

Relativistic calculations of screening parameters and atomic radii of neutral atoms

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Abstract

Calculations of the effective nuclear charge for elements with $1 \leq Z \leq 118$ have been performed in a Dirac-Fock approach including all relativistic effects as well as contributions from quantum electrodynamics. Maximum charge density for every subshell of every element in the periodic table were also computed in the same framework as well as atomic radii based on the total charge density. Results were compared with the extensively cited works of Clementi *et al.*, obtained in the 1960's with Roothan's self-consistent-field method.

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1. Introduction

The usefulness of simple analytical functions for describing the electronic orbitals has been demonstrated long ago [1], and even now, with very high computer processing power, some codes rely on them to obtain physical properties that range from g -factors [2] and crystallographic defect structures [3], electron-hole interactions in layered materials [4] to molecular equilibrium structures and force constants [5]. In fact, the materials science community has been very active in searching new materials with targeted properties using state-of-the-art codes that still rely on atomic radii calculated with Slater-type-orbitals (STO) [6]. Quantum chemistry program packages, for instance, rely heavily on these analytical functions, with libraries of basis sets that span over several types of wavefunctions, from single STO to linear combinations of Gaussian-Type functions [7]. Electron and photon scattering are some of the physical processes in which screening parameters are still very much in use [8], with several calculations relying on atomic radii obtained through the use of screened potentials [9]. Scattering cross sections of both neutral atoms and ions make use of the effective nuclear charge [10–12] and atomic form factors that can be evaluated in the independent particle model using screened

hydrogenic wavefunctions [13]. Tong *et al.* [14], for instance, have investigated two-photon transitions in atomic inner shells by evaluating the relativistic and atomic-screening effects on the decay rates, and found that electron screening has a sizable effect in the two-photon decay rate by expanding the outer-shell wavefunctions, hence decreasing orbital overlap. Multielectron quantum electrodynamic (QED) computations still employ several approaches that rely on the screening of the nuclear potential in order to evaluate the electron's self-energy by providing a scaling factor to the hydrogenic expression [15–17]. Also, atomic and covalent radii are usually calculated on several codes with simple analytical expressions based on Z_{eff} [18, 19], which also renders this method as a versatile tool for teaching periodic trends in the classroom [20].

The definition of atomic radii varies greatly within the literature (see [15] and references therein), dating back as far as the work of Slater which noticed a correlation between the maximum charge density of the outermost electron shell and ionic radius of an atom [21]. Other authors, however, have chosen either the simple [22] or weighted [23] mean radius of specific orbitals to infer the atom size, while others defined it as half the distance of the molecular single bond length in heteronuclear diatomic molecules [18, 24]. One particular use of the atomic radii is in the measuring of drift times of atoms in gases, which are used in velocity filters, for example in the isolation of superheavy elements in buffer-gas traps [25, 26]. Although the radii data of superheavy elements is scarce, the experimental results of the ionic radii of Am^+ , Pu^+ , Fm^+ and Cf^+ seem to favor a mean spherical radius description over the mean atomic radius based on the total electronic density [15].

The STO functions, which are still very useful, especially in codes that calculate solid state properties, due to the large number of evaluated orbitals, can be written as

$$\psi_{n,l,m} = N r^{n-1} e^{-\frac{Z_{\text{eff}}}{n} r} Y_{l,m}(\theta, \phi), \quad (1)$$

where n, l, m are, respectively, the principal, orbital and magnetic hydrogenic quantum numbers, N is the normalization factor, and $Y_{l,m}(\theta, \phi)$ are the spherical harmonics, whereas the effective nuclear charge Z_{eff} is just the nuclear charge, Z , minus the screening constant $\sigma_{n,l}$

$$Z_{\text{eff}} = Z - \sigma_{n,l}. \quad (2)$$

Clementi *et al.* [27, 28] performed a systematic calculation in the 1960's of the screening constants of all orbitals for neutral atoms with $1 \leq Z \leq 86$, using Roothan-Hartree-Fock (RHF) Self-Consistent-Method with STO, and their results are still a reference, as seen not only by the sheer number of citations of both papers (more than 3000), but also their substantial increase in the last decade. The fact that the STO, used in their HF computation, lack radial nodes by definition, will result in drastically different average radius and Z_{eff} for wavefunctions with one or more nodes.

In this work, we have calculated screening constants, effective nuclear charges and atomic radii for every shell of neutral atoms with $1 \leq Z \leq 118$, using the relativistic Multiconfiguration Dirac-Fock General Matrix Element (MCDFGME) program package [29, 30] in the monoconfiguration mode. The effective nuclear charge is obtained through a routine that evaluates many-electron radiative corrections, being Z_{eff} deduced from the comparison of the mean value of the radius of the Dirac-Fock (DF) orbital with that of the hydrogenic one [31]. These QED contributions, such as the vacuum polarization and electron self-energy, contribute to the high reliability of this code in obtaining high precision level energies and radiative and radiationless transition rates [32, 33]. The fact that our calculations are fully relativistic result in wavefunctions that, in some orbitals, deviate quite strongly from the non-relativistic calculations of Clementi

et al. [27, 28], which in turn result in different atomic radii and screening constants.

2. The Dirac-Fock method

The Dirac-Fock (DF) method implemented in the MCDFGME computer code has been thoroughly described elsewhere [29, 30, 34], and it starts from a no-pair Hamiltonian, which is written as

$$\mathcal{H}^{\text{nopair}} = \sum_{i=1}^N \mathcal{H}_D(r_i) + \sum_{i<j} v(r_{ij}), \quad (3)$$

where \mathcal{H}_D is a one-electron Dirac operator and v is an operator that represents the two-body interaction and r_{ij} is the inter-electronic distance ($|r_i - r_j|$). The operator v combines the Coulomb potential with retardation and magnetic interactions, as well as projection operators [35]. The total wave function is then calculated with the help of the variational principle. We can write the total energy of the ion as

$$\mathcal{H}^{\text{nopair}} |\Psi_{\Pi,J,M}(r_1, \dots, r_N)\rangle = E_{\Pi,J,M} |\Psi_{\Pi,J,M}(r_1, \dots, r_N)\rangle, \quad (4)$$

where Π is the parity, J is the total angular momentum eigenvalue, and M is the eigenvalue of its projection on the z axis. **The DF method is then defined by the particular choice of the trial functions to solve Eq. (4), which are linear combinations of configuration state functions (CSF):**

$$|\Psi_{\Pi,J,M}\rangle = \sum_{\nu} c_{\nu} |\nu\Pi JM\rangle. \quad (5)$$

These trial functions are eigenfunctions of the parity Π , the total angular momentum J^2 , and its projection J_z . The label ν stands for all other numbers (principal quantum number,...) necessary to define unambiguously the CSF. The c_{ν} are called the mixing coefficients and are obtained by diagonalization of the Hamiltonian matrix coming from the minimization of the energy in Eq. (4) with respect to the c_{ν} . In the present work, since we remain at the DF level, those coefficients will only account for the intermediate coupling. The configuration state functions are antisymmetric products of one-electron wave functions expressed as a linear combination of Slater determinants of Dirac 4-spinors, whose coefficients, d_i , are evaluated by requiring that the configuration state functions are eigenstates of J^2 and J_z ,

$$|\nu\Pi JM\rangle = \sum_i d_i \begin{vmatrix} \Phi_1^i(r_1) & \cdots & \Phi_N^i(r_1) \\ \vdots & \ddots & \vdots \\ \Phi_1^i(r_N) & \cdots & \Phi_N^i(r_N) \end{vmatrix}, \quad (6)$$

where the Φ -s are the one-electron wave functions. In the relativistic case, the one-electron wave functions are the Dirac four-component spinors:

$$\Phi_{n\kappa\mu}(r) = \frac{1}{r} \begin{bmatrix} P_{n\kappa}(r)\chi_{\kappa\mu}(\theta, \phi) \\ iQ_{n\kappa}(r)\chi_{-\kappa\mu}(\theta, \phi) \end{bmatrix}, \quad (7)$$

where $\chi_{\kappa\mu}(\theta, \phi)$ is a two component Pauli spherical spinors [36], and $P_{n\kappa}(r)$ and $Q_{n\kappa}(r)$ are the large and the small radial components of the wave function, respectively. The functions $P_{n\kappa}(r)$ and $Q_{n\kappa}(r)$ are the solutions of coupled integro-differential equations obtained by minimizing the energy as defined in Eq. (4) with respect to each radial wave function.

2.1. Atomic radii and effective atomic number definitions

There are several different ways of defining the atomic radius of an atom or ion. In this work, we follow the definitions of Indelicato *et al.* [15], where besides the usual definition of the mean radius of a specific orbital as

$$\langle r \rangle_{\text{DF}} = \langle \Phi_{n\kappa\mu} | r | \Phi_{n\kappa\mu} \rangle, \quad (8)$$

and the radii of maximum density of every subshell, r_{max} , defined as the radius where $|r\Psi(r)|^2$ is maximum, we also define the mean atom radius $\langle r_{\text{at.}} \rangle$ and the mean spherical radius $\sqrt{\langle r_{\text{at.}}^2 \rangle}$. These two last quantities are based on the total electronic density, and the radial electronic density is defined as [15]

$$r^2\rho(r) = \int r^2\rho(r, \Omega)d\Omega = \sum_{i \in \text{occ. orb.}} \varpi_i (P_i(r)^2 + Q_i(r)^2), \quad (9)$$

where P and Q are defined in Eq. (7). The ϖ_i are the orbital effective occupation numbers,

$$\varpi_i = \sum_j c_j^2 n_j^i, \quad (10)$$

and the sum extends over all configurations containing orbital i , whereas n_j^i is the occupation number of this orbital in the configuration j . The total density is then normalized to

$$\int_0^\infty r^2\rho(r)dr = N_e, \quad (11)$$

where N_e is the number of electrons in the atom. In this way, the mean atom radius is represented as

$$\langle r_{\text{at.}}^{(p)} \rangle = \frac{1}{N_e} \int_0^\infty r^{(p)} r^2\rho(r)dr, \quad (12)$$

for $p = 1$, and the mean spherical radius is obtained as $\sqrt{\langle r_{\text{at.}}^{(p)} \rangle}$ for $p = 2$.

The effective nuclear charge, Z_{eff} , is then calculated by solving numerically the equation

$$\langle r \rangle_{\text{DF}} = \langle r(Z_{\text{eff}}) \rangle_{\text{Hydr.}}, \quad (13)$$

where the point-nucleus, Dirac hydrogenic mean orbital radius is given by [37–40]

$$\langle r(Z) \rangle_{\text{Hydr.}} = \frac{(n - |\kappa| + \gamma)(3N^2 - \kappa^2) - \kappa N}{2ZN}, \quad (14)$$

with $\gamma = \sqrt{\kappa^2 - (Z\alpha)^2}$ and $N = \sqrt{(n - |\kappa| + \gamma)^2 + (Z\alpha)^2}$.

3. Results and discussion

3.1. Effective Z

The effective nuclear charge values of all subshells of elements with $1 \leq Z \leq 118$ obtained in this work are presented in Fig. 1. The overall behavior is as expected, with inner shells presenting smoother curves as higher shells become filled. As can be seen from Fig. 2, where the Z_{eff} for the L_1 , L_2 and L_3 subshells are plotted as a function of Z , there are some differences that can be quite substantial in the obtained Z_{eff} when compared to the results of Clementi *et al.* These differences arise not only from the fact that their calculation is nonrelativistic, resulting in orbitals that deviate

strongly from the DF orbitals due to the relativistic contraction of the wavefunction, but also because they employed minimum basis sets composed of nodeless STO. **Although each STO is a nodeless function, due to orthogonality constraints in the SCF computation, the wavefunctions of the same symmetry are coupled. This means that the final wavefunctions will have the right number of nodes, but at the same time it means that the correspondence between the optimized orbital exponents and the screening parameters cannot be performed unambiguously. This is one of the reasons for the discrepancies observed in Fig. 3** where the Z_{eff} for every subshell of Kr, Xe and Rn are compared to the results of Clementi *et al.*. The DF calculated Z_{eff} values for every subshell of elements with $1 \leq Z \leq 118$ are presented in Tables 1, 2 and 3. QED contributions such as self-energy and vacuum polarization were included self consistently in our calculations, however their effect is almost negligible, contributing only with a maximum of 0.2% to the Z_{eff} . Note, however, that we have used configurations corresponding to the ground state of a Dirac-Fock calculation (i.e., without correlation), as given in [41]. For a few elements, the ground state configuration, as given for example on several databases, can be different. Nevertheless, as shown in [15] the lack of electronic correlation can give rise to variations in the atomic radii of less than 1% for high-Z elements.

3.2. Atomic radii

The use of nodeless wavefunctions combined with the absence of relativistic effects in their calculation will result in radii of maximum charge density of the valence orbital that deviate from our results by around 3% for He, increasing to 40% for Hg. For La and Ce the radii show an even greater disparity, which is due to the fact that Clementi *et al.* had convergence problems due to the extreme broadness of the 4f orbital. For individual orbital radii, the differences can be even higher for some subshells, especially those which are not well described by a single STO. The divergence of results can be traced to the quality of the wavefunctions, as demonstrated in Fig. 4 since the definition of the r_{max} is the same in both works. In Table A the average radii of the 1s, 2s, $2p_{1/2}$, $2p_{3/2}$ and 3s subshells of Rn calculated both with the DF and HF methods, are also compared with the $\langle r \rangle$ values of STO calculated with the optimized exponents of Clementi *et al.*, illustrating not only the effect of the relativistic treatment of the wavefunction, but also the quality of the STO wavefunctions.

As can be readily seen, the $2p_{3/2}$ wavefunction is very similar in both the DF and HF approach calculated in this work, and, since it has no nodes, it compares very well to the results of Clementi *et al.*. However for subshells in which the relativistic contraction is more evident and especially for wavefunctions with one or more nodes, the results deviate substantially. The radii of maximum charge density of every subshell, r_{max} , are also presented in Tables 4, 5 and 6. In Table 7 we present the ground state mean atomic radius and the atom mean spherical radius, calculated using the total charge distribution, as described by Indelicato *et al.* [15] for all the elements of the periodic table. These values can be important in atomic drift spectroscopy as the drift time of an atom in a gas cell depends on its initial velocity and its charge distribution radius, while the maximum charge density of the valence orbital is of greater importance in molecular bonding for example.

In Fig. 5 we present the comparison of the average radius of the outer shell of neutral elements and compare them with the results of Clementi *et al.* as well as with the covalent radii values of Cordero *et al.* [19] and Pyykkö *et al.* [24]. The covalent radii of Cordero *et al.* and Pyykkö *et al.* were calculated having in mind the additive rule of the atomic radii for bond length calculation, and hence their results are highly dependent on the orbital hybridization,

$\langle r \rangle$	DF	HF	[28]
1s	0.0151	0.0176	0.0176
2s	0.0626	0.0738	0.0788
2p _{1/2}	0.05126	0.06235	
2p _{3/2}	0.06062	0.06235	0.0614
3s	0.1625	0.1846	0.1389

Table A

Average radius, $\langle r \rangle$ for the 1s, 2s, 2p_{1/2}, 2p_{3/2} and 3s orbitals of Rn calculated using the Dirac-Fock and Hartree-Fock methods. The $\langle r \rangle$ values are also compared with the ones computed with a single STO with orbital exponents obtained by Clementi *et al.*.

molecular bond type, coordination number and whether they show a single- or multiple-bond character [24, 42]. In fact, it is well known that the cationic radius of an element is typically at least 0.5 Å shorter than its covalent radius, while the opposite situation occurs with the anionic radii, showing how strongly the effective radius of an atom depends on its electronegativity relative to that of the atom to which it is bonded. Note, however, that our values correspond to the radial values of the probability distribution maximum of the valence shell, which should not be compared directly with the covalent radii of every element, as the features described above can be very important for different groups of elements in the periodic table. For instance, the bonds between transition metals and halides tend to show a multiple-bond character, and in fact the triple bond covalent radii of Pyykkö *et al.* [42] show better agreement with our halogens results than their single bond data [24]. We emphasize the fact that we have used the configurations corresponding to the ground state of a DF calculation (i.e., without correlation), as given in [41]. For some elements, the ground state configuration obtained through HF calculations as well as those given for example on the NIST database [43], can be different.

As can be readily seen in Fig. 5, our results compare very well with the values of Cordero *et al.* except for the halogens and rare gases, for which our values are substantially lower. This can be traced back to the fact that the molecular bonds of compounds with such elements do not usually involve one single orbital, having a multiple-bond character. On the other hand, our results for the radii of alkali metals are somewhat higher than those of both Cordero *et al.* and Pyykkö *et al.* but lower than those of Clementi *et al.* which can be attributed to relativistic effects. For Pd (Z=46) we notice a huge discrepancy with all the compared results, as the DF ground state configuration, which also corresponds to the one presented in the NIST database [43], is $[Kr]4d^{10}$ in contrast with its neighbors which have in their ground state configuration the 5s orbital. In fact, the calculated ground state configuration with the HF method of Clementi *et al.* [28] is $[Kr]4d^85s^2$, which explains the smoothness of their curve near Z=46. The fact that the covalent radii also presents the same smoothness, is due to the mixing of the 4d and 5s orbitals in the HOMO of Pd complexes [44].

4. Conclusions

We have calculated the effective nuclear charge and maximum charge density radii for every subshell of elements with $1 \leq Z \leq 118$ using the Dirac-Fock formalism. The results were compared to those of Clementi *et al.* which have been ubiquitously used for several decades, and the radii data with the results of Cordero *et al.* and Pyykkö *et al.*. Deviations up to 30% were found in the Z_{eff} , especially for the multiple node orbitals of high Z elements, and differences up to 40% have been found in the radii of maximum charge density of the valence orbitals with the results of Clementi *et al.*, while greater disparities were found in the comparison with the covalent radii of Cordero *et al.* and Pyykkö *et al.*. Although some of the physical properties that are usually calculated from the parameters discussed in this work are not very sensitive to changes in the mentioned parameters, others (such as covalent radii, vacuum polarization, self-energy, etc.) can be severely influenced by changes in the Z_{eff} and r_{max} . Consequently, we believe that our work could prove invaluable in updating the databases in which the Clementi *et al.* results are used.

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Figures

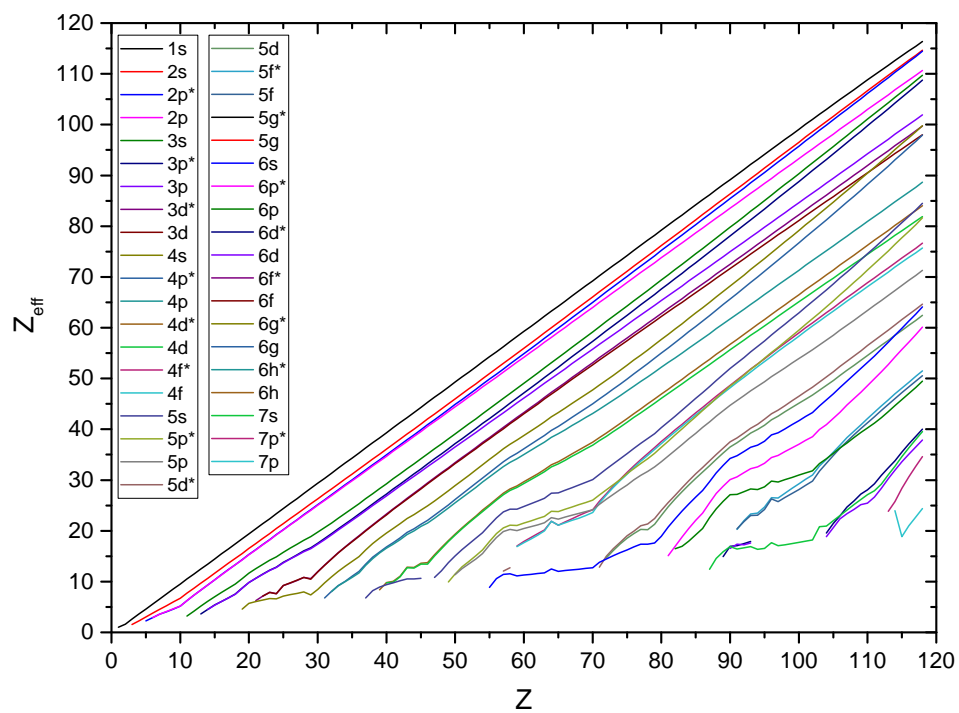


Fig. 1: Dirac-Fock effective nuclear charge as a function of the atomic number. Stared orbital labels correspond to the orbital with $j = l - \frac{1}{2}$.

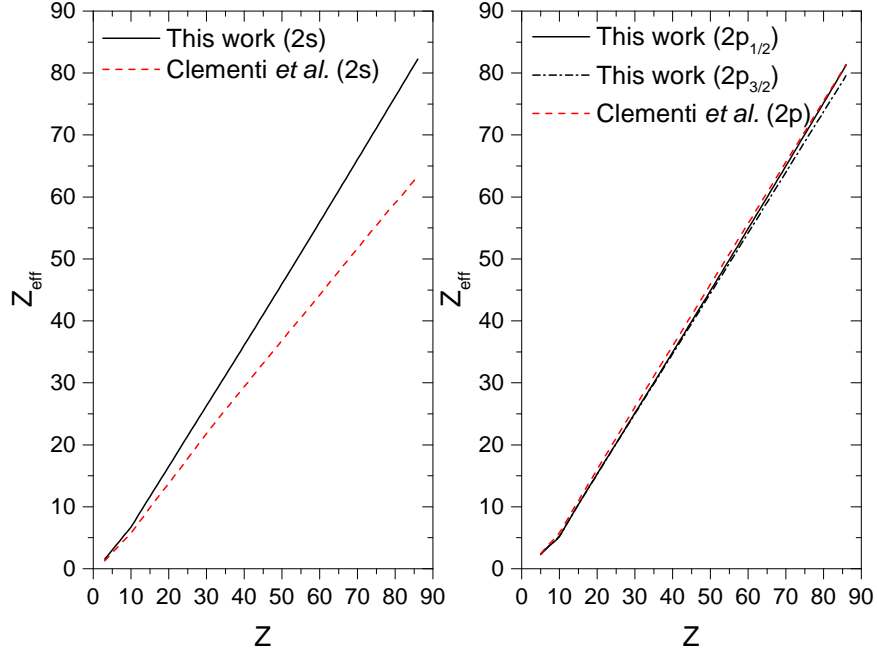


Fig. 2: Comparison of the effective nuclear charge calculated in this work with the results of Clementi *et al.* [27, 28]

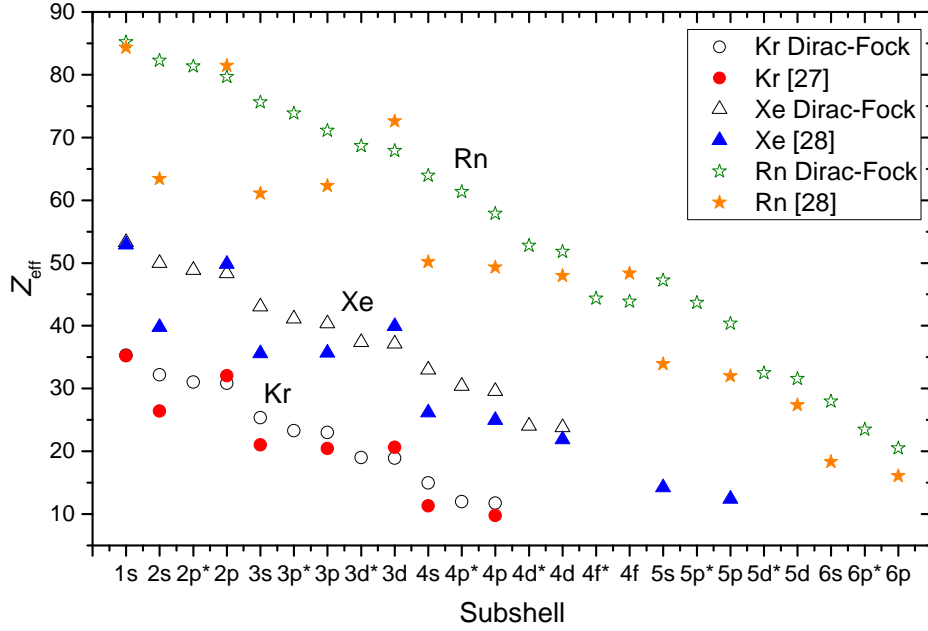


Fig. 3: Comparison of the Dirac-Fock and Hartree-Fock effective nuclear charge calculated in this work for all subshells of Kr, Xe and Rn. Stared orbital labels correspond to the orbital with $j = l - \frac{1}{2}$.

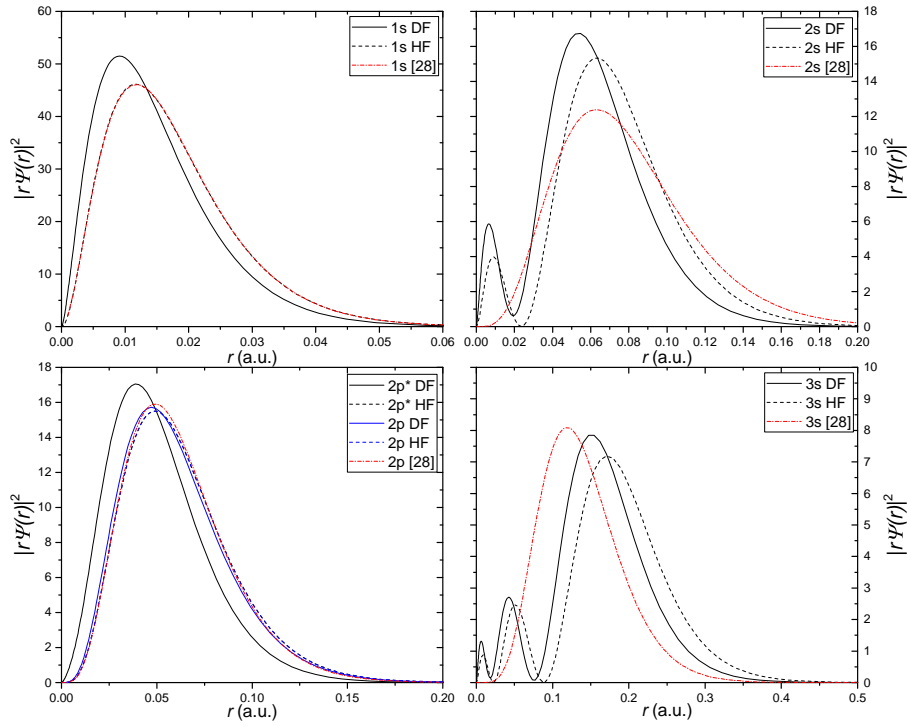


Fig. 4: Hartree-Fock and Dirac-Fock radial probability densities for the $1s$, $2s$, $2p_{1/2}$, $2p_{3/2}$ and $3s$ subshells of Rn. Shown also are STO with the orbital exponents obtained by Clementi *et al.* [27, 28].

Explanation of Tables

Table 1. Effective nuclear charge calculated with the Dirac-Fock method for subshells ranging from $1s$ to $4s$.

Configuration	Ground state configuration, obtained from reference [41], for which the calculation was performed.
$1s, 2s, 2p_{1/2}, \dots$	Atomic subshells.

Table 2. Effective nuclear charge calculated with the Dirac-Fock method for subshells ranging from $4p_{1/2}$ to $5f_{7/2}$.

Configuration	Ground state configuration, obtained from reference [41], for which the calculation was performed.
$4p_{1/2}, 4p_{3/2}, 4d_{3/2}, \dots$	Atomic subshells.

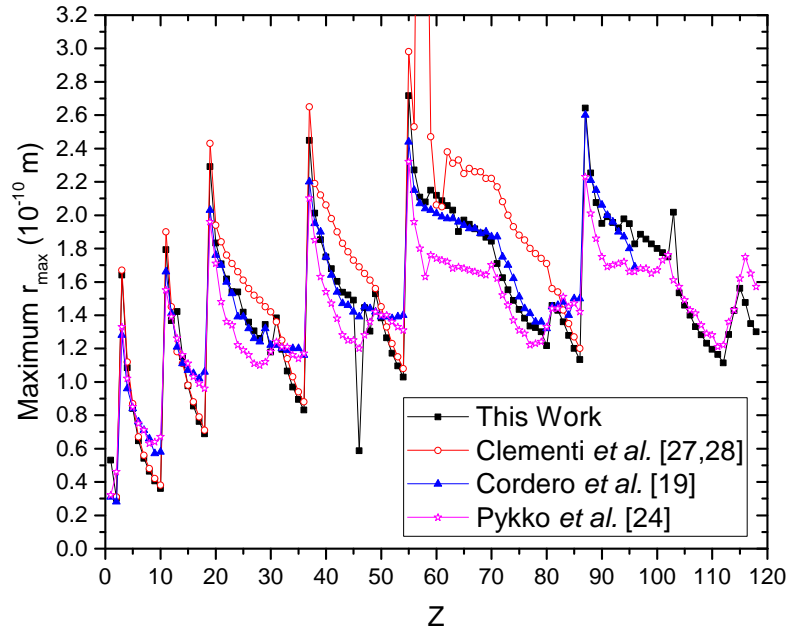


Fig. 5: Radii of maximum charge distribution, r_{\max} , of the valence shell of elements with $1 \leq Z \leq 118$. Presented also are the maximum charge distribution values of Clementi *et al.* [27, 28], as well as the single-bond covalent radii of Cordero *et al.* [19] and Pykko *et al.* [24].

Table 3. Effective nuclear charge calculated with the Dirac-Fock method for subshells ranging from $6s$ to $7p_{3/2}$.

Configuration	Ground state configuration, obtained from reference [41], for which the calculation was performed.
$6s, 6p_{1/2}, 6p_{3/2}, \dots$	Atomic subshells.

Table 4. Radii of maximum charge density, r_{\max} , calculated with the Dirac-Fock method for subshells ranging from $1s$ to $4s$ (a_0).

Configuration	Ground state configuration, obtained from reference [41], for which the calculation was performed.
$1s, 2s, 2p_{1/2}, \dots$	Atomic subshells.

Table 5. Radii of maximum charge density, r_{\max} , calculated with the Dirac-Fock method for subshells ranging from $4p_{1/2}$ to $5f_{7/2}$ (a_0).

Configuration	Ground state configuration, obtained from reference [41], for which the calculation was performed.
$4p_{1/2}, 4p_{3/2}, 4d_{3/2}, \dots$	Atomic subshells.

Table 6. Radii of maximum charge density, r_{\max} , calculated with the Dirac-Fock method for subshells ranging from $6s$ to $7p_{3/2}$ (a_0).

Configuration	Ground state configuration, obtained from reference [41], for which the calculation was performed.
$6s, 6p_{1/2}, 6p_{3/2}, \dots$	Atomic subshells.

Table 7. Atom mean radius, $\langle r_{\text{at}} \rangle$, using the total charge density and atom mean spherical radius, $\sqrt{\langle r_{\text{at}}^2 \rangle}$, of neutral elements with $1 \leq Z \leq 118$ (a_0).

Configuration	Ground state configuration, obtained from reference [41], for which the calculation was performed.
Atom mean radius	Atomic radius, $\langle r_{\text{at}} \rangle$, using total charge density from Eq. (12)
Atom mean spherical radius	Atomic spherical radius obtained as $\sqrt{\langle r_{\text{at}}^2 \rangle}$ from Eq. (12)

Explanation of Graphs

Graph 1. Dirac-Fock effective nuclear charge.

Z	Atomic number
Z_{eff}	Effective nuclear charge
*	Stared orbital labels correspond to the orbital with $j = l - \frac{1}{2}$.

Graph 2. Comparison of the $n = 2$ Z_{eff} with the results of Clementi *et al.* [27, 28].

Z_{eff}	Effective nuclear charge.
Z	Atomic number.

Graph 3. Comparison of the Dirac-Fock and Hartree-Fock effective nuclear charge calculated in this work for all subshells of Kr, Xe and Rn.

Z_{eff}	Effective nuclear charge.
*	Stared orbital labels correspond to the orbital with $j = l - \frac{1}{2}$.

Graph 4. Hartree-Fock and Dirac-Fock wavefunctions for the $1s$, $2s$, $2p_{1/2}$, $2p_{3/2}$ and $3s$ subshells of Rn.

$ r\Psi(r) ^2$	Radial probability density.
r	Distance to the nucleus in a_0
DF, HF	Dirac-Fock and Hartree-Fock values, respectively.

Graph 5. Radii of maximum charge distribution of the valence shell of elements with $1 \leq Z \leq 118$.

Z	Atomic number
Maximum r_{max}	Maximum charge density of the valence shell.

Table 1

Effective nuclear charge calculated with the Dirac-Fock method for subshells ranging from 1s to 4s.

Atomic Number	Configuration	1s	2s	2p _{1/2}	2p _{3/2}	3s	3p _{1/2}	3p _{3/2}	3d _{3/2}	3d _{5/2}	4s
1	1s	1.000									
2	1s ²	1.618									
3	1s ² 2s	2.617	1.550								
4	1s ² 2s ²	3.614	2.266								
5	1s ² 2s ² 2p	4.603	3.036	2.268							
6	1s ² 2s ² 2p ²	5.587	3.776	2.916	2.916						
7	1s ² 2s ² 2p ³	6.570	4.505	3.547	3.546						
8	1s ² 2s ² 2p ⁴	7.553	5.257	4.022	4.082						
9	1s ² 2s ² 2p ⁵	8.534	5.997	4.598	4.615						
10	1s ² 2s ² 2p ⁶	9.515	6.731	5.185	5.175						
11	[Ne]3s	10.498	7.708	6.268	6.256	3.211					
12	[Ne]3s ²	11.484	8.699	7.306	7.292	4.156					
13	[Ne]3s ² 3p	12.471	9.686	8.343	8.313	5.201	3.642				
14	[Ne]3s ² 3p ²	13.459	10.669	9.362	9.323	6.127	4.543	4.541			
15	[Ne]3s ² 3p ³	14.448	11.648	10.355	10.332	6.999	5.386	5.376			
16	[Ne]3s ² 3p ⁴	15.437	12.625	11.356	11.321	7.863	6.041	6.083			
17	[Ne]3s ² 3p ⁵	16.427	13.599	12.357	12.306	8.699	6.789	6.781			
18	[Ne]3s ² 3p ⁶	17.417	14.572	13.340	13.304	9.519	7.541	7.502			
19	[Ar]4s	18.408	15.550	14.330	14.289	10.601	8.726	8.685			4.593
20	[Ar]4s ²	19.400	16.530	15.320	15.275	11.677	9.837	9.791			5.709
21	[Ar]3d4s ²	20.394	17.510	16.303	16.262	12.549	10.673	10.633	6.216		6.090
22	[Ar]3d ² 4s ²	21.388	18.490	17.286	17.244	13.377	11.471	11.415	7.146	7.139	6.388
23	[Ar]3d ³ 4s ²	22.382	19.468	18.270	18.222	14.189	12.251	12.178	7.904	7.845	6.664
24	[Ar]3d ⁴ 4s	23.378	20.446	19.264	19.193	14.875	12.864	12.774	7.631	7.608	6.615
25	[Ar]3d ⁵ 4s ²	24.372	21.421	20.244	20.165	15.776	13.766	13.666	9.246	9.225	7.161
26	[Ar]3d ⁶ 4s ²	25.368	22.397	21.218	21.140	16.578	14.537	14.422	9.670	9.779	7.442
27	[Ar]3d ⁷ 4s ²	26.364	23.373	22.194	22.112	17.370	15.297	15.163	10.240	10.300	7.699
28	[Ar]3d ⁸ 4s ²	27.360	24.348	23.172	23.081	18.157	16.053	15.898	10.829	10.805	7.948
29	[Ar]3d ¹⁰ 4s	28.357	25.324	24.161	24.045	18.835	16.667	16.495	10.584	10.479	7.351
30	[Ar]3d ¹⁰ 4s ²	29.353	26.298	25.137	25.011	19.719	17.544	17.356	11.978	11.886	8.416
31	[Ar]3d ¹⁰ 4s ² 4p	30.349	27.273	26.114	25.976	20.623	18.468	18.257	13.272	13.228	9.782
32	[Ar]3d ¹⁰ 4s ² 4p ²	31.345	28.248	27.093	26.942	21.549	19.420	19.184	14.501	14.457	10.934
33	[Ar]3d ¹⁰ 4s ² 4p ³	32.341	29.224	28.071	27.909	22.489	20.370	20.136	15.694	15.613	11.995
34	[Ar]3d ¹⁰ 4s ² 4p ⁴	33.337	30.201	29.051	28.876	23.439	21.342	21.083	16.822	16.751	13.023
35	[Ar]3d ¹⁰ 4s ² 4p ⁵	34.333	31.179	30.033	29.844	24.397	22.324	22.037	17.929	17.850	14.003
36	[Ar]3d ¹⁰ 4s ² 4p ⁶	35.329	32.158	31.014	30.813	25.359	23.298	23.012	19.018	18.924	14.950
37	[Kr]5s	36.325	33.139	31.998	31.782	26.331	24.283	23.977	20.096	19.998	16.241
38	[Kr]5s ²	37.322	34.121	32.983	32.752	27.308	25.271	24.946	21.166	21.062	17.503
39	[Kr]4d5s ²	38.318	35.105	33.968	33.723	28.285	26.261	25.914	22.234	22.117	18.574
40	[Kr]4d ² 5s ²	39.315	36.089	34.955	34.695	29.264	27.250	26.882	23.290	23.160	19.575
41	[Kr]4d ⁴ 5s	40.311	37.074	35.943	35.666	30.240	28.238	27.848	24.335	24.199	20.463
42	[Kr]4d ⁵ 5s	41.308	38.060	36.932	36.637	31.220	29.226	28.812	25.361	25.232	21.403
43	[Kr]4d ⁵ 5s ²	42.305	39.048	37.922	37.610	32.203	30.214	29.776	26.384	26.247	22.398
44	[Kr]4d ⁷ 5s	43.302	40.036	38.912	38.582	33.181	31.201	30.738	27.406	27.251	23.217
45	[Kr]4d ⁸ 5s	44.299	41.025	39.903	39.555	34.163	32.189	31.698	28.416	28.253	24.107
46	[Kr]4d ¹⁰	45.296	42.014	40.897	40.528	35.143	33.176	32.656	29.429	29.271	24.893
47	[Kr]4d ¹⁰ 5s	46.294	43.005	41.890	41.501	36.127	34.164	33.614	30.430	30.264	25.864
48	[Kr]4d ¹⁰ 5s ²	47.291	43.997	42.884	42.474	37.113	35.152	34.573	31.428	31.254	26.871
49	[Kr]4d ¹⁰ 5s ² 5p	48.288	44.989	43.879	43.447	38.100	36.144	35.531	32.424	32.242	27.877
50	[Kr]4d ¹⁰ 5s ² 5p ²	49.285	45.982	44.875	44.421	39.088	37.138	36.491	33.419	33.228	28.896
51	[Kr]4d ¹⁰ 5s ² 5p ³	50.282	46.976	45.872	45.395	40.079	38.131	37.454	34.413	34.212	29.922
52	[Kr]4d ¹⁰ 5s ² 5p ⁴	51.280	47.971	46.869	46.370	41.071	39.129	38.416	35.405	35.195	30.951
53	[Kr]4d ¹⁰ 5s ² 5p ⁵	52.277	48.967	47.868	47.345	42.064	40.128	39.379	36.396	36.176	31.984
54	[Kr]4d ¹⁰ 5s ² 5p ⁶	53.274	49.963	48.867	48.319	43.060	41.128	40.344	37.387	37.157	33.017
55	[Xe]6s	54.271	50.960	49.867	49.295	44.058	42.130	41.309	38.377	38.137	34.056
56	[Xe]6s ²	55.269	51.958	50.868	50.270	45.058	43.134	42.275	39.368	39.116	35.099
57	[Xe]5d6s ²	56.266	52.957	51.870	51.245	46.059	44.140	43.241	40.358	40.094	36.137
58	[Xe]4f ⁵ 6s ²	57.264	53.960	52.876	52.225	47.065	45.148	44.211	41.345	41.078	37.059
59	[Xe]4f ³ 6s ²	58.262	54.966	53.886	53.209	48.075	46.159	45.186	42.322	42.068	37.871
60	[Xe]4f ⁴ 6s ²	59.259	55.971	54.894	54.189	49.081	47.170	46.153	43.308	43.044	38.775
61	[Xe]4f ⁵ 6s ²	60.257	56.976	55.903	55.170	50.089	48.182	47.120	44.294	44.016	39.676
62	[Xe]4f ⁶ 6s ²	61.255	57.982	56.913	56.151	51.097	49.195	48.085	45.280	44.985	40.572
63	[Xe]4f ⁷ 6s ²	62.252	58.989	57.924	57.132	52.106	50.214	49.047	46.275	45.944	41.464
64	[Xe]4f ⁷ 5d6s ²	63.250	59.994	58.933	58.111	53.112	51.224	50.008	47.253	46.907	42.431
65	[Xe]4f ⁹ 6s ²	64.248	61.004	59.947	59.095	54.126	52.238	50.978	48.220	47.886	43.263
66	[Xe]4f ¹⁰ 6s ²	65.246	62.013	60.960	60.077	55.137	53.253	51.942	49.196	48.851	44.158
67	[Xe]4f ¹¹ 6s ²	66.244	63.023	61.973	61.058	56.150	54.272	52.904	50.176	49.811	45.055

Table 1 (continued)

Atomic Number	Configuration	1s	2s	2p _{1/2}	2p _{3/2}	3s	3p _{1/2}	3p _{3/2}	3d _{3/2}	3d _{5/2}	4s
68	[Xe]4f ¹² 6s ²	67.241	64.033	62.988	62.040	57.163	55.292	53.865	51.156	50.768	45.953
69	[Xe]4f ¹³ 6s ²	68.239	65.044	64.003	63.021	58.178	56.313	54.825	52.138	51.723	46.851
70	[Xe]4f ¹⁴ 6s ²	69.237	66.055	65.019	64.003	59.193	57.337	55.785	53.121	52.674	47.748
71	[Xe]4f ¹⁴ 5d6s ²	70.234	67.065	66.034	64.983	60.207	58.356	56.742	54.092	53.626	48.697
72	[Xe]4f ¹⁴ 5d ² 6s ²	71.232	68.076	67.049	65.962	61.222	59.377	57.699	55.062	54.578	49.659
73	[Xe]4f ¹⁴ 5d ³ 6s ²	72.230	69.087	68.065	66.941	62.238	60.398	58.656	56.032	55.528	50.633
74	[Xe]4f ¹⁴ 5d ⁴ 6s ²	73.227	70.099	69.082	67.919	63.256	61.421	59.613	57.001	56.479	51.617
75	[Xe]4f ¹⁴ 5d ⁵ 6s ²	74.225	71.111	70.099	68.897	64.275	62.446	60.569	57.969	57.430	52.610
76	[Xe]4f ¹⁴ 5d ⁶ 6s ²	75.222	72.123	71.116	69.875	65.296	63.473	61.527	58.939	58.379	53.610
77	[Xe]4f ¹⁴ 5d ⁷ 6s ²	76.220	73.136	72.135	70.853	66.318	64.501	62.484	59.908	59.329	54.618
78	[Xe]4f ¹⁴ 5d ⁹ 6s	77.217	74.149	73.154	71.830	67.341	65.531	63.440	60.876	60.279	55.629
79	[Xe]4f ¹⁴ 5d ¹⁰ 6s	78.214	75.163	74.173	72.807	68.367	66.563	64.398	61.844	61.228	56.648
80	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	79.211	76.177	75.193	73.784	69.395	67.598	65.356	62.813	62.177	57.676
81	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p	80.208	77.192	76.214	74.761	70.425	68.635	66.314	63.782	63.125	58.708
82	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	81.204	78.207	77.236	75.737	71.456	69.674	67.273	64.752	64.074	59.745
83	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	82.200	79.222	78.258	76.713	72.489	70.715	68.233	65.721	65.022	60.788
84	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	83.197	80.238	79.281	77.689	73.525	71.759	69.193	66.691	65.970	61.836
85	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	84.192	81.254	80.304	78.665	74.562	72.805	70.154	67.661	66.918	62.888
86	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	85.186	82.270	81.329	79.640	75.601	73.853	71.115	68.632	67.866	63.945
87	[Rn]7s	86.182	83.287	82.354	80.615	76.642	74.904	72.076	69.602	68.814	65.008
88	[Rn]7s ²	87.176	84.304	83.380	81.590	77.685	75.957	73.038	70.573	69.762	66.076
89	[Rn]6d ² 7s ²	88.171	85.322	84.406	82.565	78.730	77.012	74.001	71.545	70.709	67.147
90	[Rn]6d ² 7s	89.165	86.340	85.434	83.539	79.777	78.069	74.963	72.516	71.657	68.222
91	[Rn]5f ² 6d7s ²	90.158	87.359	86.463	84.515	80.827	79.130	75.927	73.487	72.606	69.296
92	[Rn]5f ³ 6d7s ²	91.149	88.376	87.492	85.489	81.876	80.193	76.891	74.459	73.554	70.375
93	[Rn]5f ⁴ 6d7s ²	92.143	89.397	88.523	86.464	82.930	81.258	77.855	75.431	74.502	71.459
94	[Rn]5f ⁵ 7s ²	93.134	90.416	89.554	87.439	83.985	82.327	78.819	76.403	75.450	72.542
95	[Rn]5f ⁷ 7s ²	94.123	91.435	90.586	88.413	85.041	83.398	79.783	77.379	76.395	73.632
96	[Rn]5f ¹⁴ 6d7s ²	95.115	92.455	91.618	89.386	86.100	84.470	80.747	78.351	77.341	74.728
97	[Rn]5f ⁹ 7s ²	96.105	93.476	92.652	90.361	87.161	85.546	81.712	79.322	78.288	75.824
98	[Rn]5f ¹⁰ 7s ²	97.093	94.495	93.687	91.334	88.222	86.624	82.676	80.295	79.233	76.924
99	[Rn]5f ¹¹ 7s ²	98.081	95.515	94.722	92.307	89.286	87.706	83.641	81.268	80.178	78.029
100	[Rn]5f ¹² 7s ²	99.067	96.534	95.757	93.279	90.352	88.789	84.605	82.242	81.122	79.137
101	[Rn]5f ¹³ 7s ²	100.051	97.552	96.794	94.251	91.418	89.876	85.570	83.215	82.066	80.249
102	[Rn]5f ¹⁴ 7s ²	101.034	98.571	97.831	95.223	92.487	90.965	86.535	84.189	83.009	81.364
103	[Rn]5f ¹⁴ 7s ² 7p	102.017	99.589	98.868	96.194	93.558	92.057	87.499	85.161	83.952	82.485
104	[Rn]5f ¹⁴ 6d ² 7s ²	102.997	100.607	99.906	97.164	94.629	93.151	88.463	86.133	84.893	83.610
105	[Rn]5f ¹⁴ 6d ³ 7s ²	103.976	101.623	100.945	98.134	95.702	94.248	89.427	87.105	85.835	84.740
106	[Rn]5f ¹⁴ 6d ⁴ 7s ²	104.941	102.631	101.982	99.103	96.770	95.346	90.391	88.077	86.776	85.868
107	[Rn]5f ¹⁴ 6d ⁵ 7s ²	105.914	103.644	103.021	100.072	97.844	96.448	91.355	89.048	87.716	87.005
108	[Rn]5f ¹⁴ 6d ⁶ 7s ²	106.884	104.656	104.061	101.040	98.920	97.554	92.319	90.020	88.656	88.147
109	[Rn]5f ¹⁴ 6d ⁷ 7s ²	107.851	105.666	105.101	102.007	99.997	98.662	93.283	90.991	89.595	89.293
110	[Rn]5f ¹⁴ 6d ⁹ 7s	108.815	106.674	106.141	102.974	101.074	99.772	94.246	91.961	90.534	90.442
111	[Rn]5f ¹⁴ 6d ¹⁰ 7s	109.775	107.680	107.181	103.940	102.151	100.886	95.209	92.932	91.472	91.595
112	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²	110.731	108.684	108.220	104.905	103.230	102.003	96.172	93.902	92.409	92.754
113	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p	111.684	109.684	109.260	105.870	104.308	103.122	97.135	94.872	93.346	93.917
114	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	112.631	110.682	110.299	106.834	105.386	104.245	98.098	95.842	94.282	95.083
115	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	113.574	111.676	111.337	107.797	106.464	105.370	99.060	96.811	95.217	96.253
116	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	114.511	112.666	112.374	108.759	107.541	106.497	100.023	97.781	96.151	97.427
117	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	115.442	113.651	113.410	109.721	108.617	107.627	100.984	98.749	97.085	98.605
118	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶	116.367	114.631	114.444	110.682	109.691	108.759	101.946	99.718	98.018	99.786

Table 2

Effective nuclear charge calculated with the Dirac-Fock method for subshells ranging from $4p_{1/2}$ to $5f_{7/2}$.

Atomic Number	Configuration	$4p_{1/2}$	$4p_{3/2}$	$4d_{3/2}$	$4d_{5/2}$	$4f_{5/2}$	$4f_{7/2}$	$5s$	$5p_{1/2}$	$5p_{3/2}$	$5d_{3/2}$	$5d_{5/2}$	$5f_{5/2}$	$5f_{7/2}$
1	1s													
2	1s ²													
3	1s ² 2s													
4	1s ² 2s ²													
5	1s ² 2s ² 2p													
6	1s ² 2s ² 2p ²													
7	1s ² 2s ² 2p ³													
8	1s ² 2s ² 2p ⁴													
9	1s ² 2s ² 2p ⁵													
10	1s ² 2s ² 2p ⁶													
11	[Ne]3s													
12	[Ne]3s ²													
13	[Ne]3s ² 3p													
14	[Ne]3s ² 3p ²													
15	[Ne]3s ² 3p ³													
16	[Ne]3s ² 3p ⁴													
17	[Ne]3s ² 3p ⁵													
18	[Ne]3s ² 3p ⁶													
19	[Ar]4s													
20	[Ar]4s ²													
21	[Ar]3d4s ²													
22	[Ar]3d ² 4s ²													
23	[Ar]3d ³ 4s ²													
24	[Ar]3d ⁵ 4s													
25	[Ar]3d ⁵ 4s ²													
26	[Ar]3d ⁶ 4s ²													
27	[Ar]3d ⁷ 4s ²													
28	[Ar]3d ⁸ 4s ²													
29	[Ar]3d ¹⁰ 4s													
30	[Ar]3d ¹⁰ 4s ²													
31	[Ar]3d ¹⁰ 4s ² 4p	6.779												
32	[Ar]3d ¹⁰ 4s ² 4p ²	8.076	8.052											
33	[Ar]3d ¹⁰ 4s ² 4p ³	9.240	9.137											
34	[Ar]3d ¹⁰ 4s ² 4p ⁴	10.076	10.000											
35	[Ar]3d ¹⁰ 4s ² 4p ⁵	11.040	10.849											
36	[Ar]3d ¹⁰ 4s ² 4p ⁶	11.983	11.731											
37	[Kr]5s	13.463	13.209					6.760						
38	[Kr]5s ²	14.798	14.531					8.210						
39	[Kr]4d5s ²	15.841	15.618	8.401				8.891						
40	[Kr]4d ² 5s ²	16.826	16.595	9.795	9.752			9.372						
41	[Kr]4d ⁴ 5s	17.663	17.374	10.028	9.859			9.751						
42	[Kr]4d ⁵ 5s	18.619	18.243	11.066	10.968			10.162						
43	[Kr]4d ⁵ 5s ²	19.666	19.265	12.824	12.730			10.498						
44	[Kr]4d ⁷ 5s	20.405	20.004	12.759	12.644			10.518						
45	[Kr]4d ⁸ 5s	21.296	20.848	13.641	13.429			10.649						
46	[Kr]4d ¹⁰	22.078	21.543	13.718	13.417									
47	[Kr]4d ¹⁰ 5s	23.060	22.493	15.295	15.038			10.848						
48	[Kr]4d ¹⁰ 5s ²	24.068	23.468	16.714	16.470			12.118						
49	[Kr]4d ¹⁰ 5s ² 5p	25.114	24.472	18.049	17.877			13.711	9.943					
50	[Kr]4d ¹⁰ 5s ² 5p ²	26.180	25.489	19.321	19.167			15.048	11.485	11.382				
51	[Kr]4d ¹⁰ 5s ² 5p ³	27.222	26.524	20.586	20.369			16.275	12.907	12.549				
52	[Kr]4d ¹⁰ 5s ² 5p ⁴	28.282	27.539	21.756	21.556			17.440	13.955	13.503				
53	[Kr]4d ¹⁰ 5s ² 5p ⁵	29.346	28.557	22.916	22.696			18.546	15.116	14.478				
54	[Kr]4d ¹⁰ 5s ² 5p ⁶	30.397	29.598	24.054	23.807			19.609	16.228	15.498				
55	[Xe]6s	31.453	30.616	25.183	24.928			21.088	17.916	17.193				
56	[Xe]6s ²	32.510	31.634	26.301	26.038			22.517	19.423	18.675				
57	[Xe]5d6s ²	33.569	32.650	27.420	27.137			23.700	20.564	19.872	12.047			
58	[Xe]4f5d6s ²	34.477	33.508	28.258	27.968			24.275	21.122	20.297	12.736	12.609		
59	[Xe]4f ³ 6s ²	35.243	34.218	28.831	28.549	16.980	16.876	24.283	21.039	20.034				
60	[Xe]4f ⁴ 6s ²	36.137	35.047	29.643	29.336	17.887	17.663	24.820	21.507	20.432				
61	[Xe]4f ⁵ 6s ²	37.027	35.870	30.448	30.113	18.686	18.371	25.352	21.967	20.823				
62	[Xe]4f ⁶ 6s ²	37.912	36.683	31.241	30.877	19.439	19.163	25.869	22.406	21.198				
63	[Xe]4f ⁷ 6s ²	38.798	37.487	32.028	31.628	20.154	20.004	26.375	22.808	21.567				
64	[Xe]4f ⁷ 5d6s ²	39.771	38.396	32.986	32.563	21.882	21.903	27.411	23.817	22.616	13.486	13.396		
65	[Xe]4f ⁹ 6s ²	40.567	39.114	33.598	33.160	21.127	21.083	27.456	23.785	22.327				
66	[Xe]4f ¹⁰ 6s ²	41.451	39.917	34.375	33.909	21.757	21.575	27.982	24.245	22.691				
67	[Xe]4f ¹¹ 6s ²	42.338	40.717	35.153	34.654	22.385	22.044	28.511	24.697	23.060				

Table 2 (continued)

Atomic Number	Configuration	$4p_{1/2}$	$4p_{3/2}$	$4d_{3/2}$	$4d_{5/2}$	$4f_{5/2}$	$4f_{7/2}$	$5s$	$5p_{1/2}$	$5p_{3/2}$	$5d_{3/2}$	$5d_{5/2}$	$5f_{5/2}$	$5f_{7/2}$
68	[Xe]4f ¹² 6s ²	43.226	41.516	35.929	35.397	23.007	22.507	29.043	25.150	23.429				
69	[Xe]4f ¹³ 6s ²	44.115	42.311	36.699	36.134	23.606	23.029	29.572	25.593	23.790				
70	[Xe]4f ¹⁴ 6s ²	45.004	43.101	37.463	36.866	24.162	23.625	30.093	26.025	24.141				
71	[Xe]4f ¹⁴ 5d6s ²	45.954	43.966	38.355	37.731	25.810	25.464	31.122	27.037	25.212	12.841			
72	[Xe]4f ¹⁴ 5d ² 6s ²	46.922	44.847	39.273	38.621	27.322	27.058	32.142	28.044	26.214	14.961	14.694		
73	[Xe]4f ¹⁴ 5d ³ 6s ²	47.902	45.741	40.208	39.528	28.764	28.514	33.164	29.083	27.185	16.475	16.107		
74	[Xe]4f ¹⁴ 5d ⁴ 6s ²	48.893	46.645	41.158	40.447	30.129	29.892	34.185	30.107	28.144	17.741	17.410		
75	[Xe]4f ¹⁴ 5d ⁵ 6s ²	49.890	47.558	42.098	41.386	31.507	31.165	35.209	31.251	29.044	19.047	18.553		
76	[Xe]4f ¹⁴ 5d ⁶ 6s ²	50.900	48.476	43.057	42.315	32.756	32.453	36.234	32.245	30.005	19.945	19.430		
77	[Xe]4f ¹⁴ 5d ⁷ 6s ²	51.917	49.400	44.021	43.250	33.989	33.687	37.253	33.268	30.933	20.991	20.284		
78	[Xe]4f ¹⁴ 5d ⁹ 6s	52.939	50.332	44.992	44.194	35.205	34.856	38.152	34.216	31.680	21.362	20.247		
79	[Xe]4f ¹⁴ 5d ¹⁰ 6s	53.966	51.265	45.956	45.159	36.401	36.019	39.146	35.269	32.569	22.394	21.296		
80	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	54.999	52.199	46.924	46.103	37.574	37.186	40.298	36.423	33.622	24.026	22.990		
81	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p	56.041	53.138	47.894	47.050	38.729	38.335	41.433	37.626	34.728	25.563	24.724		
82	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	57.089	54.080	48.866	47.998	39.871	39.470	42.581	38.864	35.843	26.997	26.271		
83	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	58.140	55.026	49.841	48.948	41.000	40.591	43.738	40.057	36.978	28.491	27.683		
84	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	59.198	55.975	50.818	49.900	42.119	41.701	44.902	41.270	38.094	29.853	29.035		
85	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	60.260	56.928	51.797	50.852	43.227	42.800	46.069	42.482	39.208	31.190	30.322		
86	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	61.327	57.883	52.777	51.806	44.325	43.890	47.240	43.686	40.349	32.483	31.563		
87	[Rn]7s	62.400	58.840	53.759	52.761	45.416	44.971	48.415	44.893	41.452	33.763	32.829		
88	[Rn]7s ²	63.477	59.798	54.742	53.717	46.500	46.046	49.594	46.099	42.550	35.022	34.074		
89	[Rn]6d7s ²	64.560	60.758	55.729	54.674	47.577	47.113	50.769	47.309	43.648	36.276	35.300		
90	[Rn]6d ² 7s ²	65.647	61.720	56.716	55.632	48.647	48.173	51.944	48.518	44.740	37.511	36.501		
91	[Rn]5f ² 6d7s ²	66.734	62.679	57.696	56.584	49.719	49.215	53.024	49.563	45.663	38.311	37.356	20.293	20.370
92	[Rn]5f ³ 6d7s ²	67.825	63.639	58.678	57.537	50.778	50.254	54.134	50.672	46.630	39.264	38.307	21.982	21.803
93	[Rn]5f ⁴ 6d7s ²	68.921	64.599	59.658	58.487	51.828	51.286	55.242	51.782	47.582	40.214	39.228	23.296	23.019
94	[Rn]5f ⁶ 7s ²	70.018	65.558	60.634	59.435	52.880	52.310	56.307	52.827	48.451	40.960	39.984	23.470	23.095
95	[Rn]5f ⁷ 7s ²	71.121	66.519	61.614	60.380	53.896	53.345	57.410	53.971	49.364	41.973	40.822	24.705	24.185
96	[Rn]5f ⁷ 6d7s ²	72.230	67.480	62.594	61.327	54.923	54.357	58.547	55.124	50.353	43.051	41.845	26.548	26.306
97	[Rn]5f ⁹ 7s ²	73.341	68.440	63.567	62.271	55.946	55.353	59.616	56.164	51.211	43.770	42.592	26.455	25.775
98	[Rn]5f ¹⁰ 7s ²	74.456	69.400	64.543	63.214	56.962	56.353	60.719	57.273	52.116	44.680	43.441	27.406	26.569
99	[Rn]5f ¹¹ 7s ²	75.576	70.360	65.519	64.154	57.969	57.350	61.824	58.391	53.014	45.596	44.277	28.341	27.351
100	[Rn]5f ¹² 7s ²	76.701	71.320	66.494	65.093	58.968	58.339	62.932	59.512	53.906	46.505	45.104	29.250	28.120
101	[Rn]5f ¹³ 7s ²	77.829	72.280	67.468	66.030	59.984	59.322	64.041	60.638	54.792	47.408	45.923	30.123	28.932
102	[Rn]5f ¹⁴ 7s ²	78.963	73.239	68.442	66.965	60.976	60.340	65.154	61.768	55.672	48.304	46.732	30.961	29.780
103	[Rn]5f ¹⁴ 7s ² 7p	80.103	74.199	69.415	67.900	61.970	61.319	66.298	62.926	56.609	49.279	47.669	32.753	31.739
104	[Rn]5f ¹⁴ 6d ² 7s ²	81.248	75.160	70.391	68.836	62.960	62.296	67.454	64.105	57.570	50.311	48.645	34.089	33.326
105	[Rn]5f ¹⁴ 6d ³ 7s ²	82.400	76.121	71.366	69.771	63.948	63.269	68.619	65.292	58.532	51.334	49.620	35.529	34.835
106	[Rn]5f ¹⁴ 6d ⁴ 7s ²	83.556	77.082	72.343	70.706	64.935	64.240	69.788	66.490	59.499	52.376	50.599	36.874	36.271
107	[Rn]5f ¹⁴ 6d ⁵ 7s ²	84.718	78.044	73.316	71.643	65.920	65.208	70.968	67.690	60.474	53.383	51.599	38.336	37.556
108	[Rn]5f ¹⁴ 6d ⁶ 7s ²	85.887	79.006	74.292	72.577	66.901	66.174	72.158	68.908	61.449	54.413	52.579	39.600	38.880
109	[Rn]5f ¹⁴ 6d ⁷ 7s ²	87.062	79.969	75.268	73.512	67.881	67.138	73.355	70.134	62.428	55.443	53.565	40.872	40.135
110	[Rn]5f ¹⁴ 6d ⁹ 7s	88.243	80.932	76.243	74.448	68.860	68.099	74.555	71.368	63.419	56.475	54.557	42.122	41.297
111	[Rn]5f ¹⁴ 6d ¹⁰ 7s	89.430	81.895	77.219	75.383	69.836	69.058	75.767	72.610	64.403	57.499	55.568	43.355	42.479
112	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²	90.624	82.858	78.194	76.316	70.811	70.015	76.992	73.862	65.380	58.520	56.549	44.571	43.688
113	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p	91.825	83.821	79.170	77.249	71.784	70.971	78.220	75.128	66.361	59.540	57.530	45.768	44.874
114	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	93.034	84.785	80.145	78.181	72.755	71.925	79.457	76.406	67.344	60.559	58.511	46.949	46.044
115	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	94.249	85.749	81.121	79.113	73.725	72.877	80.702	77.692	68.330	61.582	59.490	48.111	47.195
116	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	95.472	86.713	82.097	80.045	74.693	73.827	81.955	78.988	69.318	62.605	60.468	49.261	48.334
117	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	96.702	87.678	83.074	80.976	75.660	74.776	83.216	80.297	70.307	63.627	61.446	50.399	49.460
118	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶	97.939	88.643	84.050	81.906	76.625	75.722	84.485	81.617	71.298	64.650	62.422	51.526	50.574

Table 3Effective nuclear charge calculated with the Dirac-Fock method for subshells ranging from 6s to $7p_{3/2}$.

Atomic Number	Configuration	6s	$6p_{1/2}$	$6p_{3/2}$	$6d_{3/2}$	$6d_{5/2}$	7s	$7p_{1/2}$	$7p_{3/2}$
55	[Xe]6s	8.869							
56	[Xe]6s ²	10.612							
57	[Xe]5d6s ²	11.407							
58	[Xe]4f5d6s ²	11.539							
59	[Xe]4f ³ 6s ²	11.134							
60	[Xe]4f ⁴ 6s ²	11.285							
61	[Xe]4f ⁵ 6s ²	11.434							
62	[Xe]4f ⁶ 6s ²	11.577							
63	[Xe]4f ⁷ 6s ²	11.714							
64	[Xe]4f ⁷ 5d6s ²	12.493							
65	[Xe]4f ⁹ 6s ²	12.019							
66	[Xe]4f ¹⁰ 6s ²	12.164							
67	[Xe]4f ¹¹ 6s ²	12.310							
68	[Xe]4f ¹² 6s ²	12.457							
69	[Xe]4f ¹³ 6s ²	12.601							
70	[Xe]4f ¹⁴ 6s ²	12.741							
71	[Xe]4f ¹⁴ 5d6s ²	13.770							
72	[Xe]4f ¹⁴ 5d ² 6s ²	14.473							
73	[Xe]4f ¹⁴ 5d ³ 6s ²	15.109							
74	[Xe]4f ¹⁴ 5d ⁴ 6s ²	15.691							
75	[Xe]4f ¹⁴ 5d ⁵ 6s ²	16.244							
76	[Xe]4f ¹⁴ 5d ⁶ 6s ²	16.843							
77	[Xe]4f ¹⁴ 5d ⁷ 6s ²	17.391							
78	[Xe]4f ¹⁴ 5d ⁹ 6s	17.381							
79	[Xe]4f ¹⁴ 5d ¹⁰ 6s	17.586							
80	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	18.926							
81	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p	20.761	15.075						
82	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	22.352	16.953	16.455					
83	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	23.877	18.820	16.925					
84	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	25.293	20.474	17.956					
85	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	26.640	22.040	19.199					
86	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	27.925	23.495	20.478					
87	[Rn]7s	29.695	25.518	22.568			12.400		
88	[Rn]7s ²	31.412	27.334	24.327				14.583	
89	[Rn]6d7s ²	32.882	28.768	25.811	14.908				
90	[Rn]6d ² 7s ²	34.258	30.108	27.125	16.952	16.547			16.894
91	[Rn]5f ² 6d7s ²	34.764	30.610	27.175	17.074	17.393			16.414
92	[Rn]5f ³ 6d7s ²	35.611	31.432	27.733	17.495	17.245			16.657
93	[Rn]5f ⁴ 6d7s ²	36.442	32.223	28.272	17.903	17.599			16.897
94	[Rn]5f ⁶ 7s ²	36.857	32.609	28.144					16.345
95	[Rn]5f ⁷ 7s ²	37.658	33.288	28.666					16.553
96	[Rn]5f ⁷ 6d7s ²	38.864	34.438	29.814	18.068	17.781			17.687
97	[Rn]5f ⁹ 7s ²	39.287	34.855	29.621					17.041
98	[Rn]5f ¹⁰ 7s ²	40.086	35.598	30.076					17.270
99	[Rn]5f ¹¹ 7s ²	40.889	36.332	30.532					17.501
100	[Rn]5f ¹² 7s ²	41.695	37.068	30.980					17.735
101	[Rn]5f ¹³ 7s ²	42.498	37.797	31.411					17.964
102	[Rn]5f ¹⁴ 7s ²	43.298	38.518	31.825					18.190
103	[Rn]5f ¹⁴ 7s ² 7p	44.623	39.985	33.252					20.817
104	[Rn]5f ¹⁴ 6d ² 7s ²	45.762	40.916	34.169	19.549	18.825			20.956
105	[Rn]5f ¹⁴ 6d ³ 7s ²	47.000	42.144	35.254	21.453	20.607			22.031
106	[Rn]5f ¹⁴ 6d ⁴ 7s ²	48.231	43.322	36.329	22.973	22.386			23.018
107	[Rn]5f ¹⁴ 6d ⁵ 7s ²	49.496	44.791	37.307	24.688	23.360			24.111
108	[Rn]5f ¹⁴ 6d ⁶ 7s ²	50.745	46.003	38.357	25.886	24.188			25.151
109	[Rn]5f ¹⁴ 6d ⁷ 7s ²	52.001	47.298	39.362	27.209	25.159			26.174
110	[Rn]5f ¹⁴ 6d ⁹ 7s	53.159	48.589	40.255	28.041	25.440			27.143
111	[Rn]5f ¹⁴ 6d ¹⁰ 7s	54.386	49.929	41.222	29.284	26.668			27.870
112	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²	55.794	51.317	42.272	30.895	28.358			29.225
113	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p	57.152	52.774	43.453	32.398	30.259		23.835	
114	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	58.525	54.247	44.630	33.800	31.942		25.759	23.952
115	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	59.901	55.717	45.838	35.498	33.592		28.378	18.858
116	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	61.288	57.163	47.052	37.082	35.107		30.646	20.984
117	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	62.685	58.658	48.253	38.586	36.537		32.686	22.716
118	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶	64.092	60.136	49.485	40.026	37.895		34.580	24.366

Table 4

Radii of maximum charge density calculated with the Dirac-Fock method for subshells ranging from 1s to 4s (a_0).

Atomic Number	Configuration	1s	2s	2p _{1/2}	2p _{3/2}	3s	3p _{1/2}	3p _{3/2}	3d _{3/2}	3d _{5/2}	4s
1	1s	1.000									
2	1s ²	0.570									
3	1s ² 2s	0.364	3.101								
4	1s ² 2s ²	0.267	2.051								
5	1s ² 2s ² 2p	0.211	1.525	1.587							
6	1s ² 2s ² 2p ²	0.174	1.222	1.212	1.212						
7	1s ² 2s ² 2p ³	0.149	1.022	0.984	0.984						
8	1s ² 2s ² 2p ⁴	0.129	0.875	0.834	0.831						
9	1s ² 2s ² 2p ⁵	0.115	0.766	0.720	0.720						
10	1s ² 2s ² 2p ⁶	0.103	0.683	0.633	0.635						
11	[Ne]3s	0.093	0.607	0.548	0.550	3.387					
12	[Ne]3s ²	0.085	0.546	0.483	0.484	2.583					
13	[Ne]3s ² 3p	0.078	0.495	0.431	0.433	2.096	2.685				
14	[Ne]3s ² 3p ²	0.073	0.453	0.389	0.391	1.796	2.168	2.172			
15	[Ne]3s ² 3p ³	0.068	0.417	0.355	0.357	1.581	1.841	1.846			
16	[Ne]3s ² 3p ⁴	0.063	0.387	0.326	0.328	1.416	1.613	1.616			
17	[Ne]3s ² 3p ⁵	0.060	0.360	0.302	0.304	1.286	1.436	1.441			
18	[Ne]3s ² 3p ⁶	0.056	0.338	0.281	0.282	1.178	1.295	1.302			
19	[Ar]4s	0.053	0.317	0.262	0.264	1.077	1.158	1.164			4.328
20	[Ar]4s ²	0.050	0.299	0.246	0.248	0.992	1.048	1.054			3.463
21	[Ar]3d4s ²	0.048	0.283	0.231	0.233	0.925	0.969	0.974	1.125		3.226
22	[Ar]3d ² 4s ²	0.046	0.268	0.219	0.221	0.869	0.903	0.909	0.997	0.999	3.055
23	[Ar]3d ³ 4s ²	0.044	0.255	0.207	0.209	0.819	0.847	0.852	0.905	0.908	2.912
24	[Ar]3d ⁴ 4s	0.042	0.243	0.197	0.199	0.778	0.800	0.807	0.861	0.863	2.908
25	[Ar]3d ⁵ 4s ²	0.040	0.232	0.187	0.189	0.737	0.753	0.760	0.772	0.775	2.680
26	[Ar]3d ⁶ 4s ²	0.038	0.222	0.179	0.181	0.701	0.714	0.720	0.723	0.724	2.567
27	[Ar]3d ⁷ 4s ²	0.037	0.213	0.171	0.173	0.670	0.678	0.685	0.678	0.680	2.469
28	[Ar]3d ⁸ 4s ²	0.035	0.204	0.164	0.166	0.640	0.646	0.653	0.639	0.642	2.381
29	[Ar]3d ¹⁰ 4s	0.034	0.196	0.157	0.159	0.615	0.619	0.626	0.614	0.617	2.540
30	[Ar]3d ¹⁰ 4s ²	0.033	0.189	0.151	0.153	0.589	0.591	0.599	0.574	0.577	2.227
31	[Ar]3d ¹⁰ 4s ² 4p	0.032	0.182	0.145	0.148	0.565	0.564	0.573	0.539	0.542	1.967
32	[Ar]3d ¹⁰ 4s ² 4p ²	0.031	0.176	0.140	0.142	0.543	0.540	0.548	0.508	0.511	1.787
33	[Ar]3d ¹⁰ 4s ² 4p ³	0.030	0.170	0.135	0.138	0.522	0.518	0.526	0.480	0.483	1.648
34	[Ar]3d ¹⁰ 4s ² 4p ⁴	0.029	0.164	0.130	0.133	0.502	0.497	0.505	0.455	0.458	1.533
35	[Ar]3d ¹⁰ 4s ² 4p ⁵	0.028	0.159	0.126	0.129	0.484	0.477	0.485	0.433	0.436	1.437
36	[Ar]3d ¹⁰ 4s ² 4p ⁶	0.027	0.154	0.122	0.125	0.466	0.459	0.467	0.413	0.415	1.353
37	[Kr]5s	0.026	0.150	0.118	0.121	0.450	0.442	0.450	0.394	0.397	1.270
38	[Kr]5s ²	0.026	0.145	0.114	0.117	0.435	0.426	0.434	0.378	0.380	1.196
39	[Kr]4d5s ²	0.025	0.141	0.111	0.114	0.421	0.411	0.419	0.362	0.365	1.135
40	[Kr]4d ² 5s ²	0.024	0.137	0.107	0.111	0.408	0.397	0.406	0.348	0.350	1.082
41	[Kr]4d ⁴ 5s	0.024	0.133	0.104	0.108	0.395	0.384	0.393	0.335	0.337	1.036
42	[Kr]4d ⁵ 5s	0.023	0.130	0.101	0.105	0.383	0.372	0.380	0.322	0.325	0.992
43	[Kr]4d ⁶ 5s ²	0.022	0.126	0.098	0.102	0.372	0.360	0.369	0.311	0.314	0.952
44	[Kr]4d ⁷ 5s	0.022	0.123	0.096	0.100	0.361	0.349	0.358	0.301	0.303	0.918
45	[Kr]4d ⁸ 5s	0.021	0.120	0.093	0.097	0.351	0.339	0.348	0.291	0.293	0.885
46	[Kr]4d ¹⁰	0.021	0.117	0.091	0.095	0.341	0.329	0.338	0.281	0.284	0.856
47	[Kr]4d ¹⁰ 5s	0.020	0.114	0.089	0.093	0.332	0.320	0.329	0.273	0.276	0.827
48	[Kr]4d ¹⁰ 5s ²	0.020	0.111	0.086	0.091	0.324	0.311	0.320	0.265	0.268	0.799
49	[Kr]4d ¹⁰ 5s ² 5p	0.019	0.109	0.084	0.088	0.315	0.302	0.312	0.257	0.260	0.773
50	[Kr]4d ¹⁰ 5s ² 5p ²	0.019	0.106	0.082	0.087	0.307	0.295	0.304	0.250	0.253	0.748
51	[Kr]4d ¹⁰ 5s ² 5p ³	0.018	0.104	0.080	0.085	0.300	0.287	0.297	0.243	0.246	0.724
52	[Kr]4d ¹⁰ 5s ² 5p ⁴	0.018	0.102	0.078	0.083	0.293	0.280	0.290	0.237	0.240	0.702
53	[Kr]4d ¹⁰ 5s ² 5p ⁵	0.018	0.099	0.076	0.081	0.285	0.273	0.283	0.230	0.233	0.682
54	[Kr]4d ¹⁰ 5s ² 5p ⁶	0.017	0.097	0.075	0.079	0.279	0.266	0.276	0.224	0.228	0.662
55	[Xe]6s	0.017	0.095	0.073	0.078	0.272	0.260	0.270	0.219	0.222	0.643
56	[Xe]6s ²	0.016	0.093	0.071	0.076	0.266	0.254	0.264	0.214	0.217	0.625
57	[Xe]5d6s ²	0.016	0.091	0.070	0.075	0.260	0.248	0.259	0.209	0.212	0.609
58	[Xe]4f5d6s ²	0.016	0.089	0.068	0.073	0.255	0.242	0.253	0.204	0.207	0.594
59	[Xe]4f ³ 6s ²	0.015	0.087	0.067	0.072	0.249	0.237	0.248	0.199	0.202	0.580
60	[Xe]4f ⁴ 6s ²	0.015	0.086	0.065	0.071	0.244	0.231	0.242	0.195	0.198	0.566
61	[Xe]4f ⁵ 6s ²	0.015	0.084	0.064	0.069	0.239	0.226	0.238	0.190	0.193	0.553
62	[Xe]4f ⁶ 6s ²	0.015	0.082	0.063	0.068	0.234	0.221	0.233	0.186	0.189	0.541
63	[Xe]4f ⁷ 6s ²	0.014	0.081	0.061	0.067	0.229	0.217	0.228	0.182	0.185	0.529
64	[Xe]4f ⁷ 5d6s ²	0.014	0.079	0.060	0.066	0.224	0.212	0.224	0.178	0.182	0.517
65	[Xe]4f ⁹ 6s ²	0.014	0.078	0.059	0.065	0.220	0.208	0.220	0.175	0.178	0.506
66	[Xe]4f ¹⁰ 6s ²	0.013	0.076	0.058	0.064	0.216	0.203	0.216	0.171	0.175	0.496
67	[Xe]4f ¹¹ 6s ²	0.013	0.075	0.056	0.063	0.211	0.199	0.212	0.168	0.171	0.485

Table 4 (continued)

Atomic Number	Configuration	1s	2s	2p _{1/2}	2p _{3/2}	3s	3p _{1/2}	3p _{3/2}	3d _{3/2}	3d _{5/2}	4s
68	[Xe]4f ¹² 6s ²	0.013	0.073	0.055	0.062	0.207	0.195	0.208	0.165	0.168	0.475
69	[Xe]4f ¹³ 6s ²	0.013	0.072	0.054	0.061	0.203	0.192	0.204	0.161	0.165	0.466
70	[Xe]4f ¹⁴ 6s ²	0.012	0.071	0.053	0.060	0.200	0.188	0.201	0.158	0.162	0.457
71	[Xe]4f ¹⁴ 5d6s ²	0.012	0.070	0.052	0.059	0.196	0.184	0.198	0.156	0.159	0.448
72	[Xe]4f ¹⁴ 5d ² 6s ²	0.012	0.068	0.051	0.058	0.192	0.181	0.194	0.153	0.156	0.439
73	[Xe]4f ¹⁴ 5d ³ 6s ²	0.012	0.067	0.050	0.057	0.189	0.178	0.191	0.150	0.154	0.430
74	[Xe]4f ¹⁴ 5d ⁴ 6s ²	0.011	0.066	0.049	0.056	0.186	0.174	0.188	0.147	0.151	0.422
75	[Xe]4f ¹⁴ 5d ⁵ 6s ²	0.011	0.065	0.048	0.055	0.182	0.171	0.185	0.145	0.149	0.414
76	[Xe]4f ¹⁴ 5d ⁶ 6s ²	0.011	0.064	0.047	0.054	0.179	0.168	0.182	0.142	0.146	0.406
77	[Xe]4f ¹⁴ 5d ⁷ 6s ²	0.011	0.063	0.046	0.054	0.176	0.165	0.179	0.140	0.144	0.398
78	[Xe]4f ¹⁴ 5d ⁹ 6s	0.011	0.062	0.046	0.053	0.173	0.162	0.176	0.138	0.142	0.391
79	[Xe]4f ¹⁴ 5d ¹⁰ 6s	0.010	0.060	0.045	0.052	0.170	0.159	0.174	0.136	0.140	0.384
80	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	0.010	0.059	0.044	0.051	0.167	0.156	0.171	0.133	0.137	0.377
81	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p	0.010	0.058	0.043	0.051	0.164	0.154	0.169	0.131	0.135	0.370
82	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	0.010	0.057	0.042	0.050	0.161	0.151	0.166	0.129	0.133	0.364
83	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	0.010	0.057	0.041	0.049	0.159	0.148	0.164	0.127	0.131	0.357
84	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	0.009	0.056	0.041	0.049	0.156	0.146	0.161	0.125	0.130	0.351
85	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	0.009	0.055	0.040	0.048	0.154	0.143	0.159	0.123	0.128	0.345
86	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	0.009	0.054	0.039	0.047	0.151	0.141	0.157	0.122	0.126	0.339
87	[Rn]7s	0.009	0.053	0.038	0.047	0.149	0.139	0.155	0.120	0.124	0.333
88	[Rn]7s ²	0.009	0.052	0.038	0.046	0.146	0.136	0.153	0.118	0.123	0.327
89	[Rn]6d7s ²	0.009	0.051	0.037	0.045	0.144	0.134	0.151	0.116	0.121	0.322
90	[Rn]6d ² 7s ²	0.008	0.050	0.036	0.045	0.142	0.132	0.149	0.115	0.119	0.316
91	[Rn]5f ² 6d7s ²	0.008	0.049	0.036	0.044	0.139	0.130	0.147	0.113	0.118	0.311
92	[Rn]5f ³ 6d7s ²	0.008	0.049	0.035	0.044	0.137	0.127	0.145	0.112	0.116	0.306
93	[Rn]5f ⁴ 6d7s ²	0.008	0.048	0.034	0.043	0.135	0.125	0.143	0.110	0.115	0.301
94	[Rn]5f ⁶ 7s ²	0.008	0.047	0.034	0.043	0.133	0.123	0.141	0.109	0.113	0.295
95	[Rn]5f ⁷ 7s ²	0.008	0.046	0.033	0.042	0.131	0.121	0.139	0.107	0.112	0.291
96	[Rn]5f ⁷ 6d7s ²	0.008	0.046	0.032	0.042	0.129	0.119	0.138	0.106	0.110	0.286
97	[Rn]5f ⁹ 7s ²	0.007	0.045	0.032	0.041	0.127	0.117	0.136	0.104	0.109	0.281
98	[Rn]5f ¹⁰ 7s ²	0.007	0.044	0.031	0.041	0.125	0.115	0.134	0.103	0.108	0.277
99	[Rn]5f ¹¹ 7s ²	0.007	0.043	0.030	0.040	0.123	0.113	0.133	0.101	0.106	0.272
100	[Rn]5f ¹² 7s ²	0.007	0.043	0.030	0.040	0.121	0.111	0.131	0.100	0.105	0.268
101	[Rn]5f ¹³ 7s ²	0.007	0.042	0.029	0.039	0.119	0.110	0.129	0.099	0.104	0.264
102	[Rn]5f ¹⁴ 7s ²	0.007	0.041	0.029	0.039	0.117	0.108	0.128	0.098	0.103	0.260
103	[Rn]5f ¹⁴ 7s ² 7p	0.007	0.041	0.028	0.038	0.115	0.106	0.126	0.096	0.102	0.256
104	[Rn]5f ¹⁴ 6d ² 7s ²	0.006	0.040	0.027	0.038	0.113	0.104	0.125	0.095	0.100	0.252
105	[Rn]5f ¹⁴ 6d ³ 7s ²	0.006	0.039	0.027	0.037	0.112	0.103	0.124	0.094	0.099	0.248
106	[Rn]5f ¹⁴ 6d ⁴ 7s ²	0.006	0.039	0.026	0.037	0.110	0.101	0.122	0.093	0.098	0.244
107	[Rn]5f ¹⁴ 6d ⁵ 7s ²	0.006	0.038	0.026	0.037	0.108	0.099	0.121	0.092	0.097	0.240
108	[Rn]5f ¹⁴ 6d ⁶ 7s ²	0.006	0.037	0.025	0.036	0.106	0.098	0.119	0.091	0.096	0.236
109	[Rn]5f ¹⁴ 6d ⁷ 7s ²	0.006	0.037	0.024	0.036	0.105	0.096	0.118	0.090	0.095	0.232
110	[Rn]5f ¹⁴ 6d ⁹ 7s	0.006	0.036	0.024	0.035	0.103	0.094	0.117	0.088	0.094	0.229
111	[Rn]5f ¹⁴ 6d ¹⁰ 7s	0.005	0.035	0.023	0.035	0.102	0.093	0.116	0.087	0.093	0.225
112	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²	0.005	0.035	0.023	0.035	0.100	0.091	0.114	0.086	0.092	0.222
113	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p	0.005	0.034	0.022	0.034	0.098	0.090	0.113	0.085	0.091	0.218
114	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	0.005	0.033	0.022	0.034	0.097	0.088	0.112	0.084	0.090	0.215
115	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	0.005	0.033	0.021	0.034	0.095	0.086	0.111	0.083	0.089	0.211
116	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	0.005	0.032	0.021	0.033	0.094	0.085	0.109	0.083	0.088	0.208
117	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	0.005	0.032	0.020	0.033	0.092	0.083	0.108	0.082	0.087	0.205
118	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶	0.005	0.031	0.019	0.033	0.091	0.082	0.107	0.081	0.087	0.202

Table 5

Radii of maximum charge density calculated with the Dirac-Fock method for subshells ranging from $4p_{1/2}$ to $5f_{7/2}$ (a_0).

Atomic Number	Configuration	$4p_{1/2}$	$4p_{3/2}$	$4d_{3/2}$	$4d_{5/2}$	$4f_{5/2}$	$4f_{7/2}$	$5s$	$5p_{1/2}$	$5p_{3/2}$	$5d_{3/2}$	$5d_{5/2}$	$5f_{5/2}$	$5f_{7/2}$
1	1s													
2	1s ²													
3	1s ² 2s													
4	1s ² 2s ²													
5	1s ² 2s ² 2p													
6	1s ² 2s ² 2p ²													
7	1s ² 2s ² 2p ³													
8	1s ² 2s ² 2p ⁴													
9	1s ² 2s ² 2p ⁵													
10	1s ² 2s ² 2p ⁶													
11	[Ne]3s													
12	[Ne]3s ²													
13	[Ne]3s ² 3p													
14	[Ne]3s ² 3p ²													
15	[Ne]3s ² 3p ³													
16	[Ne]3s ² 3p ⁴													
17	[Ne]3s ² 3p ⁵													
18	[Ne]3s ² 3p ⁶													
19	[Ar]4s													
20	[Ar]4s ²													
21	[Ar]3d4s ²													
22	[Ar]3d ² 4s ²													
23	[Ar]3d ³ 4s ²													
24	[Ar]3d ⁵ 4s													
25	[Ar]3d ⁵ 4s ²													
26	[Ar]3d ⁶ 4s ²													
27	[Ar]3d ⁷ 4s ²													
28	[Ar]3d ⁸ 4s ²													
29	[Ar]3d ¹⁰ 4s													
30	[Ar]3d ¹⁰ 4s ²													
31	[Ar]3d ¹⁰ 4s ² 4p	2.615												
32	[Ar]3d ¹⁰ 4s ² 4p ²	2.234	2.257											
33	[Ar]3d ¹⁰ 4s ² 4p ³	1.984	2.011											
34	[Ar]3d ¹⁰ 4s ² 4p ⁴	1.804	1.832											
35	[Ar]3d ¹⁰ 4s ² 4p ⁵	1.658	1.689											
36	[Ar]3d ¹⁰ 4s ² 4p ⁶	1.538	1.570											
37	[Kr]5s	1.414	1.442					4.627						
38	[Kr]5s ²	1.314	1.339					3.803						
39	[Kr]4d5s ²	1.236	1.258	1.801				3.500						
40	[Kr]4d ² 5s ²	1.170	1.190	1.600	1.612			3.305						
41	[Kr]4d ⁴ 5s	1.113	1.136	1.506	1.521			3.174						
42	[Kr]4d ⁵ 5s	1.061	1.084	1.388	1.400			3.028						
43	[Kr]4d ⁵ 5s ²	1.012	1.034	1.267	1.278			2.905						
44	[Kr]4d ⁷ 5s	0.973	0.995	1.212	1.225			2.875						
45	[Kr]4d ⁸ 5s	0.934	0.956	1.144	1.157			2.818						
46	[Kr]4d ¹⁰	0.900	0.923	1.096	1.109									
47	[Kr]4d ¹⁰ 5s	0.866	0.889	1.033	1.046			2.729						
48	[Kr]4d ¹⁰ 5s ²	0.834	0.857	0.977	0.988			2.465						
49	[Kr]4d ¹⁰ 5s ² 5p	0.804	0.827	0.928	0.938			2.236	2.889					
50	[Kr]4d ¹⁰ 5s ² 5p ²	0.776	0.799	0.884	0.893			2.071	2.547	2.614				
51	[Kr]4d ¹⁰ 5s ² 5p ³	0.750	0.773	0.844	0.853			1.938	2.309	2.387				
52	[Kr]4d ¹⁰ 5s ² 5p ⁴	0.725	0.748	0.808	0.817			1.828	2.130	2.213				
53	[Kr]4d ¹⁰ 5s ² 5p ⁵	0.701	0.724	0.775	0.784			1.731	1.986	2.069				
54	[Kr]4d ¹⁰ 5s ² 5p ⁶	0.679	0.702	0.745	0.754			1.649	1.866	1.945				
55	[Xe]6s	0.659	0.681	0.718	0.726			1.562	1.738	1.808				
56	[Xe]6s ²	0.639	0.662	0.693	0.701			1.483	1.634	1.697				
57	[Xe]5d6s ²	0.621	0.644	0.669	0.677			1.419	1.552	1.610				
58	[Xe]4f5d6s ²	0.604	0.627	0.650	0.657	0.715	0.721	1.383	1.510	1.572	2.220			
59	[Xe]4f ³ 6s ²	0.590	0.613	0.634	0.642	0.697	0.702	1.370	1.501	1.573	2.115	2.144		
60	[Xe]4f ⁴ 6s ²	0.575	0.599	0.617	0.625	0.667	0.673	1.340	1.467	1.540				
61	[Xe]4f ⁵ 6s ²	0.561	0.585	0.601	0.609	0.641	0.647	1.310	1.433	1.507				
62	[Xe]4f ⁶ 6s ²	0.547	0.572	0.586	0.594	0.618	0.623	1.281	1.402	1.477				
63	[Xe]4f ⁷ 6s ²	0.535	0.560	0.571	0.580	0.597	0.601	1.254	1.373	1.449				
64	[Xe]4f ⁷ 5d6s ²	0.523	0.547	0.556	0.565	0.570	0.574	1.213	1.322	1.393	1.909	1.931		
65	[Xe]4f ⁹ 6s ²	0.511	0.537	0.545	0.553	0.560	0.564	1.201	1.313	1.394				
66	[Xe]4f ¹⁰ 6s ²	0.500	0.526	0.532	0.541	0.543	0.548	1.177	1.286	1.370				
67	[Xe]4f ¹¹ 6s ²	0.489	0.515	0.521	0.529	0.527	0.533	1.154	1.261	1.344				

Table 5 (continued)

Atomic Number	Configuration	$4p_{1/2}$	$4p_{3/2}$	$4d_{3/2}$	$4d_{5/2}$	$4f_{5/2}$	$4f_{7/2}$	$5s$	$5p_{1/2}$	$5p_{3/2}$	$5d_{3/2}$	$5d_{5/2}$	$5f_{5/2}$	$5f_{7/2}$
68	[Xe]4f ¹² 6s ²	0.479	0.505	0.510	0.518	0.512	0.518	1.131	1.235	1.322				
69	[Xe]4f ¹³ 6s ²	0.469	0.496	0.499	0.508	0.499	0.505	1.110	1.212	1.299				
70	[Xe]4f ¹⁴ 6s ²	0.459	0.487	0.489	0.497	0.486	0.492	1.089	1.189	1.278				
71	[Xe]4f ¹⁴ 5d6s ²	0.450	0.477	0.478	0.487	0.471	0.476	1.058	1.152	1.236	1.847			
72	[Xe]4f ¹⁴ 5d ² 6s ²	0.441	0.468	0.468	0.477	0.456	0.461	1.030	1.116	1.196	1.675	1.723		
73	[Xe]4f ¹⁴ 5d ³ 6s ²	0.431	0.459	0.458	0.468	0.442	0.448	1.001	1.081	1.159	1.560	1.606		
74	[Xe]4f ¹⁴ 5d ⁴ 6s ²	0.423	0.451	0.449	0.458	0.430	0.434	0.974	1.048	1.125	1.472	1.511		
75	[Xe]4f ¹⁴ 5d ⁵ 6s ²	0.414	0.443	0.440	0.449	0.417	0.422	0.949	1.016	1.093	1.396	1.436		
76	[Xe]4f ¹⁴ 5d ⁶ 6s ²	0.406	0.435	0.431	0.440	0.406	0.411	0.924	0.987	1.062	1.332	1.374		
77	[Xe]4f ¹⁴ 5d ⁷ 6s ²	0.398	0.428	0.422	0.432	0.395	0.400	0.901	0.959	1.034	1.275	1.318		
78	[Xe]4f ¹⁴ 5d ⁹ 6s	0.391	0.420	0.414	0.423	0.385	0.390	0.879	0.933	1.009	1.235	1.280		
79	[Xe]4f ¹⁴ 5d ¹⁰ 6s	0.383	0.413	0.406	0.416	0.375	0.380	0.858	0.908	0.983	1.189	1.231		
80	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	0.376	0.405	0.399	0.408	0.366	0.371	0.837	0.883	0.958	1.139	1.177		
81	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p	0.369	0.399	0.391	0.401	0.358	0.362	0.816	0.859	0.933	1.094	1.128		
82	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	0.362	0.392	0.384	0.394	0.349	0.354	0.797	0.836	0.909	1.054	1.084		
83	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	0.355	0.386	0.377	0.386	0.342	0.346	0.778	0.815	0.887	1.016	1.045		
84	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	0.349	0.380	0.370	0.380	0.334	0.338	0.759	0.793	0.865	0.982	1.010		
85	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	0.342	0.373	0.364	0.373	0.327	0.331	0.742	0.774	0.844	0.950	0.978		
86	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	0.336	0.368	0.358	0.367	0.320	0.324	0.726	0.755	0.825	0.921	0.948		
87	[Rn]7s	0.330	0.362	0.352	0.361	0.313	0.318	0.709	0.736	0.806	0.893	0.920		
88	[Rn]7s ²	0.324	0.356	0.345	0.355	0.307	0.312	0.694	0.719	0.788	0.868	0.893		
89	[Rn]6d ² 7s ²	0.318	0.351	0.340	0.349	0.301	0.305	0.678	0.703	0.771	0.844	0.869		
90	[Rn]6d ² 7s ²	0.313	0.346	0.334	0.344	0.295	0.300	0.665	0.686	0.754	0.821	0.846		
91	[Rn]5f ² 6d7s ²	0.307	0.341	0.329	0.339	0.290	0.294	0.651	0.672	0.740	0.804	0.826	1.110	1.124
92	[Rn]5f ³ 6d7s ²	0.302	0.336	0.324	0.334	0.284	0.289	0.637	0.658	0.725	0.786	0.808	1.055	1.071
93	[Rn]5f ⁴ 6d7s ²	0.297	0.331	0.319	0.328	0.279	0.284	0.625	0.644	0.712	0.769	0.790	1.012	1.028
94	[Rn]5f ⁶ 7s ²	0.292	0.326	0.314	0.323	0.274	0.279	0.613	0.630	0.698	0.753	0.775	0.985	1.001
95	[Rn]5f ⁷ 7s ²	0.287	0.321	0.309	0.319	0.270	0.274	0.601	0.617	0.687	0.736	0.760	0.950	0.966
96	[Rn]5f ⁷ 6d7s ²	0.282	0.317	0.304	0.314	0.265	0.269	0.590	0.605	0.674	0.721	0.744	0.911	0.925
97	[Rn]5f ⁹ 7s ²	0.278	0.312	0.300	0.310	0.261	0.265	0.578	0.593	0.663	0.708	0.730	0.890	0.908
98	[Rn]5f ¹⁰ 7s ²	0.273	0.308	0.295	0.305	0.256	0.261	0.568	0.581	0.652	0.694	0.717	0.865	0.882
99	[Rn]5f ¹¹ 7s ²	0.268	0.304	0.291	0.301	0.252	0.257	0.557	0.571	0.641	0.681	0.703	0.840	0.858
100	[Rn]5f ¹² 7s ²	0.264	0.300	0.287	0.297	0.248	0.253	0.547	0.559	0.631	0.668	0.691	0.818	0.836
101	[Rn]5f ¹³ 7s ²	0.259	0.296	0.283	0.293	0.244	0.249	0.537	0.549	0.620	0.656	0.679	0.797	0.814
102	[Rn]5f ¹⁴ 7s ²	0.255	0.292	0.279	0.289	0.241	0.245	0.528	0.538	0.611	0.644	0.668	0.778	0.795
103	[Rn]5f ¹⁴ 7s ² 7p	0.251	0.288	0.275	0.285	0.237	0.241	0.518	0.529	0.601	0.633	0.656	0.756	0.772
104	[Rn]5f ¹⁴ 6d ² 7s ²	0.247	0.285	0.271	0.282	0.233	0.238	0.509	0.519	0.591	0.621	0.645	0.736	0.750
105	[Rn]5f ¹⁴ 6d ³ 7s ²	0.243	0.281	0.268	0.278	0.230	0.234	0.501	0.509	0.583	0.611	0.634	0.716	0.731
106	[Rn]5f ¹⁴ 6d ⁴ 7s ²	0.239	0.278	0.264	0.275	0.227	0.231	0.492	0.500	0.574	0.600	0.623	0.698	0.712
107	[Rn]5f ¹⁴ 6d ⁵ 7s ²	0.235	0.274	0.261	0.271	0.223	0.228	0.483	0.491	0.565	0.590	0.613	0.681	0.695
108	[Rn]5f ¹⁴ 6d ⁶ 7s ²	0.231	0.271	0.257	0.268	0.220	0.225	0.475	0.481	0.556	0.580	0.603	0.665	0.678
109	[Rn]5f ¹⁴ 6d ⁷ 7s ²	0.227	0.268	0.254	0.264	0.217	0.222	0.467	0.473	0.549	0.571	0.593	0.650	0.663
110	[Rn]5f ¹⁴ 6d ⁹ 7s	0.224	0.264	0.250	0.261	0.214	0.219	0.459	0.464	0.541	0.561	0.583	0.636	0.648
111	[Rn]5f ¹⁴ 6d ¹⁰ 7s	0.220	0.261	0.247	0.258	0.211	0.216	0.451	0.456	0.533	0.552	0.575	0.622	0.635
112	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²	0.216	0.258	0.244	0.255	0.209	0.213	0.443	0.448	0.526	0.543	0.565	0.609	0.621
113	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p	0.213	0.255	0.241	0.252	0.206	0.210	0.436	0.440	0.518	0.535	0.557	0.596	0.609
114	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	0.209	0.252	0.238	0.249	0.203	0.208	0.428	0.432	0.511	0.527	0.549	0.585	0.596
115	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	0.206	0.249	0.235	0.246	0.201	0.205	0.421	0.424	0.504	0.518	0.541	0.574	0.585
116	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	0.202	0.246	0.232	0.243	0.198	0.203	0.414	0.417	0.497	0.511	0.533	0.563	0.574
117	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	0.199	0.244	0.229	0.241	0.195	0.200	0.407	0.409	0.490	0.503	0.525	0.553	0.564
118	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶	0.196	0.241	0.227	0.238	0.193	0.198	0.400	0.402	0.484	0.496	0.518	0.543	0.554

Table 6

Radii of maximum charge density calculated with the Dirac-Fock method for subshells ranging from $6s$ to $7p_{3/2}(a_0)$.

Atomic Number	Configuration	$6s$	$6p_{1/2}$	$6p_{3/2}$	$6d_{3/2}$	$6d_{5/2}$	$7s$	$7p_{1/2}$	$7p_{3/2}$
55	[Xe]6s	5.135							
56	[Xe]6s ²	4.292							
57	[Xe]5d6s ²	3.985							
58	[Xe]4f5d6s ²	3.927							
59	[Xe]4f ³ 6s ²	4.063							
60	[Xe]4f ⁴ 6s ²	4.003							
61	[Xe]4f ⁵ 6s ²	3.942							
62	[Xe]4f ⁶ 6s ²	3.886							
63	[Xe]4f ⁷ 6s ²	3.836							
64	[Xe]4f ⁷ 5d6s ²	3.592							
65	[Xe]4f ⁹ 6s ²	3.725							
66	[Xe]4f ¹⁰ 6s ²	3.674							
67	[Xe]4f ¹¹ 6s ²	3.625							
68	[Xe]4f ¹² 6s ²	3.575							
69	[Xe]4f ¹³ 6s ²	3.530							
70	[Xe]4f ¹⁴ 6s ²	3.484							
71	[Xe]4f ¹⁴ 5d6s ²	3.232							
72	[Xe]4f ¹⁴ 5d ² 6s ²	3.069							
73	[Xe]4f ¹⁴ 5d ³ 6s ²	2.932							
74	[Xe]4f ¹⁴ 5d ⁴ 6s ²	2.814							
75	[Xe]4f ¹⁴ 5d ⁵ 6s ²	2.711							
76	[Xe]4f ¹⁴ 5d ⁶ 6s ²	2.610							
77	[Xe]4f ¹⁴ 5d ⁷ 6s ²	2.522							
78	[Xe]4f ¹⁴ 5d ⁹ 6s	2.505							
79	[Xe]4f ¹⁴ 5d ¹⁰ 6s	2.458							
80	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	2.301							
81	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p	2.147	2.754						
82	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	2.026	2.495	2.699					
83	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	1.920	2.293	2.569					
84	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	1.831	2.134	2.415					
85	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	1.753	2.007	2.269					
86	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	1.683	1.902	2.144					
87	[Rn]7s	1.609	1.794	2.009			4.992		
88	[Rn]7s ²	1.541	1.703	1.898			4.258		
89	[Rn]6d7s ²	1.483	1.629	1.810	2.627		3.921		
90	[Rn]6d ² 7s ²	1.432	1.565	1.735	2.389	2.474	3.684		
91	[Rn]5f ² 6d7s ²	1.404	1.533	1.717	2.347	2.355	3.762		
92	[Rn]5f ³ 6d7s ²	1.370	1.493	1.679	2.282	2.339	3.698		
93	[Rn]5f ⁴ 6d7s ²	1.338	1.456	1.644	2.223	2.282	3.634		
94	[Rn]5f ⁶ 7s ²	1.315	1.431	1.634			3.738		
95	[Rn]5f ⁷ 7s ²	1.286	1.398	1.602			3.681		
96	[Rn]5f ⁷ 6d7s ²	1.252	1.357	1.553	2.128	2.189	3.455		
97	[Rn]5f ⁹ 7s ²	1.232	1.335	1.546			3.562		
98	[Rn]5f ¹⁰ 7s ²	1.206	1.306	1.518			3.508		
99	[Rn]5f ¹¹ 7s ²	1.182	1.277	1.494			3.455		
100	[Rn]5f ¹² 7s ²	1.158	1.252	1.469			3.402		
101	[Rn]5f ¹³ 7s ²	1.135	1.225	1.447			3.353		
102	[Rn]5f ¹⁴ 7s ²	1.113	1.201	1.425			3.305		
103	[Rn]5f ¹⁴ 7s ² 7p	1.086	1.167	1.384			2.959	3.813	
104	[Rn]5f ¹⁴ 6d ² 7s ²	1.061	1.139	1.348	1.893	2.010	2.898		
105	[Rn]5f ¹⁴ 6d ³ 7s ²	1.035	1.110	1.315	1.777	1.881	2.759		
106	[Rn]5f ¹⁴ 6d ⁴ 7s ²	1.011	1.082	1.281	1.687	1.778	2.644		
107	[Rn]5f ¹⁴ 6d ⁵ 7s ²	0.988	1.052	1.251	1.603	1.713	2.517		
108	[Rn]5f ¹⁴ 6d ⁶ 7s ²	0.965	1.025	1.221	1.540	1.640	2.422		
109	[Rn]5f ¹⁴ 6d ⁷ 7s ²	0.944	1.000	1.195	1.479	1.580	2.328		
110	[Rn]5f ¹⁴ 6d ⁹ 7s	0.924	0.975	1.169	1.436	1.532	2.258		
111	[Rn]5f ¹⁴ 6d ¹⁰ 7s	0.904	0.953	1.145	1.389	1.485	2.200		
112	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²	0.884	0.930	1.120	1.340	1.430	2.104		
113	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p	0.863	0.906	1.096	1.297	1.377	2.008	2.425	
114	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	0.846	0.885	1.072	1.259	1.330	1.922	2.273	2.697
115	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	0.827	0.864	1.049	1.217	1.286	1.835	2.097	2.949
116	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	0.810	0.844	1.028	1.184	1.247	1.768	2.002	2.790
117	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	0.793	0.825	1.006	1.147	1.212	1.701	1.894	2.551
118	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶	0.777	0.806	0.987	1.121	1.180	1.647	1.820	2.456

Table 7Atom mean radius using the total charge density and atom mean spherical radius of neutral elements with $1 \leq Z \leq 118$.

Atomic Number	Configuration	Atom mean radius (a_0)	Atom mean spherical radius (a_0)
1	1s	1.501	
2	1s ²	0.927	
3	1s ² 2s	1.673	2.492
4	1s ² 2s ²	1.532	2.081
5	1s ² 2s ² 2p	1.362	1.780
6	1s ² 2s ² 2p ²	1.191	1.516
7	1s ² 2s ² 2p ³	1.050	1.314
8	1s ² 2s ² 2p ⁴	0.951	1.181
9	1s ² 2s ² 2p ⁵	0.864	1.066
10	1s ² 2s ² 2p ⁶	0.789	0.968
11	[Ne]3s	0.985	1.570
12	[Ne]3s ²	1.020	1.569
13	[Ne]3s ² 3p	1.054	1.603
14	[Ne]3s ² 3p ²	1.033	1.517
15	[Ne]3s ² 3p ³	0.997	1.420
16	[Ne]3s ² 3p ⁴	0.966	1.349
17	[Ne]3s ² 3p ⁵	0.929	1.274
18	[Ne]3s ² 3p ⁶	0.892	1.201
19	[Ar]4s	1.021	1.638
20	[Ar]4s ²	1.059	1.677
21	[Ar]3d4s ²	1.020	1.586
22	[Ar]3d ² 4s ²	0.979	1.504
23	[Ar]3d ³ 4s ²	0.940	1.431
24	[Ar]3d ⁵ 4s	0.852	1.248
25	[Ar]3d ⁵ 4s ²	0.868	1.306
26	[Ar]3d ⁶ 4s ²	0.838	1.251
27	[Ar]3d ⁷ 4s ²	0.808	1.200
28	[Ar]3d ⁸ 4s ²	0.780	1.154
29	[Ar]3d ¹⁰ 4s	0.724	1.047
30	[Ar]3d ¹⁰ 4s ²	0.730	1.071
31	[Ar]3d ¹⁰ 4s ² 4p	0.751	1.141
32	[Ar]3d ¹⁰ 4s ² 4p ²	0.752	1.133
33	[Ar]3d ¹⁰ 4s ² 4p ³	0.747	1.110
34	[Ar]3d ¹⁰ 4s ² 4p ⁴	0.743	1.094
35	[Ar]3d ¹⁰ 4s ² 4p ⁵	0.735	1.070
36	[Ar]3d ¹⁰ 4s ² 4p ⁶	0.725	1.043
37	[Kr]5s	0.799	1.344
38	[Kr]5s ²	0.830	1.399
39	[Kr]4d5s ²	0.823	1.357
40	[Kr]4d ² 5s ²	0.810	1.312
41	[Kr]4d ⁴ 5s	0.770	1.191
42	[Kr]4d ⁵ 5s	0.755	1.149
43	[Kr]4d ⁵ 5s ²	0.768	1.195
44	[Kr]4d ⁷ 5s	0.731	1.097
45	[Kr]4d ⁸ 5s	0.719	1.071
46	[Kr]4d ¹⁰	0.682	0.967
47	[Kr]4d ¹⁰ 5s	0.694	1.023
48	[Kr]4d ¹⁰ 5s ²	0.700	1.045
49	[Kr]4d ¹⁰ 5s ² 5p	0.716	1.099
50	[Kr]4d ¹⁰ 5s ² 5p ²	0.720	1.104
51	[Kr]4d ¹⁰ 5s ² 5p ³	0.722	1.099
52	[Kr]4d ¹⁰ 5s ² 5p ⁴	0.722	1.093
53	[Kr]4d ¹⁰ 5s ² 5p ⁵	0.719	1.081
54	[Kr]4d ¹⁰ 5s ² 5p ⁶	0.715	1.065
55	[Xe]6s	0.770	1.305
56	[Xe]6s ²	0.795	1.360
57	[Xe]5d6s ²	0.792	1.331
58	[Xe]4f5d6s ²	0.778	1.303
59	[Xe]4f ³ 6s ²	0.756	1.278
60	[Xe]4f ⁴ 6s ²	0.743	1.253
61	[Xe]4f ⁵ 6s ²	0.731	1.229
62	[Xe]4f ⁶ 6s ²	0.719	1.206
63	[Xe]4f ⁷ 6s ²	0.707	1.184
64	[Xe]4f ⁷ 5d6s ²	0.704	1.163
65	[Xe]4f ⁹ 6s ²	0.685	1.142
66	[Xe]4f ¹⁰ 6s ²	0.674	1.122
67	[Xe]4f ¹¹ 6s ²	0.664	1.103

Table 7 (continued)

Atomic Number	Configuration	Atom mean radius (a_0)	Atom mean spherical radius (a_0)
68	[Xe]4f ¹² 6s ²	0.653	1.084
69	[Xe]4f ¹³ 6s ²	0.643	1.066
70	[Xe]4f ¹⁴ 6s ²	0.634	1.048
71	[Xe]4f ¹⁴ 5d6s ²	0.633	1.038
72	[Xe]4f ¹⁴ 5d ² 6s ²	0.628	1.018
73	[Xe]4f ¹⁴ 5d ³ 6s ²	0.623	0.998
74	[Xe]4f ¹⁴ 5d ⁴ 6s ²	0.618	0.979
75	[Xe]4f ¹⁴ 5d ⁵ 6s ²	0.612	0.961
76	[Xe]4f ¹⁴ 5d ⁶ 6s ²	0.607	0.945
77	[Xe]4f ¹⁴ 5d ⁷ 6s ²	0.602	0.929
78	[Xe]4f ¹⁴ 5d ⁹ 6s	0.587	0.885
79	[Xe]4f ¹⁴ 5d ¹⁰ 6s	0.582	0.872
80	[Xe]4f ¹⁴ 5d ¹⁰ 6s ²	0.584	0.882
81	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p	0.594	0.921
82	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ²	0.598	0.932
83	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ³	0.603	0.945
84	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁴	0.606	0.949
85	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁵	0.607	0.949
86	[Xe]4f ¹⁴ 5d ¹⁰ 6s ² 6p ⁶	0.607	0.944
87	[Rn]7s	0.640	1.105
88	[Rn]7s ²	0.657	1.151
89	[Rn]6d7s ²	0.660	1.143
90	[Rn]6d ² 7s ²	0.659	1.125
91	[Rn]5f ² 6d7s ²	0.648	1.102
92	[Rn]5f ³ 6d7s ²	0.641	1.084
93	[Rn]5f ⁴ 6d7s ²	0.635	1.067
94	[Rn]5f ⁶ 7s ²	0.623	1.048
95	[Rn]5f ⁷ 7s ²	0.617	1.032
96	[Rn]5f ⁷ 6d7s ²	0.616	1.022
97	[Rn]5f ⁹ 7s ²	0.605	1.002
98	[Rn]5f ¹⁰ 7s ²	0.598	0.987
99	[Rn]5f ¹¹ 7s ²	0.592	0.973
100	[Rn]5f ¹² 7s ²	0.586	0.959
101	[Rn]5f ¹³ 7s ²	0.580	0.946
102	[Rn]5f ¹⁴ 7s ²	0.574	0.933
103	[Rn]5f ¹⁴ 7s ² 7p	0.587	0.984
104	[Rn]5f ¹⁴ 6d ² 7s ²	0.574	0.922
105	[Rn]5f ¹⁴ 6d ³ 7s ²	0.572	0.910
106	[Rn]5f ¹⁴ 6d ⁴ 7s ²	0.569	0.898
107	[Rn]5f ¹⁴ 6d ⁵ 7s ²	0.567	0.890
108	[Rn]5f ¹⁴ 6d ⁶ 7s ²	0.564	0.878
109	[Rn]5f ¹⁴ 6d ⁷ 7s ²	0.561	0.868
110	[Rn]5f ¹⁴ 6d ⁹ 7s	0.555	0.849
111	[Rn]5f ¹⁴ 6d ¹⁰ 7s	0.552	0.838
112	[Rn]5f ¹⁴ 6d ¹⁰ 7s ²	0.551	0.835
113	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p	0.554	0.849
114	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ²	0.556	0.855
115	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ³	0.573	0.932
116	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁴	0.571	0.918
117	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁵	0.577	0.928
118	[Rn]5f ¹⁴ 6d ¹⁰ 7s ² 7p ⁶	0.574	0.916

