

# The Effect of Firm Cash Holdings on Monetary Policy\*

Bernardino Adão      André C. Silva

June 2020


## Abstract

Firm cash holdings increased substantially from 1980 to 2017. We study the implications of the increase in firm cash holdings on monetary policy. We introduce a model that takes the distribution of firm cash holdings as an input. We find that the interest rate channel of the transmission of monetary policy becomes more powerful, as the impact of monetary policy over real interest rates increases. The time for the real interest rate to return to its initial value increases three times. Given the current large firm cash holdings, our results imply that monetary policy changes should be made gradually.

JEL Codes: E40, E50, G12, G31

Keywords: firm cash holdings, interest rates, financial frictions, liquidity effect, monetary policy

---

\*Adão: Banco de Portugal, DEE, Av. Almirante Reis 71, 1150-021 Lisbon, Portugal, badao@bportugal.pt. Silva: Nova School of Business and Economics, Universidade NOVA de Lisboa, Campus de Carcavelos, Rua da Holanda 1, 2775-405, Carcavelos, Portugal, andre.silva@novasbe.pt. The views in this paper are those of the authors and do not necessarily reflect the views of the Banco de Portugal. We thank Heitor Almeida, Rui Albuquerque, Roc Armenter, Philippe Bacchetta, Thomas Bates, Paco Buera, Jeffrey Campbell, Isabel Correia, Igor Cunha, Carlos da Costa, Miguel Ferreira, Nir Jaimovich, Francesco Lippi, Ana Marques, Ed Nosal, Ricardo Nunes, Luigi Paciello, Bruno Sultanum, Shaofeng Xu, and participants at various seminars for valuable comments and discussions. We also thank the editor and the referees for valuable comments and suggestions. We acknowledge financial support from FCT. Silva thanks the hospitality of the Banco de Portugal, where he wrote part of this paper, and acknowledges financial support from Banco de Portugal. This work was funded by FCT-Fundação para a Ciência e a Tecnologia (PTDC/IIM-ECO/4825/2012, UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab Project 22209), and POR Norte (Social Sciences DataLab Project 22209). ©2020 This manuscript version is made available under the CC-BY-NC-ND 4.0 license  <http://creativecommons.org/licenses/by-nc-nd/4.0/>. This is a post-peer-review, pre-copyedit version of an article published in *European Economic Review*, volume 128, manuscript 103508, 2020, DOI <https://doi.org/10.1016/j.eurocorev.2020.103508>.

# 1 Introduction

Corporate cash holdings corrected for inflation increased almost six times from 1980 to 2017, from 140 to 811 billion dollars. The median cash-sales ratio increased from 3.5% in 1980 to 12.9% in 2017. The mean cash-sales ratio increased from 16% to 189% during the same period (the mean cash-sales ratio is now greater than 1). Bates et al. (2009), Bover and Watson (2005) and others notice the increase in corporate cash holdings since 1980, both in real terms and as a percentage of aggregate money. Corporate cash holdings, measured as cash and equivalents of U.S. nonfinancial firms listed in Compustat, amounted to 1.58 trillion dollars in 2010. As M1 amounted to 1.84 trillion, according to data from the St. Louis Fed, 1.58 trillion dollars corresponds to 86% of M1. The ratio of corporate cash holdings decreased from 2010 to 2017, as M1 has increased sharply more recently. Even so, corporate cash holdings amounts to 2 trillion dollars in 2017, and its ratio to M1 equals to 56%. This ratio was 30% in 1980. As the demand for money from corporations is substantial, changes in corporate cash holdings can affect monetary aggregates and monetary policy significantly.<sup>1,2</sup>

Our objective is to obtain the implications of the increase in corporate cash holdings on monetary policy. To the best of our knowledge, we are the first to study the consequences for monetary policy of the changes in the distribution of corporate cash. A related paper is Cole and Ohanian (2002), which evaluates how changes in the demand for money affect monetary policy shocks in the liquidity model and the sticky price model. We emphasize here the increase in firm cash holdings together with changes in the cross-sectional distribution of firm cash holdings.<sup>3</sup>

---

<sup>1</sup>We use cash (CH) and cash and equivalents (CHE) as measures of cash. We restrict our sample to firms with positive cash, positive assets, assets greater than cash, and positive sales. We also truncate the firms at the 1 and 99 percentiles of the cash-sales ratio. Bates et al. (2009) concentrate on the cash-assets ratio, which shows a similar increase over time. In addition to the cash-sales ratio, different measures of cash holdings, such as the cash-assets and the cash-net assets ratio have been used in the literature. As it will be clear when we introduce the model, we use the cash-sales ratio because it has a better data counterpart to the variables in the model. We discuss the data in more detail in section 2.

<sup>2</sup>M1 is defined as currency plus traveler checks plus checkable deposits. In January 2014, currency corresponds to 43.6% of M1 and checkable deposits to 56.3%. The definition of cash and equivalents in Compustat includes the components of M1 plus “securities readily transferable to cash,” which includes short term commercial paper, short term government securities, and money market funds. On average, cash represents 60% of CHE in the entire sample and 70% of CHE since 1990.

<sup>3</sup>We analyze how changes in firm cash holdings affect macroeconomic variables. Fresard (2010) and Palazzo (2012) study the real effects of cash holdings on market share and equity returns. A recent paper that studies the interaction of firm cash holdings and macroeconomic variables (labor) is Bacchetta et al. (2019). Gao and Xu (2018) study the effect of firm cash holdings on business cycles.

According to our model, in response to a monetary shock given by an increase in the nominal interest rate, the real interest rate takes 1.78 months to revert to its initial value with the distribution of money holdings of 1980, and takes 5.17 months to revert to its initial value with the distribution of money holdings of 2017. The 3.4 additional months imply an increase of almost three times of the time that it takes for the real interest rate to return to its initial value. Given the large current corporate cash holdings, an increase in interest rates today has a higher impact on real interest rates. Figure 1 shows the time that it takes for the real interest rate to return to its initial value according to our simulations for each year from 1980 to 2017.<sup>4</sup>

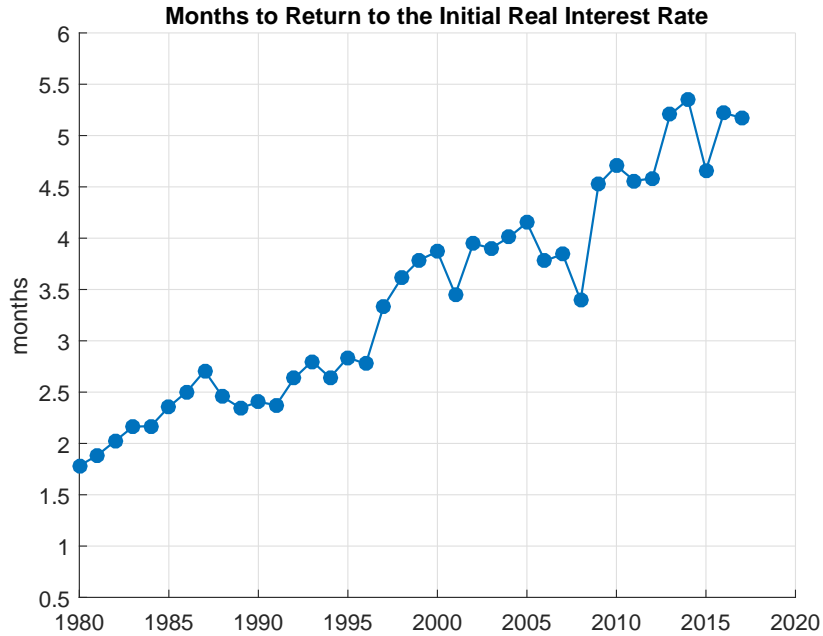


Figure 1: Simulations with the model of section 4 for a given nominal interest rate shock (see section 7 for robustness and comparison between CH and CHE as measures of cash). The simulations take into account the distribution of the cash-sales ratio for each year.

The effects of monetary policy over real interest rates are more persistent. As a consequence, to mitigate adverse effects, a monetary authority should avoid make abrupt changes in interest rates. For example, a monetary authority interested in restoring the historical levels of interest rates from their current low levels should increase interest rates gradually.<sup>5</sup>

<sup>4</sup>The initial responses of the real interest rates are identical in all periods. As the real interest rate changes, it means that the action of the monetary authority is able to affect a fundamental real variable which, in turn, can affect other real variables such as consumption and investment. The effects on consumption and investment will be larger the longer it takes for the real interest rate to return to its initial equilibrium value.

<sup>5</sup>In agreement with this view, Yellen (2017a) states that “Waiting too long to tighten policy could require

According to the traditional interest rate channel of monetary policy, central banks influence economic activity through their impact on short-term nominal interest rates, which gets passed through to real interest rates. The real rates affect investment and consumption. In the New Keynesian models the change in the nominal rate affects the real rate because prices are sticky (for example, Christiano et al. 2005). There is a distribution of prices across firms, but typically the distribution of money is degenerate. A representative agent uses all money carried from the last period to buy goods and services in the current period. The distribution of money holdings in these models does not affect the results of monetary policy. Other kinds of frictions, such as informational frictions (Mankiw and Reis 2002, Alvarez et al. 2011, Paciello and Wiederholt 2014) and menu costs (Goloso and Lucas 2007) have also been introduced to study the real effects of monetary policy.<sup>6</sup>

To take into account the effects of changes in the distribution of cash holdings, we use a model with market segmentation. The friction in this kind of model is the separation of markets for liquid and illiquid assets. These markets are separated in the sense that firms cannot exchange illiquid assets for cash instantaneously or, alternatively, without costs. Liquid assets are used for transactions while illiquid assets receive higher interest yields.

We modify the models in Alvarez et al. (2009) and Silva (2012) to match the observed distribution of firm cash holdings in the data. Alvarez et al. (2009) show that the model closely matches the short-run fluctuations in velocity. We use the model to obtain predictions about the effects of the increase in cash holdings. The predictions are obtained by calculating

---

the FOMC to eventually raise interest rates rapidly, which could risk disrupting financial markets and pushing the economy into recession. For these reasons, I consider it prudent to adjust the stance of monetary policy gradually over time.” See also Yellen (2017b).

<sup>6</sup>There are other channels of the monetary transmission mechanism. Under the balance sheet channel of monetary policy, described originally by Bernanke and Gertler (1989) and used more recently for instance by Brunnermeier and Koby (2019), a surprise increase in the nominal interest rates causes bank assets to decline by more than their liabilities, reducing their net worth and forcing banks to shrink their balance sheets. Drechsler et al. (2017) propose the deposit channel of monetary policy. According to this channel, the spread between the nominal interest rate and the deposit rates rises when the central bank increases the nominal interest rate, triggering large deposit outflows and a decrease in bank loans. Stein (1998), Kashyap and Stein (2000), and Bolton and Freixas (2006) also focus on the role of bank lending. Lagos and Zhang (forthcoming) explain the turnover-liquidity transmission mechanism of monetary policy, according to which an increase in the nominal interest rate increases the opportunity cost of holding the nominal assets used to settle financial transactions (e.g., bank reserves, money balances), making these payment instruments scarcer. This scarcity reduces the liquidity and the price of financial assets. Rocheteau et al. (2018) and Rocheteau et al. (2017) describe a different transmission mechanism. A higher nominal rate induces firms to decrease cash holdings. Firms with smaller amounts of cash have to rely more on external funding to finance investments, which is only possible at higher real interest rates.

the response of the real interest rate to a nominal interest rate shock for each year from 1980 to 2017. The shocks follow the empirical interest rate dynamics in Christiano et al. (1999) and Uhlig (2005). For each year, we recalibrate the model to fit the distribution of cash holdings. As the distribution of cash holdings changes, the response of the real interest rate changes.<sup>7</sup>

There are real effects because the behavior of firms depends on their cash holdings at the time of the shock. Firms with more cash holdings take longer to react as they can use their liquidity at the moment of the shock. If the market segmentation friction is removed, the real interest rate does not change after the shock. As we want to isolate the effects of the change in cash holdings, we eliminate other mechanisms besides market segmentation that could generate additional real effects. In particular, there are no sticky prices, output is constant, and the only change in the economy during the period is in the distribution of cash holdings. Prices are obtained in equilibrium through a general equilibrium model. The changes in firm characteristics during the period are reflected in the distribution of cash holdings.

## 2 Data: Level and Distribution of Firm Cash Holdings

Figure 2 shows the median and the mean of the corporate cash-sales ratio from 1980 to 2017. The cash-sales ratio and the cash-assets ratio have been increasing substantially over time. The cash-assets ratio indicates how firms allocate their portfolios. The cash-sales ratio indicates how much cash is held by firms with respect to their flow of resources. It has a direct interpretation in terms of the use of cash for the firms' businesses. We use the cash-sales ratio because its interpretation, cash relative to the volume of transactions, allows a better connection between model parameters and data.<sup>8,9</sup>

---

<sup>7</sup>In Silva (2012) and Adão and Silva (2017, 2020) there is an explicit cost of transferring money from the assets market to the goods market and the holding period is obtained endogenously. Here, we abstract from this cost and set the holding period exogenously, as we focus on the short-run dynamics of a change in the nominal interest rate. As the shocks considered are small, it is assumed implicitly that the holding periods do not change in an important way. Alvarez et al. (2009) and Alvarez and Lippi (2009) also keep holding periods fixed in the short run. Our assumptions allow closed-form solutions for the effects of shocks.

<sup>8</sup>Different measures of cash have been used to analyze firm cash holdings such as the cash-net assets ratio (used, for example, by Opler et al. 1999) and the cash-assets ratio (by Bates et al. 2009). The cash-sales ratio has been used, among others, by Harford (1999), Harford et al. (2008), and Bover and Watson (2005).

<sup>9</sup>We use two measures of cash, both from Compustat: (1) cash and equivalents (cash and short-term investments, code CHE), and (2) cash (code CH). As the literature uses CHE, we first report the results for CHE and report the results for CH in section 7). For sales, we use SALE from Compustat. The data is for U.S. nonfinancial firms, excluding utilities. To avoid anomalies, we consider observations with positive assets, cash and sales; and remove observations with cash greater than assets. To avoid extreme cash-sales ratios, we

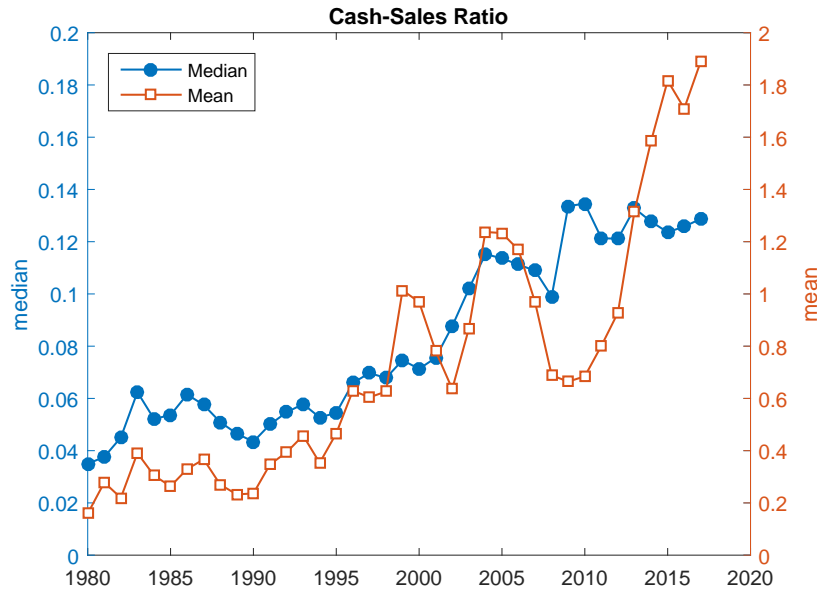


Figure 2: Mean and median of the cash-sales ratio across firms for each year. The ratio between the mean and the median of the cash-sales ratio increased from 4.5 in 1980 to 14.7 in 2017. The cash-sales ratio state how much firms maintain of their sales in cash. A cash-sales ratio of 0.1, for example, means that firms maintain 10 percent of their yearly sales, or 1.2 months of sales, in cash. Source: Compustat; see note 9 for details.

As cash is measured in dollars and sales are measured in dollars per unit of time, the cash-sales ratio is a variable given in units of time. The median cash-sales ratio of 0.13 year in 2017, for example, means that firms maintained about 1.6 months of their sales as cash in 2017. In 1980, this same ratio was only 0.03, or 11 days. The mean cash-sales ratio in the same period increased from 0.16 in 1980 to 1.89 in 2017. The distribution of the cash-sales ratio across firms is highly asymmetric as it can be inferred by the difference between its mean and median. The mean is more than four times the median during the whole period and it is more than 14 times the median in 2017.<sup>10</sup>

Usually, firms maintain cash-sales ratios smaller than one. A cash-sales ratio above one means that the firm keeps more than one year of sales in cash. The mean cash-sales ratio has

---

remove observations with cash/sales below the 1st and above the 99th percentiles of cash/sales. We correct for inflation with the CPI from the St. Louis Fed FRED dataset, CPIAUCSL, base 1982-84. Our procedure implies 203,842 firm-years or about 5,364 firms per year. Small values of sales imply large values for the cash-sales ratio, which are more frequent for small firms; we address this issue as we weight firms by their sales when we parameterize the model. We also repeated our procedure restricting further for firms with sales greater than 10 million, which implies 151,887 firm-years, and we obtained similar results.

<sup>10</sup>The values of the cash-sales ratios mean that a picture of firm cash holdings taken in a particular day in 2017, for example, implies an average of 1.6 months of sales in cash.

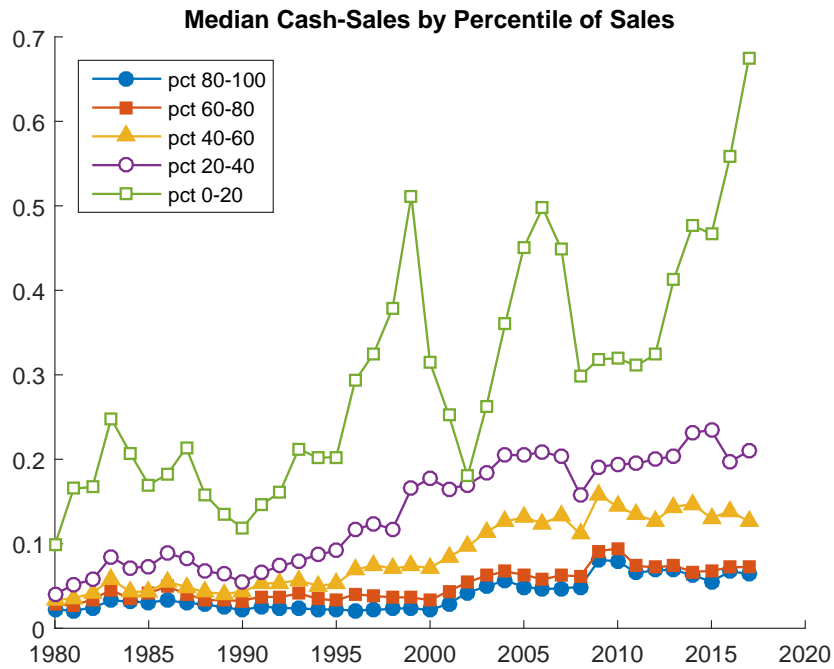


Figure 3: Median of the cash-sales ratio for different percentiles of sales. Source: Compustat; see note 9 for details.

been higher than 1 since 2013.<sup>11</sup>

Smaller firms maintain higher cash-sales ratios. Figure 3 shows the median of the cash-sales ratio over the same period for firms grouped in percentiles of sales. The cash ratio increased for all groups. Moreover, smaller firms experienced a relatively larger increase in their cash holdings. The cash ratio increased 4 times for all firms as a whole, whereas it increased 7 times for firms in the smaller percentiles. Bates et al. (2009) show a similar evolution for the cash-assets ratio.

In addition to the increase in cash-sales ratios, corporate cash holdings represent a large fraction of the monetary aggregates; and this fraction has increased substantially. From 1980 to 2010, the ratio between corporate cash holdings to M1 increased from 30 percent to 86 percent. This fraction decreased to 56 percent in 2017, which is still almost two times the ratio in 1980.

Figure 4 shows the distribution of the cash-sales ratios for each year. The distributions look symmetric because the figure shows the logs of the cash-sales ratio. The support and the median of the distribution of the cash-sales ratio increased during the period. The support

<sup>11</sup>Before this date, it was higher than 1 briefly in 1999 and from 2004 to 2006.

of the distribution increased first and the median increased later. Figures 2 and figure 4 complement each other as they show that both aggregate corporate cash holdings and the distribution of corporate cash holdings changed substantially since 1980.<sup>12</sup>

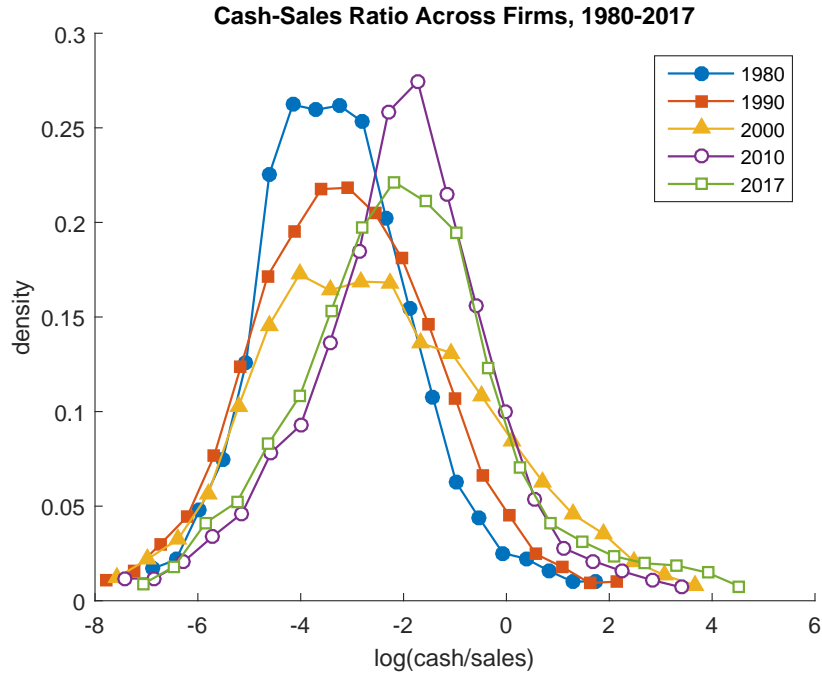


Figure 4: Distribution of the cash-sale ratio across firms from 1980 to 2017 for selected years. Each curve has the distribution for one year. The distributions are highly asymmetric; the curves look symmetric because the figure shows the logs of the cash-sales ratio. Over the years, the support and the median of the cash-sales ratio increased. Source: Compustat; see note 9 for details.

As figure 4 shows, the distribution of cash holdings across firms is not uniform; it is far from degenerate; and it has changed over time. Our objective is to calculate the predictions of the effects of monetary policy shocks under different distributions of cash holdings. In order to do so, we need a model that takes into account the different distributions of cash holdings. We introduce this model in section 4.

### 3 Discussion: The Increase in Firm Cash Holdings

We provide here a discussion based on the available literature on the reasons for the increase in the demand for money by firms. Firms hold money for three core motives: transactions, precaution, and store of value. The most important is the transactions motive, as the creation

<sup>12</sup>Figure 6 in section 5 shows the distributions of cash holdings for 1980, 2010, and 2017 together with the calibrated distributions.



of money itself is justified by the need to facilitate transactions. Firms need to make daily expenditures such as payments of wages and raw materials. As money is the best medium of exchange, firms hold money to facilitate these payments. The demand for money for transaction purposes depends upon income and the general level of business activity and the manner of the receipt of income.<sup>13</sup>

One of the main approaches to model the demand for money is to consider the problem of cash management as a problem of inventory management. This approach was introduced by Baumol (1952) and Tobin (1956), and later expanded by Miller and Orr (1966) and Frenkel and Jovanovic (1980).<sup>14</sup> According to this approach, an agent (firm or household) minimizes the cost of holding money as if money were an inventory item. As little or no interest is accrued to money, the opportunity cost of holding money is the higher interest that could be obtained with the funds held as money if these funds were used to purchase interest-bearing securities (we refer to money as cash, checking deposits and other highly liquid similar assets for which little interest is accrued). Interest-bearing securities cannot be used as a means of payment. Such securities are held until the inventory of money reaches a critical low level, which triggers the sale of a fraction of the interest-bearing securities to replenish money holdings. The optimal allocation of funds depends on the costs of selling interest-bearing securities for money and the opportunity cost of holding money.

The theory of money as inventory has two main predictions. One is that higher costs to exchange high-yielding securities for money imply higher money holdings, as replenishing the inventory of money would be made less frequent. In accordance with this prediction, Almeida et al. (2004) find that firms with more financial constraints maintain more liquid assets.<sup>15</sup> The second main prediction is that, as the opportunity cost of money decreases (that is, as the interest rate decreases), money holdings increase.

In fact, the increase in firm cash holdings can be related to the decrease in the nominal interest rates. The Aaa corporate bond yield decreased from 12% in 1980 to 3.7% in 2017

---

<sup>13</sup>Empirically, the different determinants of firm cash holdings are analyzed for instance by Kim et al. (1998) and Opler et al. (1999).

<sup>14</sup>A full general equilibrium version of the Baumol-Tobin model with optimizing infinitely-lived agents and production has been introduced by Silva (2012).

<sup>15</sup>More stringent financial constraints means that it is more difficult for firms to have access to different financial markets. This can be understood as access to the purchase of assets with higher yields, or the ability to transform the illiquid collateral of a firm into liquid funds.

and the Baa corporate bond yield for the same period decreased from 14% to 4.4%. The elasticity of real cash holdings with respect to yields is negative and highly significant. Figures C.1 and C.2 in the appendix shows corporate bond yields and various measures of real cash from 1980 to 2017 in a cross section and over time. Figure C.1, in particular, shows a strong negative relation between interest rates and cash holdings. In terms of the model, a decrease in interest rate allows firms to replenish money balances less frequently or, in an equivalent way, firms increase the interval between exchanges between illiquid and liquid assets. This action increases average money holdings.<sup>16</sup>

The inventory approach also implies that larger firms hold relatively less cash. That is, there are economies of scale in the transactions technology (smaller cash-sales ratio for larger firms). This prediction is confirmed in Mulligan (1997), Adão and Mata (1999), and Bover and Watson (2005). This is also in accordance with the evidence presented in figure 3.<sup>17</sup>

Taxes are another factor that can influence money holdings. According to Foley et al. (2007), U.S. tax on foreign earnings are equal to the difference between foreign income taxes paid and tax payments that would be due if foreign earnings were taxed in the U.S. These taxes can be deferred until earnings are repatriated. These taxes create incentives for U.S. multinationals to retain earnings abroad and hold the retained earnings as cash. Foley et al. find that the difference of taxation of foreign earnings can explain part of the increase in firm cash holdings. Faulkender et al. (2019) argue that the increase in aggregate cash of multinational firms is due to the steady decrease in foreign corporate tax rates.<sup>18</sup>

Firms might hold more money to safeguard against future uncertainties. This motive is called the precautionary motive. Balances held on account of the precautionary motive differ across firms according to their own degree of risk, access to credit and ease of conversion of financial assets into cash. For instance, Opler et al. (1999) and Kalcheva and Lins (2007) find that firms with riskier cash flows and poor access to external capital hold more cash.

---

<sup>16</sup>Azar et al. (2016) analyze further the relation between the opportunity cost of money and firm cash holdings. They conclude that the decrease of the opportunity cost was an important determinant of the long-run changes in U.S. corporate cash holdings. The opportunity cost of money decreased for two reasons: the interest rate ceilings on various types of deposit accounts were lifted starting in the late 1970s and early 1980s; there was a decline in inflation and in the T-bill rate.

<sup>17</sup>The inventory approach, more strictly, implies economies of scale if the fixed cost to exchange bonds for money is not proportional to the size of the firm.

<sup>18</sup>Armenter and Hnatkovska (2017) present a model in which the fiscal advantage of debt stimulates firms to hold financial assets.

They also find that firms with better investment opportunities hold more cash, which can be justified by the fact that adverse shocks and financial distress are more costly for them. In the same way, Acharya et al. (2007) find that firms hold more cash to hedge future investment against income fluctuations. Moreover, Acharya et al. (2013), Han and Qiu (2007) and Palazzo (2012) present evidence that firms hold more money in times of heightened aggregate risk.

Firms may also hold money to store value. A justification to use money as a store of value is asymmetry of information. Adverse selection and moral hazard make it more expensive for firms to obtain external funding. This can lead to distortions that generate underinvestment. Myers and Majluf (1984) and Jensen (1986) argue that information asymmetry implies a hierarchy in the financing alternatives. Firms finance themselves with internal resources before resorting to the market. Keeping liquid assets reduces the costs of being dependent on external financing. Dittmar et al. (2003), Pinkowitz et al. (2006), Dittmar and Mahrt-Smith (2007), Harford et al. (2008) and Yun (2009), study the relationship between corporate governance and holdings of liquid assets by firms.

Irvine and Pontiff (2008) and Bates et al. (2009) show that the increase in idiosyncratic risk moves together with an increase in cash flow volatility (Campbell et al. 2001 find a secular increase in idiosyncratic risk). These changes imply a higher volatility in non-diversifiable risks. As a result, a higher precautionary demand for cash holdings.

Zhao (2020) reports that there was a decrease in the correlation between revenues and operating expenses. Revenues in this case no longer act as a natural hedge for operating expenses. As a result, firms need more cash for the same volume of transactions. Falato et al. (2018) claim that the rise in intangible capital is important to explain the increase in corporate cash holdings. Since only tangible capital can be pledged as collateral, the rise in intangible capital decreases the debt capacity of firms and leads them to hold more cash. Gao (2018) argues that as firms adopt just-in-time production, they shift resources from inventory of goods to cash to facilitate transactions with suppliers.

As documented in the previous section and in the papers above, there was a large increase in firm cash holdings.<sup>19</sup> We report, in particular, changes in the distribution of firm cash

---

<sup>19</sup>Liquid holdings also increased among the general population. The ratio of M2 to GDP increased from 55% to more than 70% from 1980 to 2020. Even the currency component of M1 to GDP increased from 4% to 8% in the same period.

holdings. As the literature above shows, there are different explanations for the increase in cash holdings. For our purposes, it is not important the reason for the increase in firm cash holdings. What is important for us is that firms with different sizes of cash holdings react differently to a monetary policy shock. The difference in the reaction of firms will generate changes in the real interest rate.

## 4 The Model

The model combines the cash inventory framework of Baumol (1952) and Tobin (1956) with the market segmentation framework introduced by Grossman and Weiss (1983), Rotemberg (1984), and developed, among others, by Alvarez et al. (2009), Williamson (2009), and Silva (2012).

The economy is composed by heterogeneous infinitely lived entrepreneurs. Each entrepreneur owns a firm, which produces the consumption good. The entrepreneurs produce, consume, borrow and hold cash. They are heterogeneous with respect to sales, bond and cash holdings. The entrepreneurs smooth their consumption using cash and bonds. Unlike bonds, cash pays no interest, but consumption must be paid out with cash.

There is market segmentation between the goods market and the assets market. Each firm has a bank account and a brokerage account. The bank account is used to hold cash for transactions in the goods market. The brokerage account is used to hold bonds.<sup>20</sup> Market segmentation means that entrepreneurs periodically can sell bonds for money and transfer the proceeds from the assets market to the goods market, where they buy consumption goods.

As different types of firms in the economy have different average cash holdings, we allow for distinct groups of firms (or entrepreneurs) in the model, each one with a different holding period. In this way, we can match the distribution of firm cash holdings in the data, given by figure 4. The groups of firms are indexed by  $i = 1, \dots, I$ . The size of each group of firms is given by  $v_i$ , where  $\sum_{i=1}^I v_i = 1$ , and the holding period of the firms that belong to group  $i$  is denoted by  $N_i$ . The firms in group  $i$  are distributed uniformly over the interval  $[0, N_i)$ ,  $i = 1, \dots, I$ , with  $N_i < N_{i+1}$ ,  $i = 1, \dots, I - 1$ . Alvarez et al. (2009) also dispose agents uniformly

---

<sup>20</sup>The bonds should be interpreted as a generic asset that pays returns higher than the returns obtained with highly liquid risk-free assets.

over the holding period, but they consider only one group of firms, i.e., only one holding period size. In contrast, we allow for various holding periods with sizes  $\{N_i\}_{i=1}^I$ .

Time is continuous,  $t \geq 0$ . Let  $M_{0i}$  denote cash holdings at  $t = 0$  of firms in the group  $i$  and let  $B_{0i}$  denote bond holdings at  $t = 0$  of firms in the group  $i$ . Each firm,  $s_i$ , is identified by its initial portfolio, i.e.,  $s_i \equiv (M_{0i}, B_{0i})$ . Firms in group  $i$  produce  $Y_i$  goods at time  $t$  and obtain  $P(t)Y_i$  of sales at time  $t$ , where  $P(t)$  denotes the price level at time  $t$ . The proceeds of sales are deposited directly in the brokerage account and converted into bonds. The price at date 0 of a bond that pays one monetary unit at date  $t$  is given by  $Q(t)$ ,

$$Q(t) = e^{-\int_0^t r(s)ds} \equiv e^{-R(t)}, \quad (1)$$

where  $r(t)$  denotes the nominal interest rate at time  $t$ ;  $r(t) = -d \log Q(t)/dt$ . Let  $T_{ji}(s_i)$ ,  $j = 1, 2, \dots$ , denote the times of the transfers of firm  $s_i$ . Define  $T_{0i}(s_i) \equiv 0$  for all  $s_i$  to simplify notation. At  $T_{ji}(s_i)$ , firm  $s_i$  sells bonds for money and transfers the proceeds to the goods market (to its bank account). The  $j$ th holding period of firm  $s_i$  is  $[T_{j,i}(s_i), T_{j+1,i}(s_i))$ . We have  $T_{j+1,i} - T_{j,i} = N_i$ ,  $j = 1, 2, \dots$  for all firms in group  $i = 1, \dots, I$ .

Cash holdings are denoted by  $M(t, s_i)$ . The cash-sales ratio of firm  $s_i$  is then given by  $M(t, s_i)/(P(t)Y_i)$ . Cash just after a transfer is denoted by  $M^+(T_{ji}(s_i), s_i)$  and is equal to  $\lim_{t \rightarrow T_{ji}, t > T_{ji}} M(t, s_i)$ . Analogously, cash just before a transfer is denoted by  $M^-(T_{ji}(s_i), s_i)$  and is equal to  $\lim_{t \rightarrow T_{ji}, t < T_{ji}} M(t, s_i)$ . The transfer amount from the brokerage account to the bank account is given by  $M^+ - M^-$ . Similarly, bonds just before a transfer and just after a transfer are given by  $B^-(T_{ji}(s_i), s_i)$  and  $B^+(T_{ji}(s_i), s_i)$ , respectively. If the amount of cash transferred to the bank account is positive, then  $B^- > B^+$ . Cash holdings in the brokerage account are zero, as interest is not accrued to cash and it is not possible to purchase goods with the cash in the brokerage account. Firms keep bonds in the brokerage account and make periodical transfers to the bank account in order to make transactions in the goods market.<sup>21</sup>

We formalize the problem of entrepreneur  $s_i$ , who starts with assets  $(M_{0i}, B_{0i})$ , receives a flow of funds  $P(t)Y_i$  and aims to achieve an optimal amount of transactions  $c_i(t, s_i)$ . Because it simplifies the analysis, we assume that the objective function of the entrepreneur is logarithmic

---

<sup>21</sup>Adão and Silva (2020) consider cash and credit goods, which addresses the possibility of using illiquid assets to purchase goods. Here, we focus on the version with cash goods only to simplify the model and characterize the transition after shocks.

in the amount of transactions. The logarithmic utility allows us to obtain analytical solutions for the dynamics of the real interest rate after shocks.

The problem of entrepreneur  $s_i$  is to choose transactions  $c_i(t, s_i)$ , cash  $M_i(t, s_i)$ , and bonds  $B_i(t, s_i)$  such that

$$\max_{c_i, B_i, M_i} \sum_{j=0}^{\infty} \int_{T_{j,i}(s_i)}^{T_{j+1,i}(s_i)} e^{-\rho t} \log(c_i(t, s_i)) dt \quad (2)$$

subject to

$$M_i^+(T_{ji}(s_i)) + B_i^+(T_{ji}(s_i)) = M_i^-(T_{ji}(s_i)) + B_i^-(T_{ji}(s_i)), \quad j = 1, 2, \dots, \quad (3)$$

$$\dot{B}_i(t, s_i) = r(t)B_i(t, s_i) + P(t)Y_i, \quad t \geq 0, \quad t \neq T_{1i}(s_i), T_{2i}(s_i), \dots, \quad (4)$$

$$\dot{M}_i(t, s_i) = -P(t)c_i(t, s_i), \quad t \geq 0, \quad t \neq T_{1i}(s_i), T_{2i}(s_i), \dots, \quad (5)$$

with  $M_i(t, s_i) \geq 0$ ,  $c_i(t, s_i) \geq 0$ , given  $M_{0i}(s_i)$  and  $B_i(t, s_i)$ , and where  $\rho > 0$  is the rate of intertemporal discounting. At  $t = T_{1i}(s_i), T_{2i}(s_i), \dots$ , we have  $\dot{B}_i(T_{ji}(s_i), s_i)^+ = r(t)B_i^+(T_{ji}(s_i), s_i) + P(t)Y_i$ , where  $\dot{B}_i(T_{ji}(s_i), s_i)^+$  is the right derivative of  $B_i(t, s_i)$  with respect to time at  $t = T_{ji}(s_i)$ . Similarly, at  $t = T_{1i}(s_i), T_{2i}(s_i), \dots$ ,  $\dot{M}_i(T_{ji}(s_i), s_i)^+ = -P(t)c_i^+(T_{ji}(s_i), s_i)$ , where  $\dot{M}_i(t, s_i)^+$  is the corresponding right derivative for cash and  $c_i^+(T_{ji}(s_i), s_i)$  are transactions just after the transfer. The solution to this problem minimizes the cost of holding cash over holding periods.

Using (4), we can write  $B_i^-(T_{ji})$  as a function of the interest payments accrued during  $[T_{j-1}, T_j]$ . Substituting recursively in (3) and using the no-Ponzi condition  $\lim_{j \rightarrow +\infty} Q(T_j) \times B_i^+(T_{ji}) = 0$ , we obtain the present value budget constraint

$$\sum_{j=1}^{\infty} Q(T_{ji}(s_i)) M_i^+(T_{ji}(s_i), s_i) \leq \sum_{j=1}^{\infty} Q(T_{ji}(s_i)) M_i^-(T_{ji}(s_i), s_i) + W_{0i}(s_i), \quad (6)$$

where  $W_{0i}(s_i) \equiv B_{0i}(s_i) + \int_0^{\infty} Q(t)P(t)Y_i dt$ . Constraint (6) states that the present value of cash transfers is equal to initial bonds plus the present value of deposits in the brokerage account.

To minimize the cost of holding money, firms make transfers and use cash during the holding periods so that  $M_i^-(T_{j+1,i}) = 0$ . Cash transfers are just enough for the transactions during the holding period. Only  $M_i^-(T_{1,i})$  might be positive because  $M_{0i}$  is given. As  $M_i^-(T_{ji}) = 0$ ,

for  $j \geq 2$ , then, from (5), cash at time  $t$  is given by  $M_i(t, s_i) = \int_t^{T_{j+1,i}(s_i)} P(\tau) c_i(\tau, s_i) d\tau$ , for  $T_{ji}(s_i) \leq t < T_{j+1,i}(s_i)$ ,  $j = 1, 2, \dots$ . Cash at the beginning of a holding period is given by

$$M_i^+(T_{ji}(s_i), s_i) = \int_{T_{j,i}(s_i)}^{T_{j+1,i}(s_i)} P(\tau) c_i(\tau, s_i) d\tau, \quad j = 1, 2, \dots \quad (7)$$

Below, instead of solving the problem of maximizing (2) subject to (3)–(5), we consider the simpler problem of maximizing (2) subject to the cash in advance constraint for the first period

$$\int_0^{T_{1,i}(s_i)} P(\tau) c_i(\tau, s_i) d\tau + M_i^-(T_{1i}(s_i)) \leq M_{0i}(s_i), \quad (8)$$

and to (6), where  $M_i^+(T_{ji}(s_i), s_i)$  is replaced by the right hand side of (7). The transactions,  $c_i(t, s_i)$  and cash  $M_i^-(T_{1i}(s_i))$  that solve this simpler problem can be replaced in (3) and (4) to obtain bonds  $B_i(t, s_i)$ .

The government executes monetary policy through open market operations in the assets market. The government supplies aggregate cash  $M(t)$ . An increase in the supply of cash generates revenue  $\dot{M}(t)/P(t)$ . We abstract from government consumption or taxes to concentrate on the effects of monetary policy. The government budget constraint is therefore given by  $B_0^G = \int_0^\infty Q(t) \dot{M}(t) dt$ , where  $B_0^G$  is the aggregate supply of government bonds.

As long as there is a positive opportunity cost of holding cash, it is optimal to start a holding period with cash and spend it gradually until the next transfer, which initiates a new holding period. Firms engage in  $(S, s)$  policies on bonds and cash. This behavior implies a nondegenerate distribution of cash holdings across firms at each point in time. The aggregate variables are obtained through the aggregation of the  $(S, s)$  policies across firms or, which is equivalent, through the aggregation of the distribution of the variable of interest across firms.

The market clearing condition for cash is given by  $\sum_i v_i \int M_i(t, s_i) dF_i(s_i) = M(t)$ , where  $F_i$  is the distribution of  $s_i$ . Similarly, the market clearing conditions for bonds and goods are given by  $B_0^G = \sum_i v_i \int B_{0i}(s_i) dF_i(s_i)$  and  $\sum_i v_i \int c_i(t, s_i) dF_i(s_i) = Y$ , respectively.

The equilibrium is defined as a vector of prices  $\{P(t), Q(t)\}$ , and allocations  $\{M_i(t, s_i), B_i(t, s_i), c_i(t, s_i)\}_{i=1}^I$  such that  $\{M_i(t, s_i), B_i(t, s_i), c_i(t, s_i)\}_{i=1}^I$  solves the maximization problems (2)–(5) given  $\{P(t), Q(t)\}$  for all  $s_i$  in the support of  $F_i(s_i)$ ; the government budget constraint holds; and the market clearing conditions for cash, bonds, and goods hold.

## 5 The Distribution of Firm Cash Holdings

We now characterize the distribution of cash holdings across firms in equilibrium. To characterize the distribution, we focus on the steady state; the equilibrium in which the inflation rate and the interest rate are constant. In the next section, we study the effects of a monetary shock when the economy is initially in the steady state.

For constant inflation and interest rate, the  $(S, s)$  policies of the firms in each group have the same pattern. At time  $t$ , although the cross section of firms show different cash holdings, the distribution of cash holdings repeats itself over time. The relevant variable to identify a firm is its position in the holding period. Let  $n_i \in [0, N_i)$  denote the position of a firm of group  $i$  in the holding period. Firm  $n_i$  makes transfers from the brokerage account to the bank account at  $T_{1,i}(n_i) = n_i$ ,  $T_{2,i}(n_i) = n_i + N_i$  and so on.

Consider the pattern of transactions for each firm. As shown in section B.1 in the appendix, the first order condition for  $c_i(t, n_i)$  of the problem of maximizing (2) subject to (6) and (8) implies

$$P(t)c_i(t, n_i) = \frac{e^{-\rho t}}{\lambda_i(n_i)Q(T_j)}, \quad (9)$$

$t \in (T_{j,i}(n_i), T_{j+1,i}(n_i))$ ,  $j \geq 1$ , where  $\lambda_i(n_i)$  is the Lagrange multiplier of (6). Let  $c_{0i}$  denote transactions at the beginning of a holding period for firms in group  $i$ . In the steady state, prices  $P$  increase at the rate  $\pi$  and bond prices  $Q$  decrease at the rate  $r$ . Therefore, transactions during holding periods of firms in group  $i$  can be written as  $c_i(t, n_i) = c_{0i}e^{(r-\pi-\rho)t}e^{-r(t-T_{j,i})}$ , for  $j$  such that  $t \in [T_{j,i}(n_i), T_{j+1,i}(n_i))$ . By integrating the expression of  $c_i(t, n_i)$  across firms, as shown in the appendix, we obtain aggregate transactions of the firms in group  $i$ ,

$$C_i(t) = c_{0i}e^{(r-\pi-\rho)t} \frac{1 - e^{-rN_i}}{rN_i}. \quad (10)$$

In the steady state, aggregate transactions are constant. We then obtain the familiar relation  $r = \rho + \pi$ . That is, the nominal interest rate is equal to the intertemporal rate of discount plus the inflation rate in a steady state equilibrium. Substituting in (10) yields

$$C_i(t) = c_{0i} \frac{1 - e^{-rN_i}}{rN_i}. \quad (11)$$



The last term in the right of (11) occurs because all firms try to avoid the opportunity cost of money. Individually, each firm sets  $c_i(t, n_i) = c_{0i}e^{-r(t-T_{j,i})}$  within holding periods. Transactions decrease at the rate  $r$ . They decrease from  $c_{0i}$  to  $c_{0i}e^{-rN_i}$  within holding periods. Although individual firms have this staggered policy for transactions, aggregate transactions are constant over time, given by (11).<sup>22</sup>

Transactions during holding periods must be equal to the cash generated by sales during the same holding period,  $\int_{T_{j,i}}^{T_{j+1,i}} c_i(t, n_i)dt = \int_{T_{j,i}}^{T_{j+1,i}} Y_i dt$ , where  $T_{j+1,i} - T_{j,i} = N_i$ . Substituting the expression of  $c_i(t, n_i)$  into the integral, given  $r = \rho + \pi$ , yields the value of transactions at the beginning of a holding period,  $c_{0,i}(1 - e^{-rN_i})/(rN_i) = Y_i$ . As we will parameterize the model using data on cash-sales ratios, it is useful to characterize the variable transactions-sales ratio. Let  $\hat{c}_i \equiv c_i/Y_i$  denote the transactions-sales ratio of firms in group  $i$ . We then have

$$\hat{c}_{0,i} \frac{1 - e^{-rN_i}}{rN_i} = 1, \quad (12)$$

which determines  $\hat{c}_{0,i}(r)$  given  $N_i$  and  $r$ . The transactions-sales ratio for  $t \in [T_{j,i}(n_i), T_{j+1,i}(n_i))$  of firms in group  $i$  is then given by  $\hat{c}_i(t, n_i) = \hat{c}_{0,i}e^{-r(t-T_{j,i})}$ .

Aggregate cash holdings are equal to  $M(t) = \sum_i v_i \frac{1}{N_i} \int M_i(t, n_i) dn_i$ , where  $M_i(t, n_i) = \int_t^{T_{j+1,i}(n_i)} P(\tau) c_i(\tau, s_i) d\tau$ , and the aggregate cash-sales ratio is  $m = M(t)/(P(t)Y)$ , which is constant at the steady state as aggregate cash holdings grow at the same rate as inflation. In appendix B, we show that the aggregate cash-sales ratio in this economy is given by

$$m = \frac{1}{Y} \sum_{i=1}^I v_i \frac{c_{0,i}}{\rho} e^{-rN_i} \left[ \frac{e^{rN_i} - 1}{rN_i} - \frac{e^{(r-\rho)N_i} - 1}{(r-\rho)N_i} \right]. \quad (13)$$

The price level at time zero is equal to  $P_0 = M_0/(mY)$ , where  $M_0$  denotes the money supply at time zero and  $Y$  denotes aggregate sales.

The cash-sales ratios of the firms in group  $i$  are given by  $m_i(n_i) = M_{0,i}(n_i)/(P_0 Y_{0,i})$ ,  $n_i \in [0, N_i)$ , where  $M_{0,i}(n_i)$  is the initial cash holdings for each firm. The  $M_{0,i}(n_i)$  compatible with an equilibrium where  $r$  and  $\pi$  are constant is obtained by requiring that  $M_{0,i}(n_i)$  is just

---

<sup>22</sup>Segniorage obtained with the depreciation of money holdings in the bank account is returned to firms. In terms of welfare, however, the staggered behavior of individual firms creates distortions. As  $r$  increases, the concentration of spending at the beginning of holding periods is more accentuated. This variation of spending decreases welfare. For further analysis of the welfare cost of inflation in related economies, see Silva (2012) and Adão and Silva (2020).

enough to cover transactions from  $t = 0$  until the first transfer of firm  $n_i$ , at  $T_{1,i}(n_i) = n_i$ . The  $M_{0,i}(n_i)$ , for  $n_i \in [0, N_i)$ , are determined in appendix B. After dividing by  $P_0 Y_i$ , we obtain the cash-sales ratio of firms  $n_i \in [0, N_i)$ ,

$$m_i(n_i) = \frac{rN_i e^{-rN_i} n_i e^{rn_i} (1 - e^{-\rho n_i})}{1 - e^{-rN_i} \rho n_i}. \quad (14)$$

The cash-sales ratios  $m_i(n_i)$  have a distribution with support  $[0, m_{H_i})$ , where  $m_{H_i} = \lim_{n_i \rightarrow N_i} m_i(n_i)$ . As firms are distributed uniformly along  $[0, N_i)$ , the density of the cash-sales ratios is given by

$$f_i(x) = \frac{1}{N_i} \frac{\partial m_i^{-1}(x)}{\partial x}, \quad (15)$$

where  $m_i^{-1}(x)$  is the value of  $n_i$  such that  $m_i(n_i) = x$ . There exists a unique value of  $m_i^{-1}(x)$ , as  $m_i(n_i)$  is strictly increasing. Therefore, from (14) and (15), the distribution of the cash-sales ratio across firms is given by

$$f_i(x) = \frac{1}{N_i} \left[ rx + \frac{rN_i}{1 - e^{-rN_i}} e^{-rN_i} e^{(r-\rho)m_i^{-1}(x)} \right]^{-1}, \quad m_i \in [0, m_{H_i}). \quad (16)$$

For the aggregate firms in the economy, the density function is  $f(x) = \sum_i v_i f_i(x) dx$ , where  $v_i$  is the fraction of firms distributed along  $[0, N_i)$ , which ensures that  $\int f(x) dx = 1$ . At any moment in time, the cross section of the cash-sales ratio is given by  $f(x)$ ,  $x \in [0, \max(m_{H_i})$ .

## 6 Parameterization

In the data, the distribution of cash holdings is concentrated on small quantities. This distribution can be approximated by a weighted combination of the distributions in (16). The parameterization is facilitated by the fact that the expression inside brackets in (16) is approximately constant, which implies that  $f_i(x) \approx \frac{1}{N_i}$  and so  $f_i(x)$  approximates a Uniform distribution. As the different firm groups in the model overlap, the distribution of cash holdings appears as shown in figure 5. As in the data, the model implies a distribution of cash-sales ratios more concentrated on small values.<sup>23</sup>

<sup>23</sup>The values of the parameterization are found from the largest to the smallest cash-sales ratio. For example, following figure 5, the highest value for the cash-sales ratio in the data implies  $m_{H_4}$  and a corresponding small value for  $v_4$ , as the height of the distribution from 0 to  $m_{H_4}$  is small. Then the values for  $m_{H_3}$  and  $v_i$  are found given that only firms in group  $i = 3$  will have cash holdings between  $m_{H_2}$  and  $m_{H_3}$ . This is done until

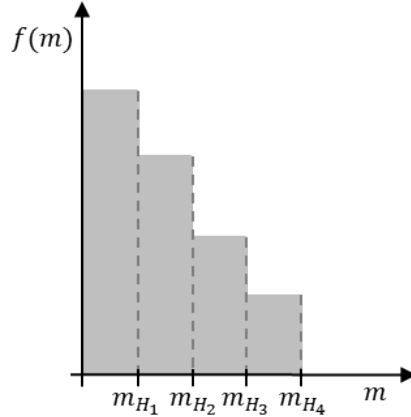


Figure 5: The parameterization is made by finding the values of  $m_{H_i}$  and  $v_i$ ,  $i = 1, \dots, I$ , so that the distribution of cash-sales ratios in the model approximates the distribution in the data. Here,  $I = 4$  to simplify the exposition.  $I$  is set to 50 in the simulations.

In the parameterization, the values of  $v_i$  and  $m_{H_i}$  are set so that the model distribution of the cash-sales ratio,  $f(x) = \sum_i v_i f_i(x) dx$ , approximates the distribution of the data (equation 16 is directly used for the parameterization, not its approximation to a uniform distribution). We use the annual interest rate and cross sectional data on cash-sales ratios to obtain a distribution for each of the years in the period from 1980 to 2017. The value of  $\rho$  is taken to be 3%, and for each year, the nominal interest rate,  $r$ , is the commercial paper interest rate of that year. The values of  $m_{H_i}$  and  $v_i$  are found to match the actual distributions of the cash-sales ratios. We set  $I = 50$  and for every year we consider the largest cash-sales ratio, which we denote as  $m_{H_{50}}$ . For every year we determine the set of equally spaced points  $\{m_{H_1}, m_{H_2}, \dots, m_{H_{50}}\}$  in the interval  $[0, m_{H_{50}}]$ . For each  $m_{H_i}$  we obtain  $N_i$  using equation (14). Once we have the  $N_i$  we get the  $f_i(x)$  using equation (16). The values  $\{v_1, v_2, \dots, v_{50}\}$  are chosen so that  $f(x) = \sum_i v_i f_i(x) dx$  closely approximates the data distribution of the cash-sales ratio and  $\sum_i v_i = 1$ .

Figure 6 shows the actual and the parameterized distributions for the years 1980, 2010, and 2017. For better visualization figure 6 uses 20 bins for the data histogram and the parameterized histogram. The distribution of the levels looks like figure 4 but it is highly asymmetric; for this reason, it is more convenient to plot the distribution of the logarithm of the cash-sales ratio.<sup>24</sup>

---

the values for  $m_{H_1}$  and  $v_1$  are found. The superimposition of the distributions for each group of firms implies the asymmetric distribution of firm cash holdings.

<sup>24</sup>For the calibration, we set  $I = 50$ . The increase in cash holdings from 1980 to 2017 is not homogeneous

In the 1980's, with higher interest rates, the values of  $m_{H_i}$  are smaller, as firms during those years held less cash holdings. For more recent years, with lower interest rates, the values of  $m_{H_i}$  are higher, as firms have larger cash holdings. These implications of the model, smaller cash holdings in the 1980's and the larger cash holdings for more recent years, are in accordance with what we observe in the data. In the next section, we simulate an interest rate shock that hits the economy given the distribution of cash holdings. In the more recent years, as firms have larger values of cash holdings, the holding periods increased and the segmentation effect is larger. As a result, a nominal interest rate shock that hits the economy with its current configuration has stronger real effects.<sup>25</sup>

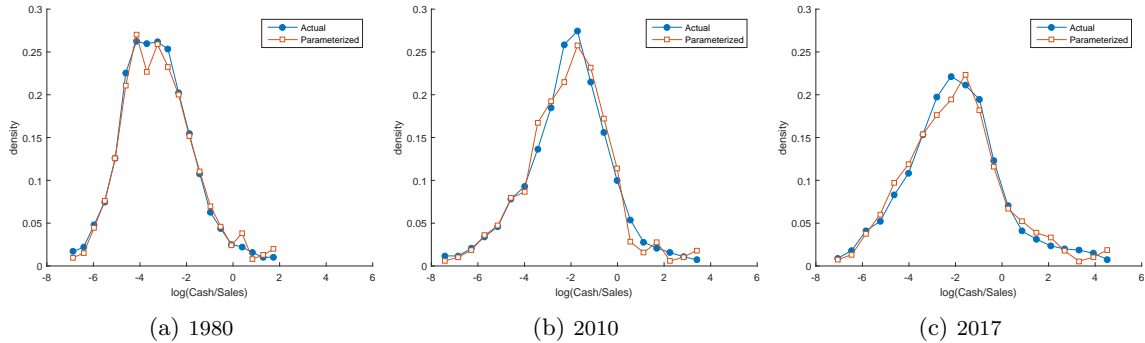


Figure 6: Actual distribution and parameterization of the cash-sales ratio for 1980, 2010, and 2017. The same pattern is obtained for all years 1980–2017. The distribution is parameterized for each year.

## 7 Monetary Policy Shocks

Let  $t = 0$  be the time of the interest rate shock. When the shock hits the economy, firms have different cash holdings. Firms with little cash, about to make a transfer, adapt to the shock promptly because they make transfers and rebalance their portfolios soon after the shock.

---

across firms. This implies changes in the parameters and in the distribution of cash holdings over time. As explained in the next section, instead of  $v_i$ , we use the fraction of sales of firms in group  $i$  with respect to total sales,  $v_{Y_i}$ , to obtain a counterpart with the data on cash-sales. Table C.1 in the appendix shows the values of  $v_{Y_i}$  and  $N_i$  for 1980 and 2010. The other years have a similar configuration. Figure C.3 shows the implied average holding period over time given by the values of  $N_i$  weighted by  $v_{Y_i}$  for each year. The data on cash-sales from 1980 to 2017 implies that the average  $N_i$  increases from 25 days to 119 days. In comparison, Alvarez et al. (2009) consider  $N$  between 24 to 36 months. Large cash holdings usually imply large holding periods in market segmentation models. See Silva (2012) for a discussion.

<sup>25</sup>In terms of the discussion in section 3, various changes in the economy such as financial constraints and production methods are summarized by larger costs of exchanging bonds for money which, in turn, would imply larger holding periods and larger segmentation. For our purposes, what is important is that the larger cash holdings imply a higher degree of segmentation.

Firms with large cash holdings take longer to make their transfer. They can adjust partially to the shock before rebalancing by managing their existing cash holdings at the moment of the shock, but they can only adjust fully to the shock when they rebalance their portfolios. Christiano et al. (1996) present evidence that firms take time to adjust their portfolios after an interest rate shock. Adjustments are not instantaneous.

The different reactions of the firms affect the real interest rate. After an increase in the nominal interest rate, the gradual reaction of firms make the price level move slower than if there were no market segmentation. As the real interest rate is equal to the difference between the nominal interest rate and the rate of inflation, the real interest rate increases together with the nominal interest rate just after a positive shock.<sup>26</sup>

The monetary policy is summarized by the nominal interest rate path  $r(t)$ ,  $t \geq 0$ . Since a change in  $r(t)$  affects cash holdings of firm  $n_i$ ,  $M_i(t, n_i)$ , when setting the interest rate path, the central bank has to adjust the money supply accordingly. The central bank supplies  $M(t)$  to satisfy the market clearing condition for cash. The interest rate path determines bond prices  $Q(t) = e^{-R(t)}$ , where  $R(t) = \int_0^t r(s)ds$ .

In the model, it is equivalent to set  $M(t)$  and obtain the equilibrium  $r(t)$  or to set  $r(t)$  and obtain the equilibrium  $M(t)$ . However, it is computationally simpler to set  $r(t)$  to obtain the equilibrium prices through the relevant equilibrium equations. Moreover, the evidence suggests that the practice of central banks is to set monetary policy through the interest rate. By focusing on  $r(t)$  as the target for the monetary policy, we follow the literature; for example, Woodford (2003).

Let  $\theta(t)$  denote the real interest rate and  $\pi(t)$  denote the rate of inflation,  $\pi(t) \equiv \dot{P}(t)/P(t)$ . The real interest rate at each time  $t$  is given by  $\theta(t) = r(t) - \pi(t)$ ,  $t \geq 0$ . To obtain  $\theta(t)$ , we have to determine the price level at each time  $P(t)$ . The shock occurs when the economy is in a steady state with constant interest rate  $r$  and constant inflation  $\pi$ . Before the shock, the real interest rate is  $\rho$  and  $r = \rho + \pi$ . Cash and bond holdings of firm  $n_i$  at the time of the shock,  $M_{0,i}(n_i)$  and  $B_{0,i}(n_i)$ , are the steady state values corresponding to the nominal interest rate  $r$ . These cash holdings represent the choices before the shock.

---

<sup>26</sup>A slow response of prices and an increase in the real interest rate after an increase in the nominal interest rate is found in many empirical studies. Among others, Cochrane (1994) and Christiano et al. (1999), Kahn et al. (2002), Bernanke et al. (2005), and Uhlig (2005).

The problem of the entrepreneur is extended to take into account two states of nature. In one state of nature, the economy continues in the steady state. In the other state of nature, the nominal interest rate follows  $r(t)$ . The new interest rate path  $r(t)$  represents the monetary policy shock. The state with the monetary policy shock has a small probability of occurrence, which allows us to approximate the state without the shock to the steady state. As Grossman (1987), we assume that bonds  $B_{0,i}(n_i)$  are contingent to the shock, but cash is not. The problem with state contingent bonds is stated and solved formally in appendices A and B.<sup>27</sup>

The equilibrium price level is obtained through the market clearing condition for goods. At time  $t$ , not long after the shock, there will be firms in each group  $i$  that have made a transfer already, and other firms that have not made a transfer yet. Firms that have not made the transfer yet, must do transactions using what is left out of  $M_{0,i}(n_i)$ . Firms that have already made the transfer are firms with smaller values of  $n_i \in [0, N_i)$ , as they make the first transfer at  $T_{1,i} = n_i$ . Aggregate transactions for all firms in group  $i$  are given by

$$C_i(t) = \frac{1}{N_i} \int_0^t \frac{e^{-\rho t}}{\lambda_i(n_i)Q(T_{1,i}(n_i))P(t)} dn_i + \frac{1}{N_i} \int_t^{N_i} \frac{e^{-\rho t}}{\mu_i(n_i)P(t)} dn_i, \quad 0 \leq t < N_i, \quad (17)$$

where  $\lambda_i(n_i) = 1/(P_0 c_{0,i}(n_i))$  is the Lagrange multiplier associated with the intertemporal budget constraint and  $\mu_i(n_i)$  is the Lagrange multiplier associated with the cash in advance constraint of the first period; the value of  $\mu_i(n_i)$  depends on  $M_{0,i}(n_i)$  and it is determined in the appendix. The second term in the right hand side of equation (17) explains most of the transactions when  $t$  is close to zero; and the first term determines most of the transactions when  $t$  is close to  $N_i$ . The interpretation is that when most firms have not yet reacted to the shock, i.e.,  $t$  is close to zero, the value of  $\mu(n_i)$  is important to determine consumption and ultimately to determine prices.

Proposition 1 describes the solution for prices obtained from equation (17). The equilibrium price path  $P(t)$  is obtained by equating aggregate demand,  $\sum v_i C_i(t)$ , to aggregate sales,  $Y$ . The logarithm utility allows us to solve for  $P(t)$ . As we use data on cash-sales ratios, we rewrite the equation in terms of the transactions-sale ratio,  $\hat{c}_{0,i}$ , and the fraction of sales of firms in group  $i$  with respect to total sales, which we denote by  $v_{Y_i}$ .<sup>28</sup> The logarithmic function

<sup>27</sup>The government budget constraint is restored as bonds are contingent on the shock.

<sup>28</sup> $v_i \equiv n_i / \sum n_i$  is the fraction of firms in group  $i$  whereas  $v_{Y_i} \equiv n_i Y_i / Y$  is the fraction of sales in group  $i$ . Both  $\sum v_i = \sum v_{Y_i} = 1$ . We have  $v_{Y_i} = v_i Y_i / \bar{Y}$ , where  $\bar{Y} = Y / \sum n_i$ . The advantage of using  $v_{Y_i}$  is that it

simplifies the analysis. Apart from the integral in  $e^{R(n)}$ , which can be solved efficiently with numerical methods, we have closed-form solutions for the price level for any  $r(t)$ .<sup>29</sup>

**Proposition 1** (Prices after shocks). *The equilibrium price level  $P(t)$  after a nominal interest rate shock with path  $r(t)$ ,  $t \geq 0$ , is given by*

$$P(t) = \sum_{i=1}^I v_{Y_i} \frac{P_0}{N_i} k \hat{c}_{0,i} e^{-\rho t} \left[ \int_0^t e^{R(n_i)} dn_i + \frac{1 - e^{-r(N_i-t)}}{r N_i} \right], \quad 0 \leq t < N_1, \quad (18a)$$

$$P(t) = v_{Y_1} \frac{P_0}{N_1} k \hat{c}_{0,1} e^{-\rho t} \int_{t-N_1}^t e^{R(n_1)} dn_1 \\ + \sum_{i=2}^I v_{Y_i} \frac{P_0}{N_i} k \hat{c}_{0,i} e^{-\rho t} \left[ \int_0^t e^{R(n_i)} dn_i + \frac{1 - e^{-r(N_i-t)}}{r N_i} \right], \quad N_1 \leq t < N_2, \quad (18b)$$

⋮

$$P(t) = \sum_{i=1}^I v_{Y_i} \frac{P_0}{N_i} k \hat{c}_{0,i} e^{-\rho t} \int_{t-N_i}^t e^{R(n_i)} dn_i, \quad t \geq N_I, \quad (18c)$$

where  $k \equiv \bar{Y}/Y$  and  $R(n_i) \equiv \int_0^{n_i} r(s) ds$ .<sup>30</sup>

As  $M_{0,i}(n_i)$  are cash holdings in the initial steady state, they can be too large for the new interest rate path  $r(t)$ . After the shock, firms may choose, to make a transfer before their cash vanishes, i.e.,  $M_i^-(T_{1,i}(n_i), n_i) > 0$ . In the proof of proposition 1, we show that this does not happen, i.e.,  $M_i^-(T_{1,i}(n_i), n_i) = 0$  for any  $r(t)$ . When there is a shock, firms adapt to the shock by changing transactions rather than choosing  $M_i^-(T_{1,i}(n_i), n_i) > 0$ .

Proposition 1 implies that monetary policy affects real interest rates. According to the Fisher relation  $\theta(t) = r(t) - \pi(t)$ , the real interest rate changes after a nominal interest rate shock if inflation and the nominal interest rate move differently after the shock. In a standard cash-in-advance model,  $\pi(t)$  changes instantaneously after a shock to  $r(t)$  and so  $\theta(t)$  remains constant. Here,  $\pi(t)$  remains constant just after the shock and changes gradually because of the market segmentation. As a result, the real interest rate increases with the increase in the

---

allows the characterization of the equilibrium variables in terms of the cash-sales ratio.

<sup>29</sup>In particular, as we can isolate  $P(t)$ , we don't need to assume an arbitrary initial path for the price level  $P^0(t)$ ,  $t \in [0, +\infty)$ , and iterate  $P^j(t)$  until convergence. This would greatly slow down the solution. The Lagrange multipliers are nominal variables that allow the determination of a unique equilibrium. See Adão et al. (2011) for more on the uniqueness of equilibrium with interest-rate rules.

<sup>30</sup>The constant  $k$  appears because we write (18) in terms of variables with respect to sales ( $v_{Y_i}$  and  $\hat{c}_0$  instead of  $v_i$  and  $c_0$ ). This constant does not affect our results because we use (18) to calculate the inflation rate,  $\pi(t) = \dot{P}(t)/P(t)$ .

nominal interest rate.

We determine the effects of market segmentation using (18). Suppose, for example, that the shock is a permanent increase of the nominal interest rate from  $r_1$  to  $r_2$ . Before the shock, inflation is equal to  $r_1 - \rho$  and the real interest rate is equal to  $\rho$ . We have  $e^{R(t)} = e^{r_2 t}$ . Solving for  $\dot{P}(t)/P(t)$ , we obtain that inflation just after the shock is equal to  $r_1 - \rho$ , its value before the shock, and the real interest rate increases to  $\rho + r_2 - r_1$ . For  $t \geq \max(N_i) = N_I$ , we have  $P(t) = ke^{(r_2 - \rho)t}$ , where  $k$  is a positive constant. Only after  $N_I$ , when all firms have made their first bond trade after the shock, does inflation increase to  $r_2 - \rho$ . If  $N_I$  is a large number, it will take longer for inflation to converge to its value at the new steady state and the effects on the real interest rate will be more prolonged. A nominal shock, however, cannot affect real variables indefinitely. As time goes by, the real interest decreases gradually to its steady state value,  $\rho$ .

Proposition 2 establishes these two results: (1) just after the shock the real interest rate increases by the same amount of the nominal interest rate and (2) the real interest rate does not move if there is no market segmentation.

**Proposition 2** (Slow reaction of prices). *For any interest rate path  $r(t)$  announced at time  $t = 0$ , the price level and the inflation rate do not move just after the shock, that is,  $P(0) = P_0$ ,  $\pi(0) = r - \rho$ , where  $r$  is the steady state nominal interest rate. Moreover, the change in the real interest rate at  $t = 0$  is equal to the change in the nominal interest rate,  $\theta(0) - \rho = r(0) - r$ . If  $N_i \rightarrow 0$ , for all  $i = 1, \dots, I$ , the real interest rate is constant and equal to  $\rho$  for any  $r(t)$  and all  $t \geq 0$ .*

Consider now a monetary policy shock as the one estimated by Uhlig (2005). According to figure 2, plot 6, in Uhlig, reproduced in figure 7, a monetary policy shock, described as an increase in the federal funds rate, initially increases the interest rate 0.3 percentage points and gradually decreases the interest rate towards its initial value. On average, the interest rate returns to its initial value in about 2 years and stays below its initial value for some time until it returns to zero.

We approximate this shock with the process for the interest rate given by

$$r(t) = r_1 + (r_2 - r_1 + Bt)e^{-\eta t}, \quad (19)$$



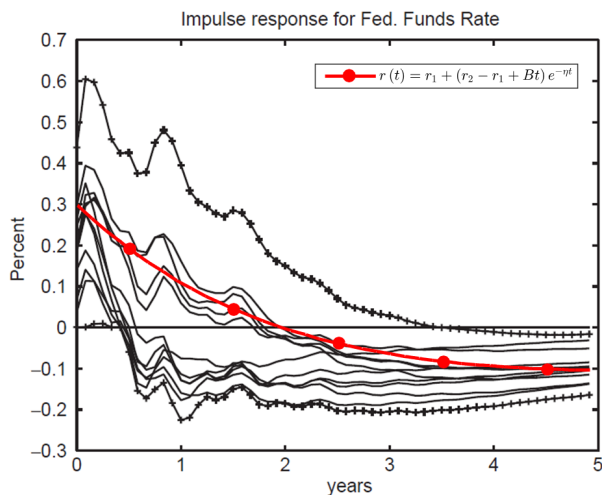


Figure 7: Process for the nominal interest rate path,  $r(t) = r_1 + (r_2 - r_1 + Bt)e^{-\eta t}$ , with the parameters  $B$  and  $\eta$  used in the simulations,  $B = -0.15\%$  and  $\eta = 0.30$ , for  $r(t)$  given in percentage per year. The parameters  $B$  and  $\eta$  were chosen to approximate the impulse-response function for the monetary policy shock estimated in Uhlig. This figure reproduces Uhlig (2005), Fig 2, plot 6, with the process  $r(t)$  for the nominal interest rate path added to the figure.

also depicted in figure 7, where  $r_2 - r_1 = 0.3$  percentage points per year. We use the results in Uhlig to set  $B$  and  $\eta$  so that  $r(t)$  approximates the estimated average impulse response of the federal funds rate. We set  $\rho = 3$  percent per year. The estimation in figure 7, as stated in Uhlig, uses a range of OLS estimates of a VAR. We later use different estimates for the monetary policy shock for comparison.<sup>31</sup>

Given the distributions of the cash-sales ratio from 1980 to 2017, we hit the economy with the shock  $r(t)$ , given by (19), and obtain the real interest rate path using proposition 1. As explained in section 5, the cash-sales distribution for each year is obtained by determining the values of  $v_{Y_i}$  and  $N_i$  so that the distribution of the cash-sales ratio from the model approximates the actual distribution of the cash-sales ratio given by Compustat data. According to proposition 1, the real interest rate  $\theta(t)$  implied by the shock to  $r(t)$  depends on the distribution of the cash-sales ratio across firms. The paths for the real interest rates for each year are our predicted effects of shocks to  $r(t)$  given the distributions of the cash-sales ratio.

Figure 8 shows the resulting dynamics of the real interest rate obtained from the model

<sup>31</sup>The expression of  $r(t)$  is the result of the differential equation  $m\ddot{r}(t) + c\dot{r}(t) + kr(t) = 0$ ,  $\eta = c/(2m)$ , which describes a dampened shock. We set  $r_1 = 3\%$  p.a. and  $r_2 = 3.3\%$  p.a. To approximate the shock, we target  $\eta$  and  $B$  so that  $r(t_1) = r_1$  and  $r(t_2) = r_1 - r_\Delta$ . For the OLS estimate,  $t_1 = 2$  years,  $t_2 = 5$  years and  $r_\Delta = 0.1\%$ . Figure 7 expresses the results as the difference from the initial value of the nominal interest rate. In our simulations,  $t$  denotes one day and we divide the year in 360 days.  $B = -0.15\%$  and  $\eta = 0.30$ , for  $r(t)$  given in percentage per year.

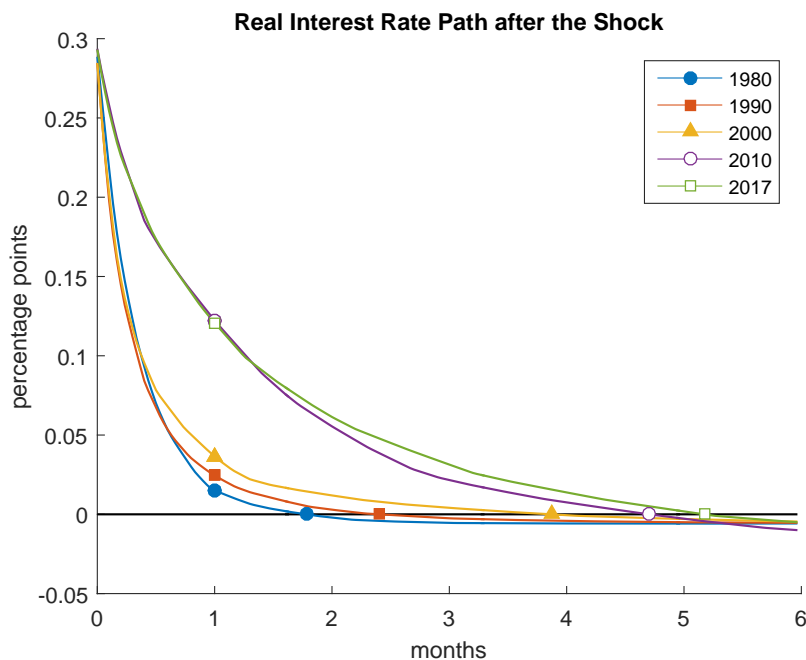


Figure 8: Response of the real interest rate for selected years given the nominal interest rate shock of figure 7. Results from simulations. The distribution of cash holdings is determined with data for each year. The markers in the horizontal axis show the time that it takes for the real interest rate to return to its initial value. The values are 1.78, 2.40, 3.87, 4.70, 5.17 months for the selected years. Figure 1 shows the values for all years.

given the cash-sales distributions for a selection of years. The vertical axis shows the difference in percentage points from the initial real interest rate. A standard cash-in-advance model would imply a horizontal line for the dynamics of  $\theta(t)$  after the shock,  $\dot{\theta}(t) = 0$ , as the standard cash-in-advance model implies an instantaneous reaction of prices and no change in real interest rates. Here, with market segmentation, the real interest rate increases after the nominal interest rate shock and returns gradually to its initial value.

We measure the effect of monetary policy by the time that it takes for the real interest rate to reach its initial value. Denote this value by  $t^*$ . In figure 8, we have, for example, that  $t^* = 1.78$  months given the cash distribution of 1980. Given the cash distribution of 2017, we have  $t^* = 5.17$  months. The values of  $t^*$  for all years from 1980 to 2017 obtained through the simulations are in figure 1.<sup>32</sup>

In addition to the time that it takes for the real interest rate to return to its initial value,  $t^*$ , figure 9 shows two additional measures to quantify the persistence of the monetary policy

<sup>32</sup>Adão et al. (2004) also evaluate the strength of monetary policy through the effects on the real interest rate.

shock. It might be case that  $t^*$  is large for a certain year, but the real interest rate is close to its initial equilibrium value during the transition. To circumvent this possibility, we calculate the area under the dynamics of the real interest rate after a monetary policy shock for each year (the area under the impulse-response functions, as shown in figure 8, from  $t = 0$  to  $t = t^*$ ). A large value of  $t^*$  together with a real interest rate close to its initial equilibrium value would imply a small area under the IRF. Moreover, we calculate the half life of the dynamics of the real interest for each year. As shown in figure 9, all measures indicates more persistence in the effects of a monetary policy shock. The measures imply an increase of about three times relative to the initial year.<sup>33</sup>

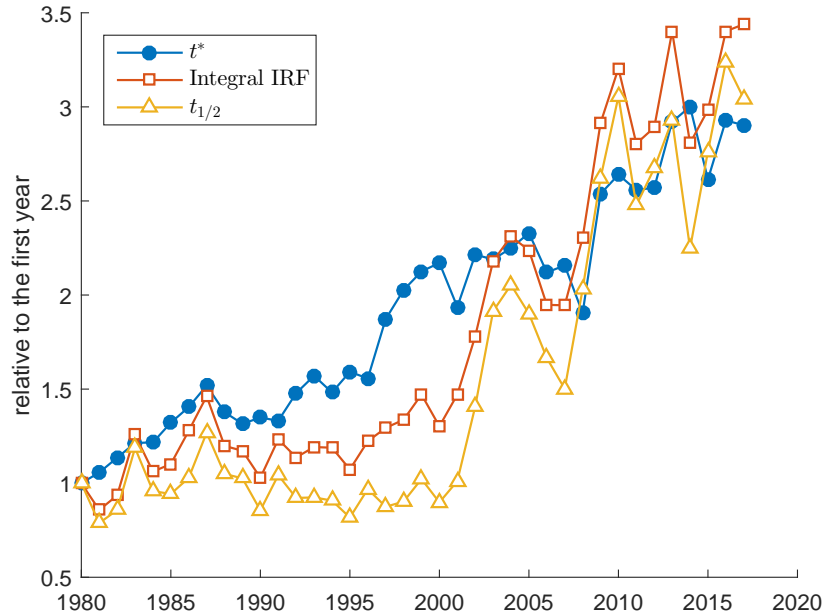


Figure 9:  $t^*$ : time for the real interest rate to return to its initial value. Integral IRF: integral of the impulse-response functions from  $t = 0$  to  $t = t^*$  (see figure 8).  $t_{1/2}$ : half life of the IRF for each year. Values relative to the results for the first year. The three measures of the monetary policy transmission yield similar results.

The effect on the real interest rate implied by the model changes as the distribution of cash-sales ratio changes. The real interest rate takes longer to return to its initial value with the more recent distributions of cash-sales. The monetary authority is now able to affect the real interest for a longer period.

Another way of understanding the causes of the real effects of monetary shocks in the

<sup>33</sup>The value of  $t^*$  increases 2.9 times, the value of the half life increases 3.0 times, and the value of the area under the impulse-response functions increases 3.4 times.

model is through the infrequent trades of bonds for cash. As firms do not rebalance their portfolios instantaneously, they do not change their behavior immediately when there is a monetary policy shock. This delayed effect is more pronounced when firms maintain large amounts of cash.<sup>34</sup>

In the data, there is a nondegenerate cross-sectional distribution of cash. The characteristics of firms, such as their business and corporate governance, are reflected in their behavior toward cash management. Heterogeneity across cash holdings changes the speed and the size of the adjustment to the shock. If all firms held the same amount of cash, the mean level, for instance, then monetary shocks would have different quantitative real effects. This property is not unique to our model, the new Keynesian Phillips curve model shares this property. Carvalho and Nechio (2011) show that heterogeneity in the price setting behavior of firms implies aggregate dynamics different from the case when all firms have the same price setting behavior. Here, after the shock hits the economy, the initial phase of the adjustment process is driven mainly by the set of firms with less cash. The later part of the adjustment process is dominated by the set of firms with larger stocks of cash.

To check the robustness of our results, we simulate the economy with different paths for the monetary policy shock and with different cash aggregates. We use other identification methods of the shock, recalculate the parameters  $B$  and  $\eta$  of the process for  $r(t)$ , and obtain the effect of the shock for the different estimates.

Besides using different identification methods for the monetary policy shock, we verify our results with different cash aggregates. Our results in figure 1 use cash and equivalents (CHE) for the distribution of cash across firms, as CHE is the variable usually used for firm cash holdings.<sup>35</sup> It may be argued, however, that CHE contains variables that are not in traditional monetary aggregates such as short-term marketable security, which is part of CHE but not of M1.<sup>36</sup> To check whether we maintain our results with a more restricted variable for firm cash holdings, we repeat the exercise using only the cash component of cash and

---

<sup>34</sup>The impact of infrequent portfolio rebalancing has also been studied by Bacchetta and van Wincoop (2010) to analyze the forward premium puzzle. Herrenbrueck (2019), using a model with random access to the asset market, also finds that the distribution of liquid and illiquid assets affects the results of monetary policy shocks.

<sup>35</sup>Cash and equivalents is used, for example, by Almeida et al. (2004), Bover and Watson (2005), Bates et al. (2009), Bacchetta et al. (2019), among others.

<sup>36</sup>A substantial part of CHE, in any case, is comprised by cash (CH), which “represents any immediately negotiable medium of exchange or any instruments normally accepted by banks for deposit and immediate credit to a customer’s account” (Compustat). This definition closely resembles the definition of M1.

equivalents (CH instead of CHE).

We use three forms of identification of the monetary policy shock, provided by Uhlig (2005). In the first, used to obtain the results of figure 1, Uhlig generates impulse-response functions, obtained from an OLS estimate of a VAR, that satisfies sign restrictions for the monetary policy shock and the price level for six months after the shock. Figure 7, reproduced from figure 2 of Uhlig, contains the results of this identification exercise. The figure has ten random draws of the impulse responses that satisfy the sign restrictions and the upper and lower bounds of ten thousand random draws of impulse responses. We added the process  $r(t)$  used in our simulations, with the parameters  $B$  and  $\eta$  chosen to approximate the impulse-response function of the shock.

The second method of identification follows a conventional identification procedure found, for example, in Christiano et al. (1999). This method uses a standard Cholesky decomposition with no imposition of sign restrictions. The third method, called pure-sign-restriction approach by Uhlig, imposes sign restrictions for the identification and uses Bayesian methods. The OLS estimate and the pure-sign-restriction approach produce similar results, although the pure-sign-restriction approach satisfies additional technical requirements. The conventional identification implies a larger increase of the interest rate at the time of the shock and a more persistent shock as compared with the OLS estimate. The pure-sign-restriction approach implies a smaller shock at the time of the shock and a somewhat more persistent shock. The shock identified with the OLS estimate is in between the pure-sign-restriction approach and the conventional identification.<sup>37</sup>

With the three different identification methods for the monetary policy shock and the two variables for cash holdings, we have a total of six different simulations. The results of these simulations are in figure 10. For comparison, the results in figure 1 are repeated in the first plot of figure 10 for the case with CHE.

In all simulations, the time that it takes for the real interest rate to return to its initial value increases as we consider more recent cross-sectional distributions of cash holdings. The pure-sign-restriction approach implies a smaller monetary policy shock. As a result, the simulations yield smaller effects on the real interest rate. The time during which the real

---

<sup>37</sup>For the OLS estimate, conventional identification, and pure-sign-restriction approach:  $B = -0.150$ ,  $\eta = 0.3008$ ;  $B = -0.158$ ,  $\eta = 0.4497$ ; and  $B = -0.167$ ,  $\eta = 0.3852$ .

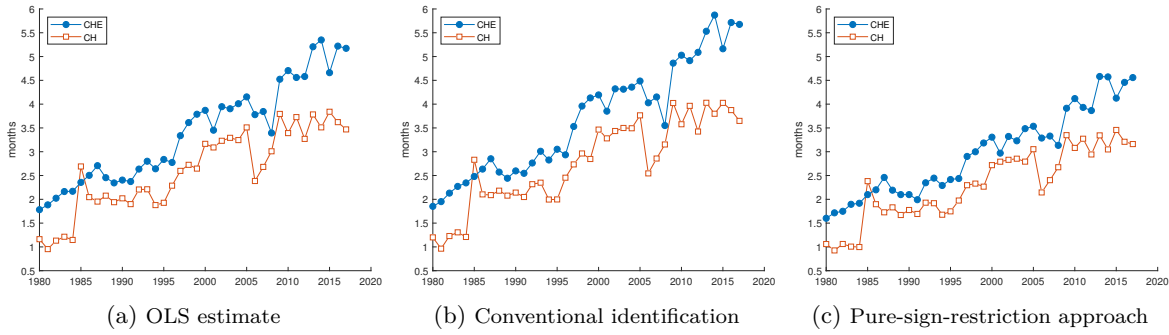


Figure 10: Robustness check. Number of months that it takes to return to the initial interest rate with different identification methods for  $r(t)$  and different cash aggregates. Different estimates for  $r(t)$  from Uhlig (2005). OLS estimate: VAR with sign restrictions obtained by OLS and random draws of possible impulse-response functions. Conventional identification: conventional VAR without sign restrictions. Pure-sign-restriction approach: VAR with sign restrictions obtained with Bayesian methods. CHE: simulations with cash and equivalents. CH: simulations with the cash portion of cash and equivalents. The results in figure 1 are repeated in the first plot with CHE. For all cases, the time to return to the initial value of the real interest rate increases substantially.

interest rate is above its initial value increases from 1.6 months in 1980 to 4.6 months in 2017, using the CHE aggregate, compared with an increase from 1.78 months to 5.17 months with the OLS estimate. The increase in cash holdings would therefore imply an increase on the effects of the real interest rate of 3 months according to the pure-sign-restriction and 3.4 according to the OLS estimate. On the other hand, the conventional identification method for the monetary policy shock implies a larger shock and a more persistent interest rate shock. The simulations then yield longer effects on the real interest rate. They also yield a larger difference between the duration in 1980 and 2017. The real interest rate takes 1.9 months in 1980 and 5.7 months in 2017 to return to its initial value. Therefore, with the conventional identification, the effect of the increase in cash holdings is 3.8 months.

The use of CH instead of CHE for the cash aggregate, also implies a more prolonged effect of monetary policy under the cash distribution of recent periods. CHE implies larger effects of monetary policy, although the effects in percentage terms are larger with CH. The predictions about the large increase in the time for the real interest rate to return to its initial value are valid for both CH and CHE.<sup>38</sup>

Our simulations have the objective of isolating the effect of the changes in the level of cash

<sup>38</sup>We also used different ways of treating the data, using different constraints on minimum cash holdings, truncation values for the cash-sales ratio, and minimum sales. These modifications do not change conclusions in a significant way.

holdings and in the cross-sectional distribution of cash holdings. Our point is that, according to our simulations, the increase from 1.78 months in 1980 to 5.17 months in 2017 can be attributed to the changes in the behavior toward cash holdings over the period. The model used by us, with market segmentation and a non-degenerate distribution of cash holdings, is particularly useful to obtain these predictions.

The exercises with alternative calibration methods, different interest rate shocks and different cash aggregates shocks, summarized in figure 10, show that our results are robust. When we allow for changes around the set up of our first results, we still find that the recent changes in the distribution of cash holdings generate a longer period during which the real interest rate is affected by monetary policy. Given the current large firm cash holdings, the conclusion is that current monetary policy shocks can have stronger effects in the economy.<sup>39</sup>

## 8 Conclusions

The increase in cash holdings by firms from 1980 to 2017 has strong macroeconomic consequences. We find that it affects the response of the real interest rate to nominal interest rate shocks. The effect of firm cash holdings on monetary policy is substantial. According to our predictions, the changes in the distribution of cash holdings imply that the time that it takes for the real interest rates to return to its initial value after a shock increases almost three times: from 1.78 months in 1980 to 5.17 months in 2017.

The current distribution of cash holdings implies that changes in monetary policy have more prolonged effects. There is a current debate about how central banks should increase nominal interest rates back to normal values. An implication of our results is that these changes in interest rates should be made gradually. Given the high current values of the cash-sales distribution as compared to past values, changes in nominal interest rates will have stronger effects in the economy.

---

<sup>39</sup>Small movements in the nominal interest would have larger than usual effects on the real interest rate. A policy implication is that the changes in the nominal interest rate have to be done gradually. Allowing firms time to adapt to the new interest rate. Non-conventional policies, that is, policies that do not use an increase in interest rates, could also be used to avoid a strong reaction after interest rate changes. For example, forward guidance before any increase in interest rates would allow firms to adapt to the new policy.

## Appendix A The Problem for the Transition

There are two states for the interest rate path,  $s = 1, 2$ , and there are two contingent bonds. In state 1, the nominal interest rate path is the constant initial steady state interest rate  $r$ . In state 2, the nominal interest rate is different; it is equal to the unexpected path  $r(t)$ . Let  $\xi$  denote the probability of state 1. For  $s = 1, 2$ , let  $c(t, n_i; s)$  denote consumption of entrepreneur  $n_i$  at date  $t$  in state  $s$ , and  $T_{j,i}(n_i; s)$  denote the date of the  $j$ th transfer of entrepreneur  $n_i$  in state  $s$ . As money is not contingent on the states, entrepreneur  $n_i$  must use the initial stock of money  $M_{0,i}(n_i)$  from  $t = 0$  until the first transfer  $T_{1,i}(n_i; s)$ . In this framework, from  $t = 0$  to  $T_{1,i}(n_i; s)$ , each entrepreneur has two cash-in-advance constraints, one for each state,

$$\int_0^{T_{1,i}(n_i; s)} P(t; s) c_i(t, n_i; s) dt + M_{0,i}^-(n_i; s) = M_{0,i}(n_i), \quad s = 1, 2. \quad (\text{A.1})$$

After  $T_{1,i}(n_i; s)$ , on the other hand, there is just one intertemporal budget constraint, because entrepreneurs use contingent bonds to transfer resources between states.

The maximization problem of each entrepreneur is

$$\max \quad \xi \sum_{j=0}^{\infty} \int_{T_{j,i}(n_i; 1)}^{T_{j+1,i}(n_i; 1)} e^{-\rho t} u(c_i(t, n_i; 1)) dt + (1 - \xi) \sum_{j=0}^{\infty} \int_{T_{j,i}(n_i; 2)}^{T_{j+1,i}(n_i; 2)} e^{-\rho t} u(c_i(t, n_i; 2)) dt \quad (\text{A.2})$$

subject to

$$\sum_s \sum_{j=1}^{\infty} Q(T_{j,i}(n_i; s)) \int_{T_{j,i}(n_i; s)}^{T_{j+1,i}(n_i; s)} P(t; s) c_i(t, n_i; s) dt \leq \sum_s Q(T_{1,i}(n_i; s)) M^-(n_i; s) + W_0(n_i), \quad (\text{A.3})$$

$$\int_0^{T_{1,i}(n_i; s)} P(t; s) c_i(t, n_i; s) dt + M_{0,i}^-(n_i; s) = M_{0,i}(n_i), \quad (\text{A.4})$$

where  $W_{0,i}(n_i) \equiv B_{0,i}(n_i) + \sum_{s=1,2} \int_0^{\infty} Q(t; s) P(t; s) Y dt$ .

The first order conditions with respect to  $c_i(t, n_i)$  in the state 2 imply, for  $j \geq 2$ ,

$$c^+(T_{j,i}(n_i), n_i) [R(T_{j,i}(n_i)) - R(T_{j-1,i}(n_i))] = r(T_j(n_i)) \int_{T_{j,i}(n_i)}^{T_{j+1,i}(n_i)} \frac{P(t) c_i(t, n_i)}{P(T_{j,i}(n_i))} dt, \quad (\text{A.5})$$



where

$$c^+(T_{j,i}(n_i), n_i) = \left[ \lambda(n_i) e^{\rho T_{j,i}(n_i)} Q(T_{j,i}(n_i)) P(T_{j,i}(n_i)) \right]^{-1}. \quad (\text{A.6})$$

For  $T_1(n_i)$ , the first order conditions imply

$$c^+(T_1, n_i) R(T_1(n_i)) - \log \frac{\lambda(n_i)}{\mu(n_i)} + \frac{r(T_1(n_i)) M_i^-(n_i)}{P(T_1(n_i))} = r(T_1(n_i)) \int_{T_1(n_i)}^{T_2(n_i)} \frac{P(t) c(t, n_i)}{P(T_1(n_i))} dt. \quad (\text{A.7})$$

## Appendix B Proofs

### B.1 Aggregate transactions $C_i(t)$

Let  $\lambda_i(n_i)$  and  $\mu_i(n_i)$  denote the Lagrange multipliers on (6) and (8). The first order conditions imply

$$P(t) c(t, n_i) = \frac{e^{-\rho t} e^{R(T_{ji})}}{\lambda(n_i)}, \quad t \in (T_{ji}, T_{j+1i}), \quad (\text{B.8})$$

$$P(T_{ji}) c^+(T_{ji}, n_i) = \frac{e^{-\rho T_{ji}} e^{R(T_{ji})}}{\lambda(n_i)}, \quad t \rightarrow T_{ji}, \quad t > T_{ji}, \quad (\text{B.9})$$

$$P(T_{j+1i}) c^-(T_{j+1i}, n_i) = \frac{e^{-\rho T_{j+1i}} e^{R(T_{ji})}}{\lambda(n_i)}, \quad t \rightarrow T_{j+1i}, \quad t < T_{j+1i}, \quad (\text{B.10})$$

$j = 1, 2, \dots, i = 1, \dots, I$ . Similarly,  $P(t) c(t, n_i) = \frac{e^{-\rho t}}{\mu(n_i)}$ ,  $t \in (0, T_{1i})$ ,  $P(0) c^+(0, n_i) = \frac{1}{\mu(n_i)}$ , and  $P(T_{1i}) c^-(T_{1i}) = \frac{e^{-\rho T_{1i}}}{\mu(n_i)}$ . The first transfer occurs at  $T_{1i} = n_i$ . For  $M_i^-(n_i)$ ,  $Q(T_{1i}) \lambda(n_i) - \mu(n_i) \leq 0$ , with equality if  $M_i^-(n_i) > 0$ . Therefore, the first order conditions for transactions imply that nominal transactions  $P(t) c(t, n_i)$  decrease at the rate  $\rho$  within holding periods. Together with the constraints (6) and (8), the first order conditions imply

$$\lambda(n_i) = \frac{1}{W_{0i}(n_i) + Q(T_{1i}) M_i^-(n_i)} \frac{e^{-\rho n_i}}{\rho}, \quad (\text{B.11})$$

$$\mu(n_i) = \frac{1}{M_{0i}(n_i) - M_i^-(n_i)} \frac{1 - e^{-\rho n_i}}{\rho}. \quad (\text{B.12})$$

The values of  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  such that the economy is in an equilibrium with constant interest rate at  $t = 0$  are such that (1) nominal transactions  $P(t) c(t, n_i)$  evolve at the steady state rate and (2) all firms start a holding period with transactions  $c_{0i}$ , excluding the shorter holding period from  $t = 0$  to  $t = n_i$ . The first order conditions imply  $\pi(t) + \frac{\dot{c}(t, n_i)}{c(t, n_i)} = -\rho$ .

So, spending decreases at the rate  $\rho$  and, in the steady state, transactions decrease at the rate  $\rho + \pi = r$ . For an arbitrary firm  $n_i$ , nominal transactions at  $t = 0$  are  $P_0 c(0, n_i) = P_0 c_{0i} e^{-r(N_i - n_i)}$ , where  $P_0$  is the price level at  $t = 0$  in the steady state before the shock hits the economy. The value  $c_{0i} e^{-r(N_i - n_i)}$  implies that firm  $n_i$  makes transactions  $c_{0i}$  just after the first bond trade. Therefore, from  $\int_0^{n_i} P(t) c(t, n_i) dt + M_i^-(n_i) = M_{0i}(n_i)$ , imposing  $M_i^-(n_i) = 0$ , we obtain

$$M_{0i}(n_i) = P_0 c_{0i} e^{-r(N_i - n_i)} \frac{1 - e^{-\rho n_i}}{\rho}. \quad (\text{B.13})$$

Analogously,  $W_{0i}(n_i) = \sum_{j=1}^{\infty} Q(T_{ji}) \int_{T_{ji}}^{T_{j+1i}} P(t) c(t, n_i) dt$ . We have  $T_{ji} = n_i + N_i(j - 1)$ ,  $j = 1, 2, \dots$ , for the times of the transfer periods. As  $Q(T_{ji}) = e^{-rT_{ji}}$  and transactions decrease at the rate  $\rho$  at the steady state, then

$$W_{0i}(n_i) = P_0 c_{0i} \frac{e^{-\rho n_i}}{\rho}. \quad (\text{B.14})$$

Using constraints (6) and (8) with  $M_i^-(T_{1i} n_i) = 0$  and the first order conditions, we obtain  $\mu(n_i) = \frac{1 - e^{-\rho n_i}}{\rho M_{0i}(n_i)}$  and  $\lambda(n_i) = \frac{e^{-\rho n_i}}{\rho W_{0i}(n_i)}$ . Substituting  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  implies  $\mu(n_i) = \frac{e^{r(N_i - n_i)}}{P_0 c_{0i}}$  and  $\lambda_i(n_i) = \frac{1}{P_0 c_{0i}}$ . The condition to verify whether  $M_i^-(T_{1i}(n_i)) = 0$  is  $\mu(n_i) > Q(T_1) \lambda(n_i)$ , which holds as  $e^{rN_i} > 1$ . Having obtained  $W_{0i}(n_i)$ , we obtain  $B_{0i}(n_i) = W_{0i}(n_i) - \int_0^{\infty} Q(t) P(t) Y_i dt$ .

To obtain aggregate transactions, suppose an arbitrary  $t \geq N_i$  (the argument is similar for  $t < N_i$ ). As  $t \geq N_i$ , we know that firm  $n_i$  has already made the first transfer. We have  $c(t, n_i) = c_{0i} e^{(r - \pi - \rho)t} e^{-r(t - T_{ji}(n_i))}$ , for the highest  $j(n_i)$  such that  $T_{ji}(n_i) \leq t < T_{j+1i}(n_i)$ . Firms with  $n_i \in [0, t - jN_i)$  are in their  $(j+1)$ th holding period while firms with  $n_i \in [t - jN_i, N_i)$  are in their  $j$ th holding period. Aggregate transactions are then given by

$$\frac{1}{N_i} \int_0^{t - jN_i} c_{0i} e^{(r - \pi - \rho)t} e^{-r(t - T_{j+1i}(n_i))} dn_i + \frac{1}{N_i} \int_{t - jN_i}^{N_i} c_{0i} e^{(r - \pi - \rho)t} e^{-r(t - T_{ji}(n_i))} dn_i. \quad (\text{B.15})$$

Changing variables to  $s_i \equiv T_{j+1i} = n_i + jN_i$  and  $s_i \equiv T_{ji} = n_i + (j - 1)N_i$  in the first and second integrals, we obtain  $C_i(t) = \frac{1}{N_i} \int_{t - N_i}^t c_{0i} e^{(r - \pi - \rho)t} e^{-r(t - s_i)} ds_i$ . With another change of variables,  $C_i(t) = \frac{1}{N_i} \int_0^{N_i} c_{0i} e^{(r - \pi - \rho)t} e^{-rx} dx$ , which implies  $C_i(t) = c_{0i} e^{(r - \pi - \rho)t} \frac{1 - e^{-rN_i}}{rN_i}$ . In the steady state,  $r = \rho + \pi$ , and then  $C_i(t) = c_{0i} e^{(r - \pi - \rho)t} \frac{1 - e^{-rN_i}}{rN_i}$ .

## B.2 Cash-sales ratio $m$ (equation 13)

To obtain the cash-sales ratio, denoted by  $m = \frac{M(t)}{P(t)Y}$ , first note that aggregate cash holdings grows at the same rate of inflation in the steady state. Therefore, the cash-sales ratio is constant in the steady state. In particular,  $m = \frac{M(0)}{P_0Y}$ . At time zero, aggregate cash holdings are equal to  $M(0) = \frac{1}{N_i} \int_0^{N_i} M_0(n_i) dn_i$ . Substituting the values found for  $M_0(n_i)$  and dividing by  $P_0Y$ , we obtain

$$m_i = \frac{e^{-rN_i}}{\rho} \frac{rN_i}{1 - e^{-rN_i}} \left[ \frac{e^{rN_i} - 1}{rN_i} - \frac{e^{(r-\rho)N_i} - 1}{(r - \rho)N_i} \right]. \quad (\text{B.16})$$

Finally, as  $M(0)$  and  $Y$  are normalized to 1, we obtain  $P_0 = 1/m$ . With this final step, we obtain all equilibrium prices and quantities for the steady state.

## B.3 Proposition 1 (Prices after shocks)

*Proof.* First, we prove that all firms choose  $M_i^-(n_i) = 0$  under the new interest rate path  $r(t)$ , given the initial cash and bond holdings  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  of the first steady state. As a result the Lagrange multipliers  $\lambda(n_i)$  and  $\mu(n_i)$  do not change with the shock. To show this statement, we have to show that the sufficient condition for  $M_i^-(n_i) = 0$ , given by  $\mu(n_i) > Q(n_i)\lambda(n_i)$ , holds for every  $n_i$ . We have

$$\lambda(n_i) = \frac{e^{-\rho n_i}}{\rho[W_0(n_i) + Q(n_i)M_i^-(n_i)]}, \quad (\text{B.17})$$

$$\mu(n_i) = \frac{1 - e^{-\rho n_i}}{\rho[M_0(n_i) - M_i^-(n_i)]} \quad (\text{B.18})$$

together with the first order conditions and the budget constraints. Substituting the values of  $M_{0i}(n_i)$  and  $W_{0i}(n_i)$  for the initial equilibrium, we have that the condition for  $M_i^-(n_i) = 0$  holds if and only if  $e^{r(N_i - n_i)} > Q(n_i)$ , which is always true as  $Q(n_i) < 1$  (moreover,  $\mu(n_i) = Q(n_i)\lambda(n_i)$  cannot hold for  $M_i^-(n_i) > 0$ ).

We obtain the price level at each time with the market clearing condition for transactions. For  $t \geq N_i$ , all firms in group  $i$  have already made their first bond trade. Working similarly as

above, substituting  $\lambda(n_i) = \frac{1}{P_0 c_{0i}}$ , aggregate transactions for all firms in group  $i$  are given by

$$C_i(t) = \frac{P_0 c_{0i}}{N_i} \int_0^{t-jN_i} \frac{e^{-\rho t} e^{R(T_{j+1})}}{P(t)} dn_i + \frac{P_0 c_{0i}}{N_i} \int_{t-jN_i}^{N_i} \frac{e^{-\rho t} e^{R(T_j)}}{P(t)} dn_i. \quad (\text{B.19})$$

For  $t \geq N_i$ , firms  $n_i \in [0, t - jN_i)$  are in their  $(j + 1)$ th holding period and firms with  $n_i \in [t - jN_i, N_i)$  are in their  $j$ th holding period. We have, therefore,

$$C_i(t) = \frac{P_0 c_{0i}}{N_i} \int_{t-N_i}^t \frac{e^{-\rho t} e^{R(n_i)}}{P(t)} dn_i. \quad (\text{B.20})$$

For  $0 \leq t < N_i$ , firms with  $n_i \in [0, t)$  have already made their first bond trade and firms with  $n_i \in [t, N_i)$  are in the short holding period from zero to  $t = n_i$ . Let real transactions of these two groups be denoted by  $C_i^1(t) = \frac{P_0 c_{0i}}{N_i} \int_0^t \frac{e^{-\rho t} e^{R(n_i)}}{P(t)} dn_i$  and  $C_i^0(t) = \frac{1}{N_i} \int_t^{N_i} \frac{e^{-\rho t}}{\mu(n_i) P(t)} dn_i$ . Aggregate real transactions are then  $C_i(t) = C_i^1(t) + C_i^0(t)$ . As  $t \rightarrow N_i$ , the firms in group  $i$  that have not made a transfer decrease, and so  $C_i^0(t)$  decreases to zero. Substituting  $\mu(n_i) = \frac{e^{r(N_i - n_i)}}{P_0 c_{0i}}$ , we obtain

$$C_i^0(t) = \frac{P_0 c_{0i} e^{-\rho t} (1 - e^{-r(N_i - t)})}{P(t) r N_i}, \quad (\text{B.21})$$

where  $r$  is the nominal interest rate before the shock. Using  $\sum v_i C_i(t) = Y$ , we obtain the  $P(t)$  in the statement of the proposition.  $\square$

#### B.4 Proposition 2 (Slow reaction of prices)

*Proof.* Without loss of generality we assume  $I = 1$ . We get  $P(0) = P_0$  by using the formula of  $P(t)$  for  $t = 0$ . Also,  $\lim_{t \rightarrow 0} P(t) = P_0$ , which shows that  $P(t)$  is continuous at  $t = 0$ , and so does not jump at the time of the shock. When  $t < N_1$  the derivative of  $P(t)$  with respect to  $t$  is  $\dot{P}(t) = k \left[ -\rho e^{-\rho t} \int_0^t e^{R(n)} dn + e^{-\rho t} e^{R(t)} - \frac{\rho e^{-\rho t} + (r - \rho) e^{(r - \rho)t} e^{-rN}}{r} \right]$ , where  $k$  is a constant. So, inflation just after the shock remains equal to inflation before the shock,  $\pi(0) = r - \rho = \pi$  for any  $r(t)$ . As the real interest rate before the shock is  $\rho$ , we have  $\theta(0) - \rho = r(0) - r$ . We have

$$\theta(t) = r(t) - \pi(t) \Rightarrow \theta(t) = \rho + r(t) - \frac{e^{R(t)} - e^{R(t - N_1)}}{\int_{t - N_1}^t e^{R(n_i)} dn_i}, \quad (\text{B.22})$$

using the formula of  $P(t)$  for  $t \geq N_I$ . We obtain  $\lim_{N \rightarrow 0} \theta(t) = \rho + r(t) - r(t) = \rho$ , which implies that the real interest rate is constant for any  $r(t)$  if there is no market segmentation and, consequently, no heterogeneity in the distribution of cash holdings.  $\square$

## Appendix C Additional Figures and Tables

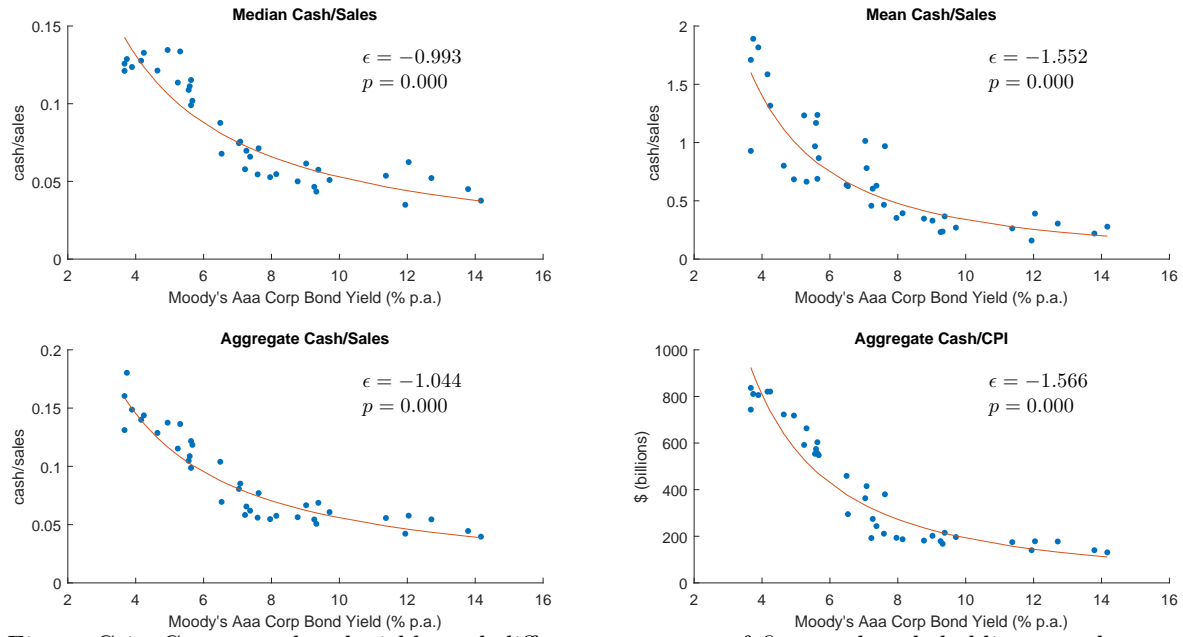


Figure C.1: Corporate bond yields and different measures of firm real cash holdings.  $\epsilon$  denotes elasticities and  $p$  denotes p-values. Annual data 1980-2017, Aaa corporate bonds. Data on yields and CPI from the St. Louis Fed FRED dataset. Data on cash and sales from Compustat.

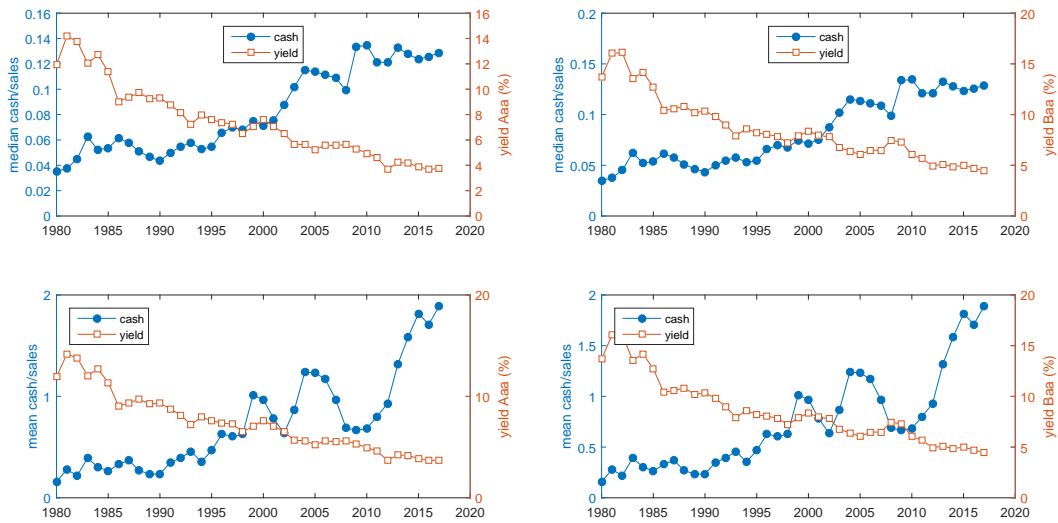


Figure C.2: Firm cash-sales ratio over time and Aaa and Baa Moody's corporate bond yields.

Table C.1: Values of  $v_{Y_i}$  and  $N_i$  for each percentile  $i = 1, \dots, 50$  of  $m_{H_i}$  for 1980 and 2010. These values were used to generate the cash-sales distributions for 1980 and 2010 in figure 6. Analogous values were calculated for the other years. The average  $N$  is a weighted average of  $N_i$  using  $v_{Y_i}$  as weights.

1980														
$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)
1	0.8	0.00	11	4.3	6.36	21	9.3	0.00	31	0.1	0.03	41	50.2	0.92
2	1.2	0.00	12	4.6	0.00	22	10.0	0.00	32	0.1	0.01	42	57.3	1.62
3	1.6	0.00	13	5.1	0.00	23	10.7	10.72	33	0.1	0.00	43	65.8	3.54
4	1.9	0.00	14	5.4	3.78	24	11.6	0.00	34	0.1	0.07	44	76.8	0.99
5	2.3	0.00	15	5.8	0.00	25	12.6	0.38	35	0.1	0.02	45	91.9	1.49
6	2.7	0.00	16	6.2	4.56	26	13.5	5.91	36	0.1	0.15	46	120.5	0.31
7	3.0	0.00	17	6.8	1.37	27	14.6	0.75	37	0.1	0.01	47	157.6	0.32
8	3.3	0.00	18	7.3	0.97	28	15.7	8.07	38	0.1	0.01	48	245.0	0.26
9	3.7	0.00	19	7.9	3.21	29	17.1	0.00	39	0.1	0.04	49	499.1	0.07
10	4.0	0.00	20	8.5	0.76	30	18.4	7.60	40	0.1	0.02	50	2030	0.04

Average  $N = 25.2$  days.

2010														
$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)	$N_i$ (days)		$v_{Y_i}$ (%)
1	0.7	0.00	11	12.6	13.35	21	37.5	0.00	31	75.5	0.00	41	196.1	0.00
2	1.4	0.91	12	14.7	0.00	22	40.0	0.00	32	80.9	10.43	42	219.5	9.52
3	2.3	0.26	13	16.7	2.79	23	42.5	7.42	33	87.9	6.51	43	257.9	0.12
4	3.2	0.00	14	19.0	0.00	24	45.7	0.00	34	96.1	1.94	44	298.3	2.52
5	4.2	0.55	15	21.1	0.90	25	48.5	3.45	35	105.0	0.84	45	361.6	0.86
6	5.2	3.43	16	23.4	1.71	26	51.9	5.60	36	114.9	3.08	46	462.9	1.91
7	6.5	2.59	17	26.1	0.00	27	56.4	0.00	37	127.7	0.00	47	633.9	0.20
8	8.0	0.00	18	28.5	0.00	28	60.4	2.64	38	140.3	1.67	48	1111	0.12
9	9.7	0.00	19	30.8	3.02	29	65.4	0.00	39	155.0	1.56	49	2722	0.04
10	11.2	0.00	20	34.1	0.00	30	69.9	5.04	40	171.2	5.03	50	59636	0.01

Average  $N = 99.2$  days.

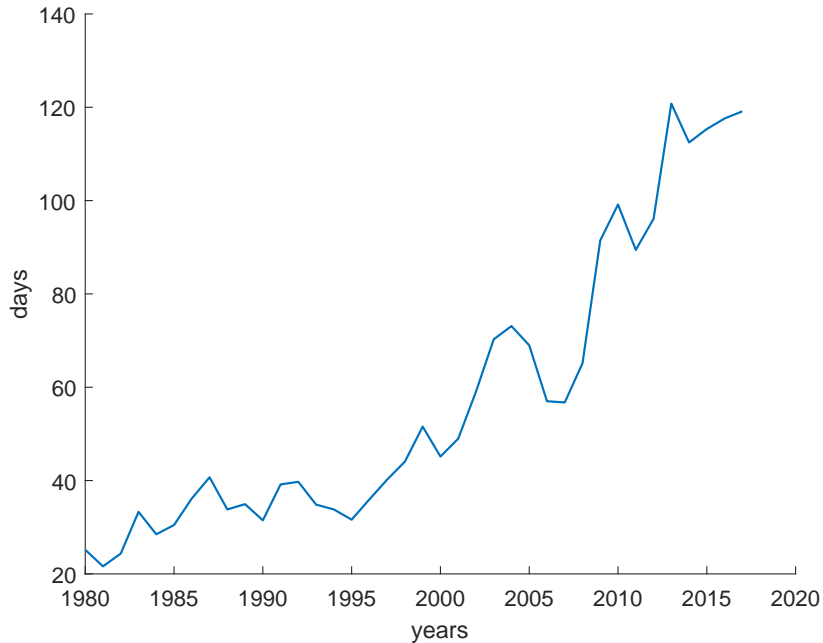


Figure C.3: Average holding period  $N$  over time (weighted average of  $N_i$  using  $v_{Y_i}$  as weights).

## References

- Acharya, Viral V., Heitor Almeida, and Murillo Campello (2007). Is cash negative debt? a hedging perspective on corporate financial policies. *Journal of Financial Intermediation* 16(4), 515–554.
- Acharya, Viral V., Heitor Almeida, and Murillo Campello (2013). Aggregate risk and the choice between cash and lines of credit. *The Journal of Finance* 68(5), 2059–2116.
- Adão, Bernardino, Isabel Correia, and Pedro Teles (2004). The monetary transmission mechanism: Is it relevant for policy? *Journal of the European Economic Association* 2(2-3), 310–319.
- Adão, Bernardino, Isabel Correia, and Pedro Teles (2011). Unique monetary equilibria with interest rate rules. *Review of Economic Dynamics* 14(3), 432–442.
- Adão, Bernardino and José Mata (1999). Money demand and scale economies: evidence from a panel of firms. Working Paper.
- Adão, Bernardino and André C. Silva (2017). Financial frictions and interest rate shocks. Working Paper.
- Adão, Bernardino and André C. Silva (2020). Government financing, inflation, and the financial sector. *Economic Theory*. Conditionally accepted.
- Almeida, Heitor, Murillo Campello, and Michael S. Weisbach (2004). The cash flow sensitivity of cash. *The Journal of Finance* 59(4), 1777–1804.
- Alvarez, Fernando, Andrew Atkeson, and Chris Edmond (2009). Sluggish responses of prices and inflation to monetary shocks in an inventory model of money demand. *The Quarterly Journal of Economics* 124(3), 911–967.
- Alvarez, Fernando and Francesco Lippi (2009). Financial innovation and the transactions demand for cash. *Econometrica* 77(2), 363–402.
- Alvarez, Fernando E., Francesco Lippi, and Luigi Paciello (2011). Optimal price setting with observation and menu costs. *The Quarterly Journal of Economics* 126(4), 1909–1960.



- Armenter, Roc and Viktoria Hnatkowska (2017). Taxes and capital structure: Understanding firms' savings. *Journal of Monetary Economics* 87, 13–33.
- Azar, José A., Jean-François Kagy, and Martin C. Schmalz (2016). Can changes in the cost of carry explain the dynamics of corporate cash holdings? *The Review of Financial Studies* 29(8), 2194–2240.
- Bacchetta, Philippe, Kenza Benhima, and Céline Poilly (2019). Corporate cash and employment. *American Economic Journal: Macroeconomics* 11(3), 30–66.
- Bacchetta, Philippe and Eric van Wincoop (2010). Infrequent portfolio decisions: A solution to the forward discount puzzle. *The American Economic Review* 100(3), 870–904.
- Bates, Thomas W., Kathleen M. Kahle, and René M. Stulz (2009). Why do U.S. firms hold so much more cash than they used to? *The Journal of Finance* 64(5), 1985–2021.
- Baumol, William J. (1952). The transactions demand for cash: An inventory theoretic approach. *The Quarterly Journal of Economics* 66(4), 545–556.
- Bernanke, Ben and Mark Gertler (1989). Agency costs, net worth, and business fluctuations. *American Economic Review* 79, 14–31.
- Bernanke, Ben S., Jean Boivin, and Piotr Eliasch (2005). Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach. *The Quarterly Journal of Economics* 120(1), 387–422.
- Bolton, Patrick and Xavier Freixas (2006). Corporate finance and the monetary transmission mechanism. *The Review of Financial Studies* 19(3), 829–870.
- Bover, Olympia and Nadine Watson (2005). Are there economies of scale in the demand for money by firms? Some panel data estimates. *Journal of Monetary Economics* 52(8), 1569–1589.
- Brunnermeier, Markus and Yann Koby (2019). The reversal interest rate. Working Paper.
- Campbell, John Y., Martin Lettau, Burton G. Malkiel, and Yexiao Xu (2001). Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *The Journal of Finance* 56(1), 1–43.

- Carvalho, Carlos and Fernanda Nechio (2011). Aggregation and the PPP puzzle in a sticky-price model. *The American Economic Review* 101(6), 2391–2424.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (1996). The effects of monetary policy shocks: Evidence from the flow of funds. *The Review of Economics and Statistics* 78(1), 16–34.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (1999). Monetary policy shocks: What have we learned and to what end? In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1A, Chapter 2, pp. 65–148. Elsevier.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113(1), 1–45.
- Cochrane, John H. (1994). Shocks. *Carnegie-Rochester Conference Series on Public Policy* 41, 295–364.
- Cole, Harold L. and Lee E. Ohanian (2002). Shrinking money: the demand for money and the nonneutrality of money. *Journal of Monetary Economics* 49(4), 653–686.
- Dittmar, Amy and Jan Mahrt-Smith (2007). Corporate governance and the value of cash holdings. *Journal of Financial Economics* 83(3), 599–634.
- Dittmar, Amy, Jan Mahrt-Smith, and Henri Servaes (2003). International corporate governance and corporate cash holdings. *The Journal of Financial and Quantitative Analysis* 38(1), 111–133.
- Drechsler, Itamar, Alexi Savov, and Philipp Schnabl (2017). The deposits channel of monetary policy. *The Quarterly Journal of Economics* 132(4), 1819–1876.
- Falato, Antonio, Dalida Kadyrzhanova, Jae W. Sim, and Roberto Steri (2018). Rising intangible capital, shrinking debt capacity, and the US corporate savings glut. SSRN Electronic Journal.

- Faulkender, Michael W, Kristine W Hankins, and Mitchell A Petersen (2019). Understanding the rise in corporate cash: Precautionary savings or foreign taxes. *The Review of Financial Studies* 32(9), 3299–3334.
- Foley, C. Fritz, Jay C. Hartzell, Sheridan Titman, and Garry Twite (2007). Why do firms hold so much cash? A tax-based explanation. *Journal of Financial Economics* 86(3), 579–607.
- Frenkel, Jacob A. and Boyan Jovanovic (1980). On transactions and precautionary demand for money. *The Quarterly Journal of Economics* 95(1), 25–43.
- Fresard, Laurent (2010). Financial strength and product market behavior: The real effects of corporate cash holdings. *The Journal of Finance* 65(3), 1097–1122.
- Gao, Xiaodan (2018). Corporate cash hoarding: The role of just-in-time adoption. *Management Science* 64(10), 4858–4876.
- Gao, Xiaodan and Shaofeng Xu (2018). The role of corporate saving over the business cycle: Shock absorber or amplifier? Staff Working Papers 18-59, Bank of Canada.
- Golosov, Mikhail and Robert E. Lucas, Jr. (2007). Menu costs and Phillips curves. *Journal of Political Economy* 115(2), 171–199.
- Grossman, Sanford and Laurence Weiss (1983). A transactions-based model of the monetary transmission mechanism. *The American Economic Review* 73(5), 871–880.
- Grossman, Sanford J. (1987). Monetary dynamics with proportional transaction costs and fixed payment periods. In William A. Barnett and Kenneth J. Singleton (Eds.), *New Approaches to Monetary Economics*, pp. 3–41. Cambridge University Press.
- Han, Seungjin and Jiaping Qiu (2007). Corporate precautionary cash holdings. *Journal of Corporate Finance* 13(1), 43–57.
- Harford, Jarrad (1999). Corporate cash reserves and acquisitions. *The Journal of Finance* 54(6), 1969–1997.
- Harford, Jarrad, Sattar A. Mansi, and William F. Maxwell (2008). Corporate governance and firm cash holdings in the US. *Journal of Financial Economics* 87(3), 535–555.

- Herrenbrueck, Lucas (2019). Frictional asset markets and the liquidity channel of monetary policy. *Journal of Economic Theory* 181, 82–120.
- Irvine, Paul J. and Jeffrey Pontiff (2008). Idiosyncratic return volatility, cash flows, and product market competition. *The Review of Financial Studies* 22(3), 1149–1177.
- Jensen, Michael C. (1986). Agency costs of free cash flow, corporate finance, and takeovers. *The American Economic Review* 76(2), 323–329.
- Kahn, Michael, Shmuel Kandel, and Oded Sarig (2002). Real and nominal effects of central bank monetary policy. *Journal of Monetary Economics* 49(8), 1493–1519.
- Kalcheva, Ivalina and Karl V. Lins (2007). International evidence on cash holdings and expected managerial agency problems. *The Review of Financial Studies* 20(4), 1087–1112.
- Kashyap, Anil K and Jeremy C. Stein (2000). What do a million observations on banks say about the transmission of monetary policy? *The American Economic Review* 90(3), 407–428.
- Kim, Chang-Soo, David C. Mauer, and Ann E. Sherman (1998). The determinants of corporate liquidity: Theory and evidence. *The Journal of Financial and Quantitative Analysis* 33(3), 335–359.
- Lagos, Ricardo and Shengxing Zhang (forthcoming). Turnover liquidity and the transmission of monetary policy. *American Economic Review*.
- Mankiw, N. Gregory and Ricardo Reis (2002). Sticky information versus sticky prices: A proposal to replace the new Keynesian Phillips curve. *The Quarterly Journal of Economics* 117(4), 1295–1328.
- Miller, Merton H. and Daniel Orr (1966). A model of the demand for money by firms. *The Quarterly Journal of Economics* 80(3), 413–435.
- Mulligan, Casey B. (1997). Scale economies, the value of time, and the demand for money: Longitudinal evidence from firms. *Journal of Political Economy* 105(5), 1061–79.

- Myers, Stewart and Nicholas Majluf (1984). Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics* 13(2), 187–221.
- Opler, Tim, Lee Pinkowitz, René Stulz, and Rohan Williamson (1999). The determinants and implications of corporate cash holdings. *Journal of Financial Economics* 52(1), 3–46.
- Paciello, Luigi and Mirko Wiederholt (2014). Exogenous information, endogenous information, and optimal monetary policy. *The Review of Economic Studies* 81(1), 356–388.
- Palazzo, Bernardino (2012). Cash holdings, risk, and expected returns. *Journal of Financial Economics* 104(1), 162–185.
- Pinkowitz, Lee, René Stulz, and Rohan Williamson (2006). Does the contribution of corporate cash holdings and dividends to firm value depend on governance? A cross-country analysis. *The Journal of Finance* 61(6), 2725–2751.
- Rocheteau, Guillaume, Tsz-Nga Wong, and Cathy Zhang (2017). Lending relationships and optimal monetary policy. Working Paper.
- Rocheteau, Guillaume, Randall Wright, and Cathy Zhang (2018). Corporate finance and monetary policy. *American Economic Review* 108(4-5), 1147–86.
- Rotemberg, Julio J. (1984). A monetary equilibrium model with transactions costs. *Journal of Political Economy* 92(1), 40–58.
- Silva, André C. (2012). Rebalancing frequency and the welfare cost of inflation. *American Economic Journal: Macroeconomics* 4(2), 153–183.
- Stein, Jeremy C. (1998). An adverse-selection model of bank asset and liability management with implications for the transmission of monetary policy. *The RAND Journal of Economics* 29(3), 466–486.
- Tobin, James (1956). The interest-elasticity of transactions demand for cash. *The Review of Economics and Statistics* 38(3), 241–247.

- Uhlig, Harald (2005). What are the effects of monetary policy on output? Results from an agnostic identification procedure. *Journal of Monetary Economics* 52(2), 381–419.
- Williamson, Stephen D. (2009). Transactions, credit, and central banking in a model of segmented markets. *Review of Economic Dynamics* 12(2), 344–362.
- Woodford, Michael (2003). *Interest and Prices*. Princeton: Princeton Univ. Press.
- Yellen, Janet L. (2017a). The economic outlook and the conduct of monetary policy. Speech at the Stanford Institute for Economic Policy Research, January 19, 2017, Stanford University, Stanford, California.
- Yellen, Janet L. (2017b). Semiannual monetary policy report to the congress. Testimony by Chair Janet Yellen before the Committee on Banking, Housing, and Urban Affairs, February 14, 2017, U.S. Senate, Washington, D.C.
- Yun, Hayong (2009). The choice of corporate liquidity and corporate governance. *The Review of Financial Studies* 22(4), 1447–1475.
- Zhao, Jake (2020). Accounting for the corporate cash increase. *European Economic Review* 123, 103393.