Chapter 6
Μηδεὶς ἀγεωμέτρητος εἰσίτω

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Abstract This paper provides a discussion to which extent the Mathematician David Hilbert could or should be considered as a Philosopher, too. In the first part, we discuss some aspects of the relation of Mathematicians and Philosophers. In the second part we give an analysis of David Hilbert as Philosopher.

6.1 Philosophers as Mathematicians and Mathematicians as Philosophers

> Let us note first that it is as rare to see a mathematician in possession of a strong philosophical culture as to see a philosopher who has an extensive knowledge of mathematics; the opinions of mathematicians on topics in philosophy, even when these questions are concerned with their field, are most often opinions received at second or third hand, coming from doubtful sources. But, precisely because of this, it is these average opinions which interest the mathematical historian, at least as much as the original views of thinkers such as Descartes or Leibniz (to mention two who were also mathematicians of the first rank), Plato (who at least kept up with the mathematics of his time), Aristotle or Kant (of whom the same could not be said).
>
> Bourbaki (1994, p. 11).

In the following, we aim to isolate some criteria which characterize Philosophers as Mathematicians or Mathematicians as Philosophers.

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The title phrase was recorded at the entrance of the Platonic Academy. In a free English translation it reads: "Let none but geometers (i.e., mathematicians) enter here." The earliest references to this phrase is from the sixth century and can be found, in slight variations, in works of John Philoponus (1897, p. 117.27) and Elias (1900, p. 118, 18–19). It is in line with Diogenes Laertius (1959, VI.10; p. 384) who reports that Xenocrate, the third leader of Platon's Academy, classified Mathematics as part of the handles of philosophy.

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Philosophical Tradition and Mathematical State-of-the-Art

To bridge Philosophy and Mathematics, it is, of course, important to show respect for the other area. For Philosophers this would mean they need to have an up-to-date knowledge of the Mathematics of their time; for Mathematicians that they would need to be at home at the philosophical debates, both historically and of their times.

This criteria can be verified for Philosophers like Plato (of course), but also Avicenna[1] and Carnap, just to mention three names. It fails, on the other hand, for instance for Kant who—despite the important role Mathematics plays in his Philosophy—was not abreast with the Mathematics of his times (Volk 1925) and, in particular, for Hegel.[2]

Likewise, Gauß did not show particular respect for Philosophy[3]; and also for Euclid we do not have sources which would show that he was appreciating Philosophy. On the other hand, Descartes, Pascal, and Leibniz are examples of Mathematicians which were at the very same time Philosophers and, of course, were not only at home at the philosophical debates but shaped them. Finally, we may mention Cantor who was staying much more in the philosophical tradition—and even theological (see Tapp 2005)—than one might expect at first glance.

Philosophers as Mathematicians

To be a Mathematician (as Philosopher or anybody else), one is expected to contribute to the body of mathematical knowledge. As a matter of fact, this is hardly the case for anybody coming from Philosophy. It could, for instance, be claimed neither for Plato nor for Wittgenstein, even if both influenced Philosophy of Mathematics—more or less, respectively.

There are, of course, examples where scientists advanced in both disciplines "in parallel"—like Descartes, Pascal, and Leibniz, already mentioned, but also Bolzano[4] and Ramsey. In these cases, it is rather debatable whether it makes sense to give "preference" to one of the areas to be the one they started from to reach also the other.

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[1] Crozet (2018), Rashed (2008, 2018).

[2] Hegel's dissertation of 1801 finishes with a discussion—although not a "proof"—that there should be exactly eight planets in the solar system. His work was characterized as "*Monumentum Insaniae Saec. XIX*" by Ernest II, Duke of Saxe-Gotha-Altenburg and discussed as such by Gauß in the correspondence with his friend Schumacher in 1842 (Peters 1862, letters 763–765).

[3]
> Just look around at the modern philosophers, at Schelling, Hegel, Nees von Esenbeck and consorts—don't their definitions make your hair stand on end? Read in the history of ancient philosophy what the men of the day, Plato and others (I except Aristotle), gave as explanations. And even in Kant matters are often not much better; his distinction between analytic and synthetic propositions seems to me to be either a triviality or false.

Gauß in a letter to Schumacher on 1 November 1844; translation from (Ewald 1959, p. 293); German original in (Peters 1862, letter 944).

[4] "Bolzano, der große Gegner Kants, ist seit LEIBNITZ[sic!] der erste philosophische Mathematiker und mathematische Philosoph." ("Bolzano, the great opponent of Kant, is the first philosophical Mathematician and mathematical Philosopher since Leibniz.") (Korselt 1903, p. 405).

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Russell and Husserl were Mathematicians by education. When they turned later into Philosophers, their initial work on Mathematics should not be considered as that of a Philosopher. The additional question is whether the mathematical community recognized this education. This might not have been the case for Husserl; but it was the case for Russell. Yet when he became one of the world's most famous Philosophers of the 20th century, he became increasingly disconnected from Mathematics. Only at the time he was publishing his opus magnum, the *Principia Mathematica* written together with Whitehead, he was (still) considered as a Mathematician. Now, a closer inspection shows that despite the tremendous *logical* work expounded in the *Principia*, there is essentially no new *mathematical* inside which could be attributed to Russell, and, from a modern perspective, one could actually question his status as Mathematician.

Mathematicians as Philosophers

How a Mathematician can be considered as a Philosopher is a rather intricate question.

One could take the position that everybody is doing Philosophy, and becomes a Philosopher, as soon as (s)he is reflecting in one or another way on his or her scientific activity. In this view, nearly every Scientist would turn into a Philosopher—even against his/her explicit will, as it would probably be the case for Bourbaki.

On the other hand, one could reserve the title "Philosopher" to those who enter in a predefined philosophical career and expose his/her reflections in terms of scholarly philosophical standards. With this criterion one would probably find only a handful of distinguished Mathematicians who could be considered Philosophers (like Descartes, Pascal, and Leibniz—but not even Frege).

We would like to steer a middle course, or better: two middle courses.

First, we may qualify a Mathematician as a Philosopher if (s)he is building his/her mathematical work on a clear philosophical conviction. And this conviction should have priority in the sense that, in case of problematic mathematical results, Mathematics is rethought but not the philosophical foundation. Pythagoras falls into this category, but the prime example is probably Brouwer (van Dalen 2013).

Secondly, a Mathematician can be considered as Philosopher, if there is an echo from Philosophy proper in the specific contribution of the Mathematician; i.e., the philosophical discussion is taking up ideas from the Mathematician. And the Mathematician takes part in this discussion. Here, Frege will be the best example. Cantor, however, who tried to take part in philosophical discussions, was, at the end of the day, rather neglected by the philosophers.

The Working Mathematician

So far, we have only mentioned leading Mathematicians and their relation to Philosophy. For the average Mathematician, it is probably the case that (s)he reflects even less on philosophical issues. In a reply to Whitehead concerning Swift's description of the Mathematicians in Gulliver's voyage to Lapute, Philip Jourdain (1915, p. 638) writes:

> …Swift, like everybody else, could not doubt the usefulness, importance, and correctness of the mathematician's work, but shared, with the philosopher, a doubt of the mathematician's being able to state his principles clearly and reasonably, just as we may doubt the existence of a knowledge of thermodynamics in a man who drives a railway engine.

This might be true for the working Mathematician in the role of the engine driver. But the comparison is flawed if the Philosopher should take over the role of the Physicist studying thermodynamics: such a Physicist might not be able to drive a locomotive, in the same way as a Philosopher does not do Mathematics. But, as a rule, the theoretical study of the Physicist contributes to the development of better engines. This is, unfortunately, rarely the case with contributions of Philosophers to Mathematics.[5]

6.2 David Hilbert as Philosopher

> Don't raise great hopes from Hilbert's philosophical "results". I'm already disappointed by what he produced of it so far. […] That he feels the need for more, is quite fine, but as soon as he leaves the purely mathematical, he is simply silly.
>
> Nelson to Hessenberg, June 1905[6]

Hilbert as Mathematician

David Hilbert was one of the greatest Mathematicians of all time. While sharing the title with Henri Poincaré of leading Mathematician around the turn of the 19th to 20th century, he occupied this position unchallenged after Poincaré's death in 1912 up to the 1930s. In this time, he also promoted a foundational programme in Mathematics, which has a strong philosophical rationale. But it is clear that the authority of Hilbert was based on his mathematical work and the influence he had on the mathematical community.

In his mathematical work, two results are of importance from a philosophical perspective.

In his solution of *Gordan's Problem* Hilbert provided a *non-constructive* existence proof; i.e., he was proving an existential statement without providing a method to find a witness for it. This was quite contrary to the understanding of existential statements by that time, and the German Mathematician Gordan, who raised the problem said[7]:

[5] Von Plato (2016) pointed to such a rare case, when Kant triggered with his discussion of the equation $7 + 5 = 12$ the development of modern recursive foundations of Arithmetic through the obscure figures of Johann Schultz and Michael Ohm; via Grassmann, Hankel, Schröder, Dedekind it finally reaches Peano, Skolem, and Bernays.

[6] The translation is ours; German original in (Peckhaus 1990, p. 166): "Von Hilberts philosophischen 'Resultaten' mach' Dir nun lieber keine großen Hoffnungen. Ich bin bereits durch das, was er bisher davon produziert hat, recht enttäuscht. […] Daß er das Bedürfnis nach mehr fühlt, ist ja auch sehr schön, aber so wie er das rein Mathematische verläßt, wird er einfach albern."

[7] The anecdote, including Hilbert's relation to Kronecker, is told by Reid (1970, Ch. V); the citation of Gordan is on page 34. McLarty (2012) gives a detailed account on the story.

"That's not Mathematics. That's Theology." The use of non-constructive methods was also contrary to the philosophical standpoint of Kronecker, one of the leading Mathematicians at that time. Kronecker was already involved in a philosophical debate with Dedekind on the use of abstract methods in Mathematics, and was also famous as an opponent of Cantor's set theory. Hilbert was clearly aware of these debates, and Reid (1970, p. 31) reports that he was very much impressed by a personal discussion with Kronecker, giving him 4 pages in his notebook, while any other mathematician only had at most one page.

The second contribution was his new axiomatization of Euclidean Geometry (Hilbert 1899). This work has, in the first place, a mathematical objective: finding a new axiomatization of Euclidean Geometry which is more appropriate with respect to modern developments (including the discovery of Non-Euclidean Geometry); he highlights, in particular, Desargues's Theorem and Pascal's (Pappus's) Theorem, and the way they are derivable in the new axiomatization.[8] Conceptionally it also builds on prior work by Pasch.

Both issues, non-constructive proofs and axiomatizations, started to merge when Hilbert raised the question of consistency of axiom systems. As Geometry can be reduced to a theory of the real numbers, the consistency problem is stated for arithmetic (including analysis)—quite prominently—in the second of his famous 23 problems given at the ICM in Paris in 1900. It was Hilbert's hope that consistency proofs would not only secure mathematical reasoning, but also provide proper meaning to non-constructive features of proofs.

Hilbert's Foundational Programme

Hilbert's Foundational Programme is often caricatured as a naive formalist endeavor to prove the consistency of formal systems by finitistic means. While, of course, consistency was its driving force, and formalistic elements occupied at some stage a prominent place, the full history of the programme is much more subtle and involve various philosophical viewpoints.

Hilbert aimed to secure usual mathematical reasoning. Being aware of the failure of Frege's *Grundgesetze*, it was Logic which attracted Hilbert's attention first. In the 1910s he submitted Whitehead and Russell's *Principia* to a thorough evaluation (see Kahle 2013). By this time, one finds evidence that could allow to count Hilbert as a Logicist. However, in 1920 at latest, Hilbert came to the conviction that Russellian Logicism fails because of the problematic status of the reducibility axiom.

By that time, Hilbert's own approach, his *Beweistheorie*, already took shape. It was first sketched in his contribution to the ICM 1904 in Heidelberg, but put aside for a while, probably because of criticism by Poincaré (see Kahle 2014). Apparently, it was Brouwer who gave him the idea to overcome this criticism by distinguishing between induction on the object and on the meta level (as we would call it now).[9] With this tool at hand, he proposed the search for consistency proofs in a weak meta theory for a formalized strong object theory. While there were some promising first

[8] A detailed account to the development is given in (Toepell 1999).

[9] See (Brouwer 1927, Footnote 1).

results by Ackermann and von Neumann, Gödel's incompleteness theorems show the unfeasibility of this programme in its original form.

This negative result, however, did not stop Hilbert. In contrast to Frege with respect to his logicistic programme, he did not give up hope, but simply proposed a change of the underlying philosophical framework.[10] And, as soon as 1936, Gentzen was able to establish a consistency proof for Arithmetic which is in line with the new foundational standpoint. In fact, proof theory continues to pursue this *revised Hilbert programme* to this day (Kahle 2015).

Was Hilbert a Philosopher?

We opened this section with the disappointing report from Nelson to Hessenberg about Hilbert's "silliness" in philosophical questions. Could such a judgement be justified in view of Hilbert's importance in the Philosophy of Mathematics? We think that, indeed, it can be.

Based on textual evidence, there are two motivations for Hilbert's foundational interests: the set-theoretical paradoxes and his opposition to the *Ignorabimus* of Bois-Reymond (see, e.g., Tapp 2013).

It is important to note that, as much as the paradoxes are concerned, Hilbert's interest was mathematically motivated, not philosophically motivated. He was quite explicit that the paradoxes should be resolved by mathematical means and he aimed to "eliminate once and for all the questions regarding the foundations of mathematics" (Hilbert 1967, p. 464). At best, we have here a "meta-philosophical" position, which, in fact, tries to "overcome Philosophy" by mathematical means.[11]

When it comes to Hilbert's epistemological optimism, he exposes, however, a genuine philosophical position. Although Hilbert tried to use Mathematics (and Meta-Mathematics) to underpin this optimism, a proper defense of it would need to be provided on philosophical grounds.[12]

Let us now compare Hilbert with the criteria which we have worked out in the previous section.

The first observation is that his texts lack a proper respect for the philosophical tradition. Of course, he might have read Plato and/or Aristotle (we have no evidence for or against it, but is might well have been part of his high school education); he

[10]"Besonders interessiert hat mich der neue meta-mathematische Standpunkt, den Sie jetzt einnehmen und der durch die Gödelsche Arbeit veranlaßt worden ist." ("I was particularly interested in the new meta-mathematical standpoint which you now adopt and which was provoked by Gödel's work.") Ackermann in a letter to Hilbert, August 23rd, 1933 (Ackermann 1933, p. 1f).

[11]Gentzen expressed it in these words: (Gentzen 1938, p. 237 in the english translation)

> A foremost characteristic of *Hilbert's* point of view seems to me to be the endeavour to withdraw the problem of the foundations of mathematics from *philosophy* and to tackle it as far as in any way possible with methods proper to mathematics.

[12]This was, in fact, one of the central points of origin of Brouwer's criticism (see Brouwer 1927, Third insight).

read some work of Kant, as Reid reports (1970, p. 194): "[…] he had smilingly commented to a young relative that a lot of what Kant had said was 'pure nonsense'".[13] And Maurice Janet, who was as a student in Göttingen in 1912, recorded Hilbert's contempt of Philosophy by the following quote[14]: "[Hilbert] has uttered the thought that he would be happy if all libraries in the world burned down, 'the mathematicians alone could reconstruct the mathematics, the philosophers would be quite embarrassed.'" In his writings there are essentially no scholarly references to Philosophy. One even has to be careful with his later publications as they are, in a good part, influenced by Paul Bernays, who, indeed, had a firm philosophical background.[15]

Even in the discussions on the Foundations of Mathematics with Brouwer, he was not taking proper notice of his opponent's work. As Smoryński (1994) argues, Hilbert was rather responding to the ghost of Kronecker or to Weyl than to Brouwer. And even more, his arguments are not carefully pondered philosophical reasons but sometimes just polemic statements that could hardly stand up to scientific standards. An extreme case is reported by Reid (1970, p. 184): "'With your methods,' [Hilbert] said to Brouwer, 'most of the results of modern mathematics would have to be abandoned, and to me the important thing is not to get fewer results but to get more results.'"

Thus, even for his contributions to the Philosophy of Mathematics—which are, of course, tremendous—one cannot claim that Hilbert was properly involved in it on philosophical ground.

Insofar as a philosophical conviction is concerned, let us assume, for the sake of the argument, that Hilbert could be judged as a Finitist and Formalist. But these positions could, on no account, be considered as a primary philosophical standpoint which "goes ahead" of the mathematical results. It is known that for a certain period preceding Formalism, Hilbert had a quite strong interest in Logicism. But even at that time he was able to astonish philosophers by asking for a *reform of logic*: "To eliminate the contradiction in set theory, [Hilbert] wants to reform (not set theory but) logic."[16] His attitude towards *inhaltliche Mathematik* (contentual Mathematics) can well be described in terms of Platonism.[17] We are, in fact, confronted with a mixup

[13] For the more positive evaluation of the influence of Kant on Hilbert, see Sinaceur (2018) in this volume.

[14] French original from Janet's notebook: "[Hilbert] a émis l'idée qu'il serait heureux que toutes les bibliothéques du monde brûlassent, ≪les mathématiciens seuls pourraient reconstruire les mathématiques, les philosophes seraient bien embarrassés≫." (Mazliak 2013, p. 55).

[15] This is explicit, for instance, in (Hilbert 1967, p. 479): "I would like to note further that P. Bernays has again been my faithful collaborator. He has not only constantly aided me by giving advice but also contributed ideas of his own and new points of view, so that I would like to call this our common work." It is also known that the opus magnum of Hilbert and Bernays, the *Grundlagen der Mathematik* (Hilbert and Bernays 1934; 1939), was essentially entirely written by Bernays.

[16] "Um den Widerspruch in der Mengenlehre zu beseitigen, will er [Hilbert] (nicht etwa die Mengenlehre sondern) die Logik reformieren." Nelson in a letter to Hessenberg, June 1905, cited in (Peckhaus 1990, p. 166).

[17] See the discussion of this point by Bernays (1935).

of philosophical positions. And, after Gödel's results,[18] Hilbert had no problem at all with abandoning the original Finitism and promoting transfinite methods in Meta-Mathematics.[19]

As another example let us mention the apparent centrality of *consistency* in Hilbert's foundational programme. Kreisel (2011, p. 43) reports the following anecdote: "According to Bernays [...] Hilbert was asked [...] if his claims for the ideal of consistency should be taken literally. In his (then) usual style, he laughed and quipped that the claims serve only to attract the attention of mathematicians to the potential of proof theory."

Thus, in conclusion, Hilbert hardly fulfills any of the criteria we have described for a Mathematician being a Philosopher. But having said this, there is something which distinguishes him among other Mathematicians (and Philosophers of Mathematics): he had a deep *philosophical vision*, namely to solve philosophical questions in Mathematics by mathematical means. One may even call it a *Meta-Philosophy*. And it is due to this vision—and due to the success of this vision—that there can be no doubt that Hilbert would have been more than welcome to enter Plato's Academy.

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[18] According to Bernays (1935, p. 215) the first steps towards such a shift took already place before Gödel's result became known.

[19] For the necessary "philosophical switch" see, for instance, Bernays (1954, p. 4), and the letter of Ackermann cited in Footnote 10 above.

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