case in which N is endogenous. In this case, the demand for money decreases more strongly when inflation increases, a pattern compatible with the data. For N endogenous, financing the government with inflation or taxes have substantially different implications.

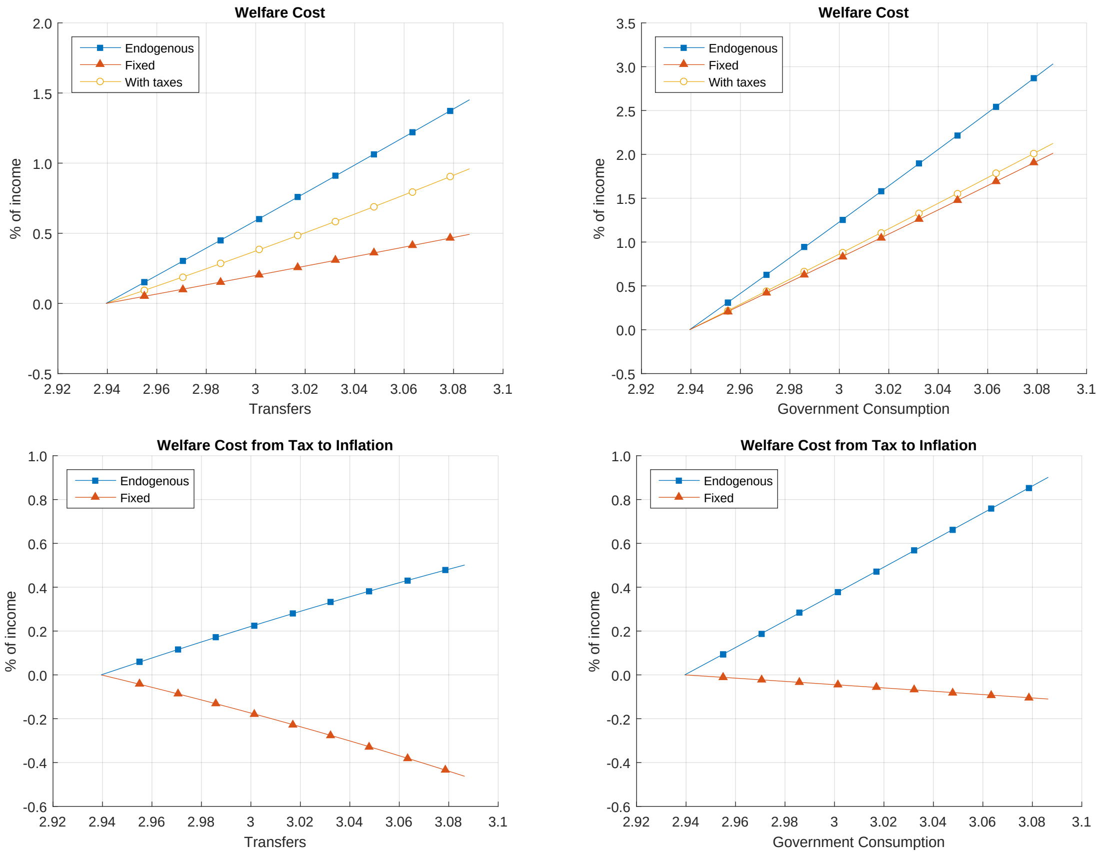
As shown in table 1, considering fixed periods underestimates the welfare cost of financing the increase in government spending with inflation. Consider the case of an increase in government spending in the form of transfers and seigniorage defined with the nominal interest rate, in the first row of table 1. Financing the increase in government spending with inflation implies a welfare cost of 1.45 percent using the model with endogenous periods. The welfare cost is only 0.49 percent with fixed intervals. The underestimation of the effects is such that a model with fixed N implies that there is a gain in financing the increase in government spending with inflation, shown by the negative value -0.46 in the table. The result is reversed with endogenous N. Considering endogenous holding periods in general increases the welfare cost by about one percentage point.

Figure 3 shows the effects on welfare in a different way.[26] The two top figures show the welfare cost associated with an increase in government spending financed with inflation for N endogenous and fixed and the welfare cost when the increase in spending is financed with taxes for N fixed (the results with N endogenous or fixed overlap when financing with taxes). The welfare cost with N endogenous is always higher than with N fixed. Moreover, the predicted welfare cost of financing with taxes is higher than with financing with inflation for N fixed. As a result, changing the fiscal policy from financing with taxes to inflation implies gains in welfare in a model with fixed N such as a cash-in-advance model.

There are two reasons for the predicted gain in welfare with N fixed. The first reason is the existence of transfers and the second is the initial value of bonds and money. About the existence of transfers, Adão and Silva (forthcoming) show that taking government transfers into account changes the optimal set of taxes for a cash-in-advance economy. Transfers constitute an untaxed source of income. When the government does not tax transfers directly, it is optimal to increase inflation to tax this additional income. As a result, the model with fixed N shows a gain of substituting labor taxes with inflation. This effect is also present with N endogenous. However, the model with N endogenous takes into account the costs from the increase in the use of financial services. These costs are greater than the gains of taxing the income from transfers. Agents use more intensively financial services with inflation to

during the interval N. However, the effects in this model and in a model in which consumption is constant during holding periods are similar.

[26]The figure concentrates on the case in which seigniorage is defined as $S = rM/P$.



(a) Increase in transfers

(b) Increase in government consumption

Figure 3: Welfare cost of changes in policy. N endogenous or fixed: financing with inflation. With taxes: financing with taxes and N fixed (endogenous or fixed N overlap in this case). Negative values imply a welfare gain of changing policy from taxes to inflation. A standard cash-in-advance model with fixed holding periods implies gains of financing an increase in spending with inflation. With endogenous holding periods, there are substantial costs of financing with inflation.

decrease their demand for money. When these costs are taken into account, the model with N endogenous predicts that it is optimal to finance the government with taxes.

The second reason for the gains in welfare with N fixed is the initial value of bonds and money. As stated in Chari et al. (1996) and in others, it is optimal to tax away through inflation any initial positive value of bonds and money. This form of taxation amounts to a lump sum tax, as the initial value of bonds and money are given. In the model, the initial aggregate demand for money is positive, as the steady state has agents in different positions of their holding periods. At any point in time, some agents have little balances whereas others have large balances. Therefore, the economy without transfers, in the right panels of figure 3, also show a gain of substituting taxes with inflation. However, this gain is small. The gain of using inflation with N fixed are larger when we consider and increase in transfers.[27]

[27] Adão and Silva (forthcoming) show that government transfers constitute a substantial fraction of GDP

Table 2: Inflation and labor tax to finance an increase in government spending

	Model and Method of Financing			
	N Endogenous		N Fixed	
	Inflation, π (% p.a.)	Labor Tax, τ_L (%)	Inflation, π (% p.a.)	Labor Tax, τ_L (%)
Transfers, Seigniorage $r \times M/P$	12.72	32.12	5.47	32.14
Government Cons, Seigniorage $r \times M/P$	9.68	31.44	4.64	31.46
Transfers, Seigniorage $\pi \times M/P$	7.82	33.37	5.49	33.37
Government Cons, Seigniorage $\pi \times M/P$	5.79	32.65	4.59	32.64

Initial inflation rate: 0.64% p.a. (percent per annum). Initial labor tax: 29.79% (case rM/P) and 30.99% (case $\pi M/P$). See table 1 for welfare cost calculations and definitions.

Phelps (1973) states that positive inflation can be optimal when only distortionary taxes are available. However, more recent results point out that the Friedman rule is optimal in cash-in-advance models with distortionary taxation.[28] This is the case of Kimbrough (1986), Correia and Teles (1996, 1999), Chari et al. (1996), de Fiore and Teles (2003) and others. Adão and Silva (forthcoming) show that the Friedman rule is not optimal in the context of Chari et al. (1996), even when both labor and consumption taxes are available, when there are positive government transfers.[29]

Table 2 and figure 4 show that considering fixed periods underestimates the required inflation to finance the increase in government spending. For the case of an increase in transfers, the required inflation is 12.72 percent per year with endogenous periods while it is 5.47 percent with fixed periods. With fixed periods, the demand for money does not change much with inflation. This implies that the government is able to finance itself with a smaller rate of inflation.

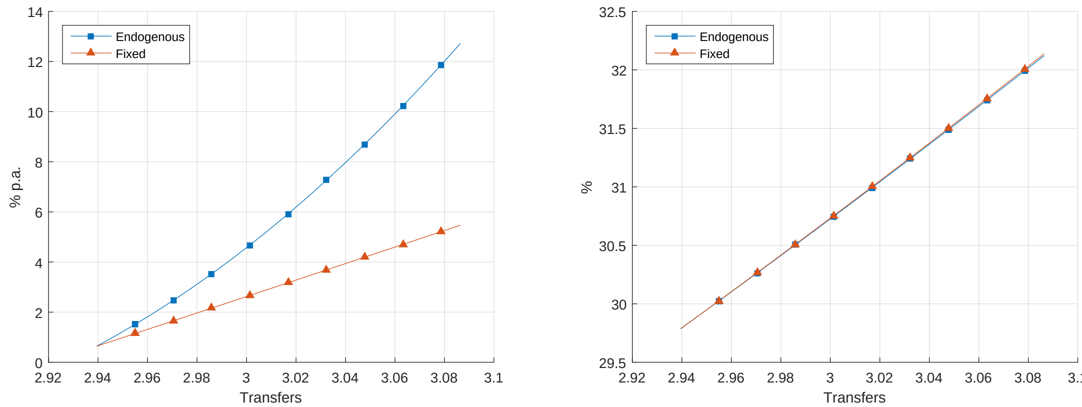
We can also see that the two models yield similar predictions when the policy change involves only an increase in taxes. In figure 4, the cases with endogenous or fixed periods yield the same required labor tax to finance the increase in spending. Table 3 shows revenues from seigniorage and from labor taxes in each case. Notice that government spending is still financed mostly with labor income taxes even when the government uses inflation to finance the increase in spending.[30]

and that they have been increasing since the 1950s.

[28]The Friedman rule (Friedman 1969), states the optimality of having nominal interest rates equal to zero.

[29]Da Costa and Werning (2008) have additional results on the Friedman rule. See Kocherlakota (2005) for a survey. Guerron-Quintana (2011) studies the welfare cost of inflation in a new Keynesian model. Wen (2015) studies the welfare cost of inflation when liquidity reduces consumption uncertainty. Our focus is on the effects of an increase in government spending financed in different ways.

[30]A different question is the transition toward the new policy. Studying the equilibrium transition of an



(a) Inflation (b) Labor tax

Figure 4: Changes in inflation π and in the labor tax τ_L to finance an increase in transfers. The model with fixed holding periods predicts a smaller increase in inflation. Financing with taxes does not imply a change in inflation. As a result, the effects on the labor tax are similar for the two models. Seigniorage defined as $S = rM/P$. The qualitative results of an increase in government consumption and with $S = \pi M/P$ are similar.

The predictions with N fixed or endogenous are different when the increase in government spending is financed with inflation. In this case, the opportunity cost of holding money increases and, therefore, the agents spend resources to decrease their real demand for money. With fixed periods, the agents can decrease slightly the demand for money by making consumption within holding periods steeper. This behavior also shows up with N endogenous, but most of the decrease in the demand for money is obtained by decreasing N. With N fixed, the money-income ratio decreases about 2 percent. With N endogenous, the money-income ratio decreases about 50%. An inflation of 12 percent p.a. implies a decrease in the money-income ratio to the values of the 1980s, corresponding to the data points on the Southeast of figure 2.

We interpret the increase in the trading frequency as an increase in the use of financial services. Let $\Gamma\frac{1}{N}$ be the measure of the financial system in the economy. Using this measure, the value of financial services as a fraction of output increases from 0.95 percent to 1.97 percent for the case of an increase in transfers financed with inflation and seigniorage defined with the nominal interest rate. With seigniorage defined with inflation, the fraction of financial services to output increases from 0.16 percent to 1.16 percent. For the four cases considered, the fraction of financial services to output increases by about 1 percentage point. English (1999) provides evidence that the share of financial services in the economy increases with inflation.[31] According to English, a 10 percent increase in inflation in U.S. would imply an increase in the financial sector of 1.3 percentage points, which agrees with our results.

aggregated (S, s) model requires techniques that are beyond the objective of this paper. Adão and Silva (2017) study the transition for a simplified version of the model. See also Gertler and Leahy (2008) and Stokey (2009).

[31] This evidence is also consistent with Aiyagari et al. (1998).

Table 3: Revenues from seigniorage and labor taxes after an increase in government spending

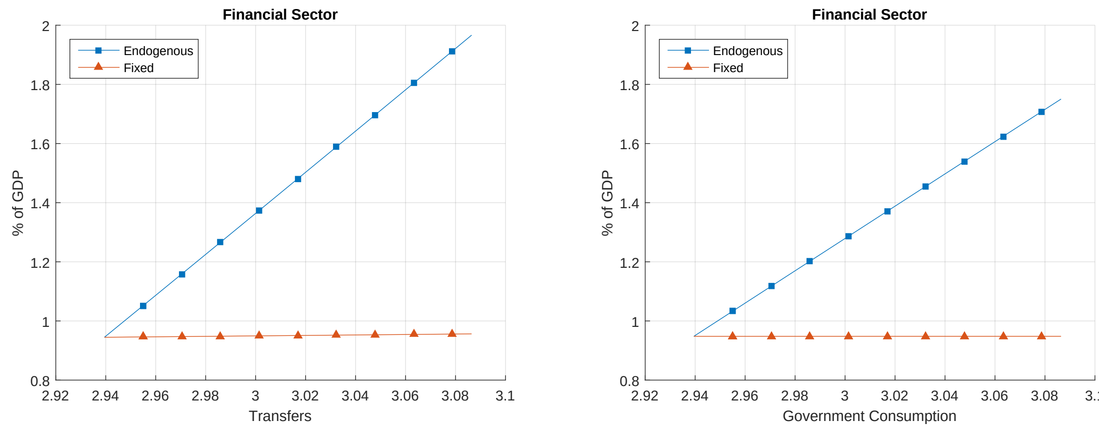
Values in % of output	Model and Method of Financing							
	N Endogenous				N Fixed			
	Inflation		Labor Tax		Inflation		Labor Tax	
	$\tau_L wH$	Seig	$\tau_L wH$	Seig	$\tau_L wH$	Seig	$\tau_L wH$	Seig
Transfers, Seigniorage $r \times M/P$	19.06	1.93	20.56	0.95	19.06	2.19	20.57	0.94
Government Cons, Seigniorage $r \times M/P$	19.06	1.71	20.12	0.93	19.06	1.94	20.13	0.92
Transfers, Seigniorage $\pi \times M/P$	19.84	1.16	21.36	0.17	19.84	1.42	21.35	0.16
Government Cons, Seigniorage $\pi \times M/P$	19.84	0.94	20.89	0.16	19.84	1.16	20.89	0.16

G: government spending in the form of transfers or government consumption. The initial value of G is such that G/Y is equal to 20%. Initial revenues from labor taxes: 19.06% (rM/P) and 19.85% ($\pi M/P$). Initial revenues from seigniorage: 0.94% (rM/P) and 0.16% ($\pi M/P$).

Figure 5 shows the size of the financial sector for each value of spending when the increase in government spending is financed with inflation. In accordance with the empirical evidence, the model with endogenous holding periods implies an increase in the financial sector as a percentage of GDP. With fixed holding period, the size of the financial sector, counterfactually, does not change. As stated above, the total increase in the financial sector implied by the model, of about 1 percentage point, corresponds to the empirical evidence on the increase in the financial sector when inflation increases 10 percentage points.[32]

An unexpected effect of considering N endogenous is the response of output after the increase in G. The response of output implies a government spending multiplier slightly above 1 for N endogenous when government spending is in the form of government consumption and when it is financed with inflation. When the increase in G is financed with taxes, agents substitute away from labor toward leisure, which decreases output. These effects can be seen in figure 6. When N is fixed, inflation and taxes have similar effects. When N is endogenous, the increase in inflation makes agents decrease the demand for money, which requires an increase in the provision of financial services. The increase in financial services implies an increase in output. This effect implies a multiplier above 1 when the increase in government spending is made through consumption and financed with inflation. When the increase in spending is in the form of transfers, the multiplier is smaller, but it is still positive. With transfers, the multiplier is equal to -1.2 with N fixed and 0.35 with N endogenous. With government consumption, the multiplier is equal to -0.01 with N fixed and 1.09 with N endogenous.

[32]The financial sector provides more services than it is implied by the model. In particular, the model abstract from risk sharing, capital allocation, and other financial services. We are concerned here with the increase in the financial sector solely given by the increase in inflation.



(a) Increase in transfers

(b) Increase in government consumption

Figure 5: Size of the financial sector when the government finances an increase in spending with inflation, $(\Gamma \times 1/N)/Y$. Inflation increases about 10 percentage points in each case (table 2). With fixed periods, the model does not predict an increase in the financial sector. With endogenous periods, the financial sector increases about 1 percentage point, which agrees with the empirical evidence in English (1999) of an increase in inflation of 10 percentage points.

A multiplier slightly above 1 is compatible with the values stated in Hall (2009) and Ramey (2011). Woodford (2011) discusses how the multiplier can be above 1 in models with sticky prices or sticky wages. Here, the multiplier is above 1 with flexible prices. The only friction in the model are financial frictions. Although output increases, the welfare cost of the economy with high inflation is large. Output increases, but the overall effect is such that welfare decreases.

Having endogenous periods is especially relevant to study policy changes that imply changes in inflation. The effects on the welfare cost, output and consumption are approximately equal with N fixed or endogenous when the increase in government spending is financed with taxes. The predicted increase in τ_L is also similar in the two cases. The models with N fixed or endogenous yield similar predictions as inflation does not change when taxes are used to cover the increase in government spending.

Endogenous periods matters crucially when the policy change involves an increase in inflation. In this case, the welfare cost of financing with inflation is larger with N endogenous. The inflation rate, output and consumption are also different with N fixed or endogenous.

A model with N endogenous implies a better match on the behavior of the demand for money and on the increase in financial services after an increase in inflation. Given the results obtained here, it is important to consider changes in the frequency of trades to evaluate the effects of inflation. Especially, for the estimates of the welfare cost of different policy changes.

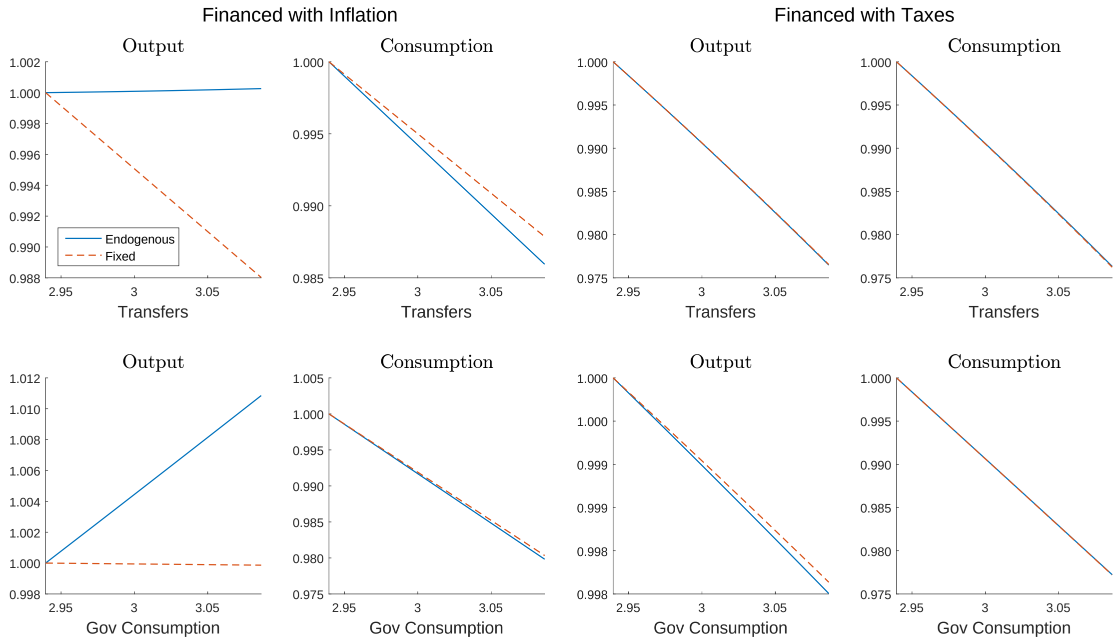


Figure 6: Effects on output and consumption relative to initial levels of an increase in spending financed in different ways. Top: increase in spending in the form of transfers. Bottom: increase in spending in the form of government consumption. Seigniorage $S = rM/P$.

5 Conclusions

We analyze the effects of an increase in government spending. We consider that the increase in government spending can take the form of transfers or government consumption and that it can be financed through taxes or through inflation. The novelty is that the economy reacts to an increase in inflation by increasing the frequency of financial trades and the size of the financial sector. The frequency of financial trades is fixed in standard cash-in-advance models.

The welfare cost of financing the government with inflation increases substantially when the frequency of financial trades is taken into account. For conventional parameters, a standard cash-in-advance model underestimates the welfare cost of inflation and implies that financing with inflation is optimal. We reverse this result when we take into account the use of financial services. Financing with taxes becomes optimal.

When the government is financed with taxes, the models with fixed or endogenous holding periods imply similar predictions. This is so because the use of financial services and the demand for money do not change, as inflation does not increase, when the government is financed through taxes. The predictions differ when the change in policy involves an increase in inflation. Higher inflation increases the use of financial services. The increase in the use of financial services appears in the model as an increase in the frequency of payments to transform bonds into money. The welfare cost of inflation increases substantially.

The effects on welfare, output and consumption change. The match between data and model predictions on the demand for money improves. The increase in the use of financial services implied by the model agrees with data. We conclude that it is important to consider the use of financial services to analyze policies that imply changes in inflation.

A Appendix

A.1 Optimal Holding Period N (equation 17)

The first order conditions with respect to $T_j(n)$, $j = 2, 3, ...$ imply

$$\frac{e^{-\rho T_j}}{P(T_j)Q(T_j)}[\log c^-(T_j) - \log c^+(T_j)] = \lambda \left\{ -r(T_j) \int_{T_j}^{T_{j+1}} \frac{P(t)c(t)}{P(T_j)} dt - c^+(T_j) \right. \\ \left. + \frac{Q(T_{j-1})c^-(T_j)}{Q(T_j)} - \Gamma \left[r(T_j) - \pi(T_j) \right] \right\}. \tag{A.1}$$

The first order conditions with respect to $c\,(t, n)$ imply

$$e^{-\rho T_j} c^+(T_j)^{-1} = \lambda Q(T_j) P(T_j), \tag{A.2}$$
$$e^{-\rho T_j} c^-(T_j)^{-1} = \lambda Q(T_{j-1}) P(T_j).$$

Therefore, $\log \frac{c^-(T_j)}{c^+(T_j)} = -rN_j$. The first order conditions with respect to $T_j\,(n)$ simplify to

$$c^+(T_j) r N_j - r \int_{T_j}^{T_{j+1}} \frac{P(t)c(t)}{P(T_j)} dt = \Gamma\,(r - \pi)\,. \tag{A.3}$$

In the steady state, $c(t) = c_0 e^{-r(t-T_j)}$, $c^+(T_j) = c_0$, and $c^-(T_j) = c_0 e^{-r(T_j - T_{j-1})} = c_0 e^{-rN_j}$. These equations imply

$$c_0 r N_j - c_0 r N_{j+1} \frac{1 - e^{-\rho N_{j+1}}}{\rho N_{j+1}} = (r - \pi)\,\Gamma. \tag{A.4}$$

Setting $N_{j+1} = N_j$, $r - \pi = \rho$, and rearranging yields equation (17) in the text.

A.2 Real Money Holdings (equation 18)

Aggregate money demand is given by $M(t) = \frac{1}{N} \int_0^{t-jN} M^1(t, n) dn + \frac{1}{N} \int_{t-jN}^{N} M^2(t, n) dn$, $t > jN$, where $M^1(t, n)$ and $M^2(t, n)$ denote, respectively, money demand of agents in their holding periods $j + 1$ and j. The transfer time T_j of agent n is $T_j \equiv n + (j - 1)N$, $j = 1, 2, \ldots$ We have $M^1(t, n) = \int_t^{T_{j+2}} P(t)c(t)dt$, $n \in [0, t - jN)$, $T_{j+1} \leq t < T_{j+2}$, where $c(t, n) = c_0 e^{-r(t - T_{j+1})}$. In the steady state, $P(t) = P_0 e^{\pi t}$. Therefore, $M^1\,(t, n) =$

$\int_t^{T_{j+2}} P_0 c_0 e^{\pi t} e^{-r(t-T_{j+1})} dt$. Analogously, $M^2(t,n) = \int_t^{T_{j+1}} P(t) c_0 e^{-r(t-T_j)} dt$, $n \in [t - jN, N)$. Therefore, aggregate money demand is given by

$$M(t) = \frac{1}{N} \int_0^{t-jN} P_0 c_0 e^{rT_{j+1}} \frac{e^{-(r-\pi)t} - e^{-(r-\pi)T_{j+2}}}{r-\pi} dn + \frac{1}{N} \int_{t-jN}^{N} P_0 c_0 e^{rT_j} \frac{e^{-(r-\pi)t} - e^{-(r-\pi)T_{j+1}}}{r-\pi} dn. \tag{A.5}$$

This expression yields

$$M(t) = \frac{P_0 c_0 e^{rt} e^{-(r-\pi)t}}{N} \int_0^N e^{-rx} \frac{1 - e^{(r-\pi)(x-N)}}{r-\pi} dx. \tag{A.6}$$

Setting $r - \pi = \rho$ and dividing by $P(t)$ yields equation (18).

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