Abstract—In this paper, we investigate the suitability of single
carrier frequency domain processing (SC-FDP) as the downlink
transmission scheme in a massive multiple input multiple output
(MIMO) system. By deriving the sum-rate of the SC-FDP
massive MIMO system theoretically, we show that this method
obtains a sum-rate similar to that of orthogonal frequency
division multiplexing (OFDM) massive MIMO. We also derive
the theoretical sum-rate of both SC-FDP and OFDM in a non-
synchronized massive MIMO scenario and show that the former
is significantly larger than the latter. Moreover, we theoretically
analyze the sum-rate of both systems in the presence of power
amplifier non-linearity. All the sum-rates are derived for both
zero forcing (ZF) and matched filter (MF) precoding schemes.
The results show that the effect of power amplifier non-linearity
on the sum-rate of both systems is similar when the number of
users is large. We also compare SC-FDP with OFDM from the
peak to average power ratio (PAPR) and complexity viewpoints.
Although the PAPR of SC-FDP signals is lower than that of
OFDM signals, for MIMO systems this difference decreases as
we increase the number of users, which means that the techniques
can have similar PAPR in massive MIMO systems. The overall
complexities are similar for SC-FDP and OFDM. Due to the
mentioned facts, we can conclude that SC-FDP is a promising
transmission scheme for the downlink of the massive MIMO
system in the presence of carrier frequency offset (CFO) and
power amplifier non-linearities.

Keywords: Single Carrier, Massive MIMO, Precoding, CFO,
PAPR.

I. INTRODUCTION

The appearance of smart phones accelerated the increasing
demand for high rate wireless communications. This trend has
motivated a great amount of research to develop schemes to
ameliorate the spectral efficiency and accommodate high rate
users more efficiently [1]. One of the promising solutions to
cope with this problem is exploiting massive multiple input
multiple output (MIMO) systems. This method enhances the
spectral and power efficiency significantly [2], [3]. However,
to get this technology to the efficient implementation stage,
new challenges such as pilot contamination in time division
duplex (TDD) mode, training and feedback overhead in fre-
quency division duplex (FFD) mode, hardware impairments
and appropriate air interface need to be encountered [4]- [8].

One of the critical issues is to examine the suitability of
different modulation schemes to be adopted in massive MIMO
systems [8]. One of the well-known modulation schemes
for wireless communications is orthogonal frequency division
multiplexing (OFDM). This method converts the frequency
selective wireless channel into flat fading channels over each
sub-carrier. Thus, the system can be analyzed in a flat fad-
ing scenario, which has been the subject of many papers
in massive MIMO [5], [9]- [14]. In [5], the fundamental
problem of pilot contamination in the downlink of a multi-
cell massive MIMO system has been analyzed and a multi-
cell precoding scheme based on minimum mean square error
(MMSE) criterion has been developed to mitigate the pilot
contamination. In [9], the authors consider the downlink (DL)
of a single cell scenario and compare the spectral and energy
efficiency of precoding schemes based on matched filter (MF)
and zero forcing (ZF) criteria. The effect of phase noise in the
uplink of a massive MIMO system with zero forcing equalizer
has been studied in [10]. The authors in [11] have considered
both uplink and downlink of a single cell massive MIMO
system and proposed an MMSE based pilot reuse algorithm
to reduce the pilot overhead. In [12], the authors analyze how
the optimal number of scheduled users depends on the number
of BS antennas and other system parameters in a multi-cell
massive MIMO systems. Efficient power and training duration
allocations have also been studied in [13] and [14]. We would
like to highlight that all the enumerated papers consider a
flat fading scenario or, equivalently, an OFDM transmission
scheme.

OFDM is a special case of generalized multi-carrier (GMC)
systems [15]. GMC air interfaces are based on utilizing fast
Fourier transform (FFT) and inverse FFT (IFFT) as efficient
tools to deal with the channel frequency selectivity. They also
allow flexible allocation of time/frequency resources in both
single user and multi-user scenarios. Another special case
of GMC, which is used as uplink transmission scheme in
fourth generation cellular communication systems, is block
wise single carrier scheme [16]. This method equalizes the
received signal in the frequency domain, thus, it is known as
single carrier frequency domain equalizer (SC-FDE). Any of
the GMC methods can be used in a massive MIMO framework.
It is noteworthy that in the massive MIMO it is usually
assumed that the base station (BS) has full channel knowledge
while the user equipments only know the channel statistics
[17], [18]. Thus, the main signal processing load is carried by
the BS. This means that in the uplink we have to equalize the
signal and in the downlink we have to precode it while both
equalization and precoding are performed in the BS. These
two processes in the block-wise single carrier systems, are
performed in the frequency domain. Therefore, in this paper
we refer to this system as single carrier frequency domain
processing (SC-FDP). We used the term processing to include both equalization and precoding.

In this paper, we focus on the downlink transmission in a massive MIMO system. We consider SC-FDP as a candidate air interface and investigate its favorable and unfavorable attributes. The research on SC-FDP, unlike OFDM, is very limited in the context of massive MIMO and is confined to [19], [20]. Contrastingly to our work, [19] considers the uplink transmission of a single cell massive MIMO system. In [19], a low complexity iterative detection algorithm for single carrier frequency division multiple access (SC-FDMA) scheme has been developed. The proposed algorithm combines a frequency domain MMSE equalization with parallel interference cancellation. As in our paper, [20] considers the downlink transmission and the BER and peak to average power ratio (PAPR) of the orthogonal frequency division multiple access (OFDMA) and SC-FDMA schemes are compared. However, this comparison is performed via simulations and there is no analytical derivation of sum-rate.

In this paper, we analytically examine the suitability of the SC-FDP method for downlink transmission of massive MIMO systems in the presence of hardware impairments. The considered hardware impairment includes the instabilities of the oscillators in the form of carrier frequency offset (CFO) and the nonlinearities of the power amplifiers. We analytically derive the achievable sum-rate of a SC-FDP massive MIMO system in the presence and absence of CFO. To put this analysis into perspective, we compare the achievable rate in both scenarios with the corresponding values of OFDM massive MIMO systems. Although the sum-rate of a fully synchronized OFDM massive MIMO system has been already derived in other papers, the literature lacks such analysis in the presence of CFO. Therefore, we derived the sum-rate of an OFDM massive MIMO system hit by CFO. Through our analysis, we show that the SC-FDP and OFDM obtain similar sum-rates in a synchronized case. In a non-synchronized system, however, we show analytically that the interference term generated due to CFO does not vanish in OFDM as the number of antennas goes to infinity while this is not the case for SC-FDP systems when the users know only the channel statistics. We have also derived the sum-rate of these two systems in presence of power amplifier non-linearity for systems with large number of users which is usually the case in the massive MIMO systems. Our analysis show that for small number of users the SC-FDP massive MIMO system has lower PAPR and out-of-band radiation compared to the OFDM massive MIMO systems. However, for large number of users the derived sum-rate, PAPR and out-of-band radiation are identical for both systems. Furthermore, as presented in our analysis in the presence of power amplifier non-linearity, the sum-rate of both systems does not increase unlimitedly as the number of BS antennas tends to infinity. We can summarize the contributions of the paper as follows:

- Deriving the sum-rate of the SC-FDP in a synchronized massive MIMO system.
- Deriving the sum-rate of the SC-FDP in a non-synchronized massive MIMO system.
- Deriving the sum-rate of the OFDM in a non-synchronized massive MIMO system.
- Deriving the sum-rate of the SC-FDP in the presence of power amplifier non-linearity.
- Deriving the sum-rate of the OFDM in the presence of power amplifier non-linearity.
- Comparing the PAPR and out-of-band radiation of the SC-FDP and OFDM massive MIMO systems.

All the sum-rate derivations are done for both MF and ZF precoding schemes. Overall, we conclude that in a massive MIMO scenario SC-FDP outperforms OFDM when hardware impairments such as CFO and power amplifier non-linearity are taken into account. This is of crucial importance specially in distributed massive MIMO systems where the base station antennas are geographically distributed over the cell [21]- [23]. In this framework, each antenna is impelled to have its own RF chain including the oscillator and power amplifier. Distributed massive MIMO is more expensive than a system where massive number of antennas are located in a single array. To make the system economically worthwhile, we have to utilize low-cost oscillators and power amplifiers. Inexpensive oscillators are imprecise and suffer from frequency instabilities. Cheap power amplifiers’s characteristic, likewise, are nonlinear which introduces in-band distortion and spectral splatter resulting in performance degradation of in-band or adjacent users.

The rest of the paper is organized as follows. Section II describes the system model of SC-FDP massive MIMO in the downlink. In section III achievable sum-rates are derived in the presence and absence of CFO and in presence of power amplifier non-linearity. Section IV compares SC-FDP from different points of view with OFDM. Section V presents the performance results and section VI concludes the paper.

II. System Model

We consider the downlink in a single cell scenario where the BS is equipped with M antennas and there are K single antenna users. All users exploit the same time and frequency resources, simultaneously. The channel coefficient of the l-th path of the frequency selective channel between the m-th antenna of the BS and k-th user is denoted by h_{m,k,l} which is a zero-mean complex Gaussian random variable with variance of \sigma^2_{k,l}. We assume the channel coefficients are independent and normalized such that \sum_{l=0}^{L-1} \sigma^2_{k,l} = 1 for all users where L is the number of paths. The n-th coefficient of the channel frequency response which is obtained as

\[ H_{m,k,n} = \sum_{l=0}^{L-1} h_{m,k,l} e^{-j2\pi nl/N} \quad (1) \]

is a zero-mean complex Gaussian random variable with variance of 1 and N is the data block length. The BS has perfect channel state information and the user equipment has the channel statistics only. In the rest of the paper, vectors and matrices are denoted by boldface letters. The vectors in the time domain are denoted by bold small letters and those in the frequency domain are denoted by bold capital letters.

In the rest of this section, we explain the details of the transmitted signal in the downlink of a massive MIMO system.
with SC-FDP modulation. We present the $q$-th symbol of the data block which should be transmitted from the BS to the $k$-th user by $x_{k,q}$ where $q = 0, ..., N - 1$. The data symbols are assumed to be independent and identically distributed random variables with zero mean and variance $\bar{P}$. By taking an $N$-point FFT of this vector, we convert the data block into the frequency domain and present the $n$-th component of the frequency domain vector by $X_{n,k}$. Then, the $n$-th data symbols of the corresponding blocks of all $K$ users are precoded in the frequency domain using the $M \times K$ precoding matrix $W_n$ as follows

$$Y_n = W_n X_n, \quad (2)$$

where $Y_n = [Y_{1,n}, Y_{2,n}, ..., Y_{M,n}]^T$, $X_n = [X_{1,n}, X_{2,n}, ..., X_{K,n}]^T$, $n = 0, ..., N - 1$ and $Y_{m,n}$ is the $n$-th symbol of data block of the $m$-th BS antenna in frequency domain. It is worth mentioning that the matrix $W_n$ is associated with each user’s channel frequency response on the $n$-th sub-carrier. Thus, we need $N$ precoding matrices to precode the whole data block of all users. After the precoding in the frequency domain, the vector obtained for each BS antenna is converted to time domain by an $N$ point IFFT as follows

$$y_{m,i} = \frac{1}{N} \sum_{n=0}^{N-1} Y_{m,n} e^{j\frac{2\pi n i}{N}}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \left( \sum_{k=1}^{K} w_{m,k,n} X_{k,n} \right) e^{j\frac{2\pi n i}{N}} \quad (3)$$

where $y_{m,i}$ is the $i$-th component of the vector transmitted from the $m$-th antenna and $w_{m,k,n}$ is element in the $m$-th row and $k$-th column of $W_n$. The generated time domain vector is preceded by cyclic prefix and transmitted over the channel. The $i$-th component of received vector at user $u$, after removing the CP, can be expressed as

$$\hat{x}_{u,i} = \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} H_{m,u,n} y_{m,i} e^{j\frac{2\pi n i}{N}} + \eta_{u,i}$$

$$= \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} \sum_{k=1}^{K} \sum_{q=0}^{N-1} w_{m,k,n} H_{m,u,n} x_{k,q} e^{j\frac{2\pi n (i-q)}{N}} + \eta_{u,i} \quad (4)$$

where $\eta_{u,i}$ is the $i$-th element of the zero-mean complex white Gaussian noise with variance of $\sigma^2$ added to the $u$-th user signal. Fig. 1 shows the block diagram of SC-FDP and OFDM Massive MIMO systems in the downlink.

**III. Achievable Sum-Rate**

In this section, we derive the achievable sum-rate in the downlink of a SC-FDP massive MIMO system taking advantage of Theorem 1 in [24]. We consider three scenarios which are presented in the following three sub-sections. In the first scenario, the system is fully synchronized and there is no power amplifier non-linearity and in the second and third one the system suffers from frequency instabilities in the form of a fixed CFO and power amplifier non-linearity, respectively.

**A. Synchronized Massive MIMO with no Power Amplifier Non-linearity**

As explained earlier, the user knowledge of the channel is confined to its statistics. Thus, to satisfy the conditions of Theorem 1 in [24] in order to derive the ergodic achievable sum-rate we can rewrite (4) as [18]

$$\hat{x}_{u,i} = \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} E \left\{ w_{m,u,n} H_{m,u,n} x_{u,i} \right\} + \xi_{u,i} \quad (5)$$

Desired signal

where $E\{\cdot\}$ is the expectation and $\xi_{u,i}$ is the effective noise defined as

$$\xi_{u,i} = \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,u,n} H_{m,u,n} x_{u,i}$$

$$- \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} E \left\{ w_{m,u,n} H_{m,u,n} x_{u,i} \right\} x_{u,i}$$

$$+ \frac{1}{N} \sum_{q=1}^{N-1} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,u,n} H_{m,u,n} x_{u,q} e^{j\frac{2\pi n (i-q)}{N}}$$

$$+ \frac{1}{N} \sum_{k=1}^{K} \sum_{q=0}^{N-1} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,k,n} H_{m,u,n} x_{k,q} e^{j\frac{2\pi n (i-q)}{N}} + \eta_{u,i} \quad (6)$$

Considering (5) and (6), we can obtain the SINR for the $u$-th user as follows

$$\text{SINR}_{u,i} = \frac{E \left\{ \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} E \left\{ w_{m,u,n} H_{m,u,n} x_{u,i} \right\}^2 \right\}}{P_{\xi_{u,i}}}$$

(7)
where $P_{\xi_{u,i}}$ is the power of the effective noise at user $u$ and is defined as

$$P_{\xi_{u,i}} = P \Var \left( \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=1}^{M} w_{m,u,n} H_{m,u,n} \right)$$

$$+ P \sum_{q=0}^{N-1} \mathbb{E} \left\{ \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=1}^{M} w_{m,u,n} H_{m,u,n} e^{j2\pi n q/N} \right\}^2$$

$$+ P \sum_{k=1}^{K} \sum_{k \neq u}^{N-1} \mathbb{E} \left\{ \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=1}^{M} w_{m,k,n} H_{m,u,n} e^{j2\pi (n-k) n/N} \right\}^2$$

$$+ \sigma^2$$

(8)

If we use MF precoding, the precoder coefficient in the $m$-th row and $u$-th column of $W_n$ can be expressed as

$$w_{m,u,n} = \frac{H_{m,u,n}^*}{\sqrt{K \sum_{m=1}^{M} |H_{m,u,n}|^2}}$$

$$\simeq \sqrt{\frac{K M E[|H_{m,u,n}|^2]}{\sum_{m=1}^{M} |H_{m,u,n}|^2}} = H_{m,u,n}^*$$

(9)

where $H_{m,u,n}^*$ is the complex conjugate of $H_{m,u,n}$ and the approximation in the above equation is due to the fact that $\frac{1}{M} \sum_{m=1}^{M} |H_{m,u,n}|^2$ can be approximated by $E[|H_{m,u,n}|^2]$ for large $M$. Substituting (9) in (7) and noting that $E[|H_{m,k,n}|^4] = 2$ for $k = 1, \ldots, K$, $m = 1, \ldots, M$ and $n = 0, \ldots, N-1$, the SINR of the $u$-th user is simplified to

$$SINR_{u,i}^{MF} = \frac{M}{K + K \frac{\sigma^2}{N}}$$

(10)

Since in (5), $E\{w_{m,u,n} H_{m,u,n}\}$ is known and $x_{u,i}$ and the effective noise can be assumed uncorrelated [18], by considering the Gaussian distribution for data symbols, we can obtain the achievable sum-rate of users as [24, Theorem 1]

$$R_{SC-FDP}^{MF} = K \log_2 \left( 1 + \frac{M}{K + K \frac{\sigma^2}{N}} \right)$$

(11)

For the ZF precoding, the precoding matrix $W_n$ is defined as

$$W_n = \frac{H_n^H (H_n H_n^H)^{-1}}{\sqrt{M - K} K I_{K \times K}}$$

(12)

where $H_n$ is a $K \times M$ matrix whose $k$-th row and $m$-th column element is $H_{m,k,n}$. In this case since $H_n W_n = \frac{M - K}{K} I_{K \times K}$, the $i$-th received symbol at user $u$ is

$$\hat{x}_{u,i} = \sqrt{\frac{M - K}{K}} x_{u,i} + n_{u,i}$$

(13)

So the SINR of the received signal at user $u$ is

$$SINR_{u,i}^{ZF} = \frac{P(M - K)}{\sigma^2}$$

(14)

and the achievable sum-rate of users is

$$R_{SC-FDP}^{ZF} = K \log_2 \left( 1 + \frac{P(M - K)}{\sigma^2} \right) .$$

(15)

**B. Non-Synchronized Massive MIMO**

In this sub-section, we investigate the achievable sum-rate in the downlink of the SC-FDP massive MIMO system when the system is hit by carrier frequency offset. Let us present the normalized CFO of the $u$-th user by $\epsilon_u$. The $i$-th component of the received signal of this user can be expressed as

$$\hat{x}_{u,i} = \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} \sum_{k=1}^{K} \sum_{q=0}^{N-1} w_{m,k,n} H_{m,u,n} x_{k,q} e^{j2\pi n (i-q)/N} e^{j2\pi n (1-q)/N}$$

$$+ \eta_{u,i}$$

(16)

The rotation of the received signal caused by CFO results in SINR and BER degradation. Since the users know the channel statistics, the received signal can be written as

$$\hat{x}_{u,i} = \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} \sum_{k=1}^{K} w_{m,k,n} H_{m,u,n} x_{k,q} + \nu_{u,i}$$

(17)

where $\nu_{u,i}$, the effective noise at $\hat{x}_{u,i}$, is defined as

$$\nu_{u,i} = \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} \sum_{k=1}^{K} \sum_{q=0}^{N-1} w_{m,k,n} H_{m,u,n} e^{j2\pi n (i-q)/N}$$

$$+ \eta_{u,i}$$

(18)

The power of $\nu_{u,i}$ is

$$P_{\nu_{u,i}} = P \left\{ \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,u,n} H_{m,u,n} e^{j2\pi n (1-q)/N} \right\}^2$$

(19)

Henceforth, the SINR for the $i$-th sample of the $u$-th user is as follows

$$SINR_{u,i}^{ZF} = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{n=1}^{M} E\{w_{m,u,n} H_{m,u,n}\} x_{u,i}^2$$

(20)
For the MF and ZF precoders, the above SINR is simplified to (21) and (22), respectively.

\[ \text{SINR}_{u,i}^{MF} = \frac{1}{2 - 2 \cos \left( \frac{2\pi u_i}{N} \right) + \frac{K}{M} + \frac{\sigma_k^2}{P_{\text{TX}}} \}, \] \quad (21) \]

\[ \text{SINR}_{u,i}^{ZF} = \frac{1}{(\cos \left( \frac{2\pi u_i}{N} \right) - 1)^2 + \sin^2 \left( \frac{2\pi u_i}{N} \right) + \frac{\sigma_k^2}{P_{\text{TX}}(M-K)}}. \] \quad (22) \]

Since

\[ E\{x_{u,i}, \xi_{u,i}\} = PE \{ \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,u,n} H_{m,u,n} \} \left( e^{j2\pi u_i} - 1 \right), \]

we can conclude that the desired signal and the effective noise are uncorrelated for small \( \epsilon_u \). Thus, regarding Theorem 1 in [24], the achievable sum-rate for the SC-FDP massive MIMO system with MF and ZF precoder for small CFO is as (23) and (24), respectively.

\[ R_{\text{SC-FDP}}^{MF} = \frac{1}{N} \sum_{u=1}^{K} \sum_{i=0}^{N-1} \log \left( 1 + \frac{1}{2 - 2 \cos \left( \frac{2\pi u_i}{N} \right) + \frac{K}{M} + \frac{\sigma_k^2}{P_{\text{TX}}}} \right), \] \quad (23) \]

\[ R_{\text{SC-FDP}}^{ZF} = \frac{1}{N} \sum_{u=1}^{K} \sum_{i=0}^{N-1} \log \left( 1 + \frac{1}{(\cos \left( \frac{2\pi u_i}{N} \right) - 1)^2 + \sin^2 \left( \frac{2\pi u_i}{N} \right) + \frac{\sigma_k^2}{P_{\text{TX}}(M-K)}} \right). \] \quad (24) \]

C. Massive MIMO with Non-linear Power Amplifier

In this subsection, we investigate the achievable sum-rate in the downlink of the SC-FDP massive MIMO system with non-linear power amplifier. We first investigate the distribution of the power amplifier input signal. The \( i \)-th sample of the power amplifier input signal of the \( m \)-th antenna in the SC-FDP massive MIMO system is

\[ y_{m,i} = \frac{1}{N} \sum_{k=1}^{K} \sum_{n=0}^{N-1} \sum_{q=0}^{N-1} w_{m,k,n} x_{k,q} e^{j2\pi (q-n) i / N}. \] \quad (25) \]

Let us define

\[ \xi_k = \sum_{n=0}^{N-1} \sum_{q=0}^{N-1} w_{m,k,n} x_{k,q} e^{j2\pi (q-n) i / N}. \]

The parameter \( \xi_k \) is not necessarily Gaussian. Since \( w_{m,k,n} \) and \( w_{m,k',n} \) and the two random variables \( x_{k,q} \) and \( x_{k',q} \) are independent for \( k \neq k' \), then \( \xi_k \)'s are also independent for different users. Thus, in general the distribution of \( y_{m,i} \) is not Gaussian. However, for large \( K \) the \( \xi_k \)'s are independent and identically distributed and we can approximate the distribution of \( y_{m,i} \) as Gaussian due to the central limit theorem. Fortunately, in the massive MIMO systems, \( K \) is usually large and the Gaussian approximation is valid. Thus, we use the Bussgang Theorem to derive the sum-rate of this system. According to the Bussgang Theorem, the non-linear power amplifier output signal is [25]

\[ \tilde{y}_{m,i} = \alpha \frac{N}{K} \sum_{k=1}^{K} \sum_{n=0}^{N-1} w_{m,k,n} X_{k,n} e^{j2\pi n i / N} + \eta_{m,i}, \] \quad (26) \]

where \( \alpha \) is the attenuation factor and \( \eta_{m,i} \) is the distortion signal modeled by a zero mean Gaussian process with variance \( \sigma_{\text{ap}}^2 \), which is independent of the signal. Consequently, the \( i \)-th sample of the received signal of \( u \)-th user is

\[ \tilde{x}_{u,i} = \alpha \frac{M}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} E\{w_{m,u,n} H_{m,u,n}\} x_{u,i} + \xi_{u,i}, \] \quad (27) \]

where \( \xi_{u,i} \) is the effective noise defined as

\[ \xi_{u,i} = \alpha \frac{M}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} E\{w_{m,u,n} H_{m,u,n}\} x_{u,i} \]

\[ - \alpha \frac{M}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} E\{w_{m,u,n} H_{m,u,n}\} x_{u,i} \]

\[ + \alpha \frac{K}{N} \sum_{k=1}^{K} \sum_{u,i}^{N} \sum_{q=1}^{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,k,n} H_{m,u,n} x_{k,q} e^{j2\pi (q-n) i / N} \]

\[ + \alpha \frac{K}{N} \sum_{k=1}^{K} \sum_{u,i}^{N} \sum_{q=1}^{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,k,n} H_{m,u,n} x_{k,q} e^{j2\pi (q-n) i / N} \]

\[ + \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} H_{m,u,n} N_{\text{ap}}^{m,n} e^{j2\pi n i / N} + \eta_{u,i}. \] \quad (28) \]

In the above equation, \( N_{\text{ap}}^{m,n} \) is the distortion noise of the \( m \)-th antenna on the \( n \)-th sub-carrier. Considering (27) and (28), we can obtain the SINR for the \( u \)-th user as follows

\[ \text{SINR}_{u,i} = \left( \frac{\alpha}{P_{\text{ef}}^{u,i}} \sum_{m=1}^{M} \sum_{n=0}^{N-1} \sum_{i=0}^{N-1} E\{w_{m,u,n} H_{m,u,n}\} x_{u,i} \right)^2, \] \quad (29) \]

where \( P_{\text{ef}}^{u,i} \) is the power of the effective noise for the \( u \)-th user defined as

\[ P_{\text{ef}}^{u,i} = P_{\text{Var}} \left[ \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,u,n} H_{m,u,n} \right] \]

\[ + P \sum_{q=0}^{N-1} E \left( \frac{1}{N} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,u,n} H_{m,u,n} e^{j2\pi (n-q) i / N} \right)^2 \]

\[ + P \sum_{k=1}^{K} \sum_{k'=1}^{K} \sum_{u,i}^{N} \sum_{q=0}^{N-1} \sum_{m=1}^{M} \sum_{n=0}^{N-1} w_{m,k,n} H_{m,u,n} e^{j2\pi (n-q) i / N} \]

\[ + M \sigma_{\text{ap}}^2 + \sigma^2. \] \quad (30) \]

For the MF precoding, the SINR is simplified to

\[ \text{SINR}_{u,i}^{MF} = \frac{M}{K + M \sigma_{\text{ap}}^2 P_{\text{TX}} + \sigma^2 K / P_{\text{TX}}}. \] \quad (31) \]

Using Theorem 1 in [24], the achievable sum-rate is

\[ R_{\text{SC-FDP}}^{MF} = K \log_2 \left( 1 + \frac{M}{K + M \sigma_{\text{ap}}^2 P_{\text{TX}} + \sigma^2 K / P_{\text{TX}}} \right). \] \quad (32) \]

If we use the ZF precoder, it yields the following SINR

\[ \text{SINR}_{u,i}^{ZF} = \frac{P \sigma^2 (M-K)}{MK \sigma_{\text{ap}}^2 + K \sigma^2}. \] \quad (33) \]
whose corresponding achievable sum-rate is
\[ R^{ZF}_{SC-OFDM} = K \log_2 \left( 1 + \frac{P\alpha^2(M - K)}{MK\sigma^2_{ap} + K\sigma^2} \right). \] (34)

IV. COMPARISON WITH THE OFDM MASSIVE MIMO

In this section, we compare different aspects of the SC-FDP massive MIMO and OFDM massive MIMO system. The system model of OFDM massive MIMO is similar to that of SC-FDP massive MIMO, except that the FFT blocks are moved from the BS to the user terminals. Fig. 1 shows the block diagram of SC-FDP and OFDM massive MIMO systems.

A. Achievable Sum-Rate

The following sub-sections discuss the achievable sum-rate of an OFDM massive MIMO system in three scenarios. In the first scenario the system is synchronized and there is no power amplifier non-linearity. In the second and third scenarios the system suffers from CFO and power amplifier non-linearity, respectively. The achievable sum-rate of the first scenario is already obtained in the literature, however, the second and third scenarios have not been investigated in the literature.

1) Synchronized Massive MIMO with no Power Amplifier Non-linearity: Using the same notation of the previous section, in an OFDM massive MIMO system, \( X_k = [X_{k,0}, ..., X_{k,N - 1}] \) is the data block to be sent to the \( k \)-th user where in \( X_{k,n} \), \( n \) denotes the subcarrier number. The \( n \)-th data symbols of all users are precoded simultaneously using the matrix \( W_n \) as explained in (2). The precoded block associated with each BS antenna is then passed through an \( N \)-point IFFT and preceded by CP. At the user terminal, after removing the CP, the signal goes through an \( N \)-point FFT block. Thus, the \( n \)-th symbol received by the \( u \)-th user is

\[
\hat{X}_{u,n} = \sum_{m=1}^{M} \sum_{k=1}^{K} H_{m,u,n} w_{m,k,n} X_{k,n} + N_{u,n}
= \sum_{m=1}^{M} \sum_{k=1}^{K} \mathbb{E}\{H_{m,u,n} w_{m,k,n}\} X_{k,n} + \Xi_{u,n},
\] (35)

where the subscripts denote the \( n \)-th sub-carrier of the \( u \)-th user. The parameter \( N_{u,n} \) is the zero mean additive Gaussian noise whose variance is \( \sigma^2 \) and \( \Xi_{u,n} \) is the effective noise defined as

\[
\Xi_{u,n} = \sum_{m=1}^{M} H_{m,u,n} w_{m,k,n} X_{k,n} - \sum_{m=1}^{M} \mathbb{E}\{H_{m,u,n} w_{m,k,n}\} X_{k,n}
+ \sum_{m=1}^{M} \sum_{k=1, k \neq u}^{K} H_{m,u,n} w_{m,k,n} X_{k,n} + N_{u,n}.
\] (36)

Therefore, the SINR of the \( u \)-th user received data over the \( n \)-th sub-carrier is

\[
SINR_{u,n} = \frac{\mathbb{E}\left\{ \sum_{m=1}^{M} \mathbb{E}\{H_{m,u,n} w_{m,k,n}\} X_{k,n} \right\}^2}{P_{\Xi_{u,n}}},
\] (37)

where \( P_{\Xi_{u,n}} \) is the power of the effective noise which is defined as

\[
\Xi_{u,n} = P\text{Var}\{\sum_{m=1}^{M} H_{m,u,n} w_{m,k,n}\}
+ P \sum_{k=1, k \neq u}^{K} \mathbb{E}\left\{ \left| \sum_{m=1}^{M} H_{m,u,n} w_{m,k,n} \right|^2 \right\} + \sigma^2.
\] (38)

For the MF and ZF precoders, the SINR is simplified to (39) and (40), respectively.

\[
SINR_{u,n}^{MF} = \frac{M}{K + K\sigma^2}, \quad (39)
\]
\[
SINR_{u,n}^{ZF} = \frac{P(M - K)}{K\sigma^2}. \quad (40)
\]

By considering Theorem 1 of [24] the corresponding sum-rates are [9]

\[
R_{OFDM}^{MF} = K \log_2 \left( 1 + \frac{M}{K + K\sigma^2} \right), \quad (41)
\]
\[
R_{OFDM}^{ZF} = K \log_2 \left( 1 + \frac{P(M - K)}{K\sigma^2} \right). \quad (42)
\]

Comparing (11) and (41) and comparing (15) and (42) shows that the achievable sum-rate in the downlink of synchronized SC-FDP and synchronized OFDM are identical for both MF and ZF precoders. This is due to the fact that in the downlink the users only know the channel statistics.

2) Non-Synchronized Massive MIMO: In the presence of carrier frequency offset, the data received on the \( n \)-th sub-carrier of the \( u \)-th user can be expressed as

\[
\hat{X}_{u,n} = \frac{1}{N} \sum_{i=0}^{N-1} \sum_{m=1}^{M} \sum_{r=0}^{K} \sum_{k=1}^{K} H_{m,u,r} w_{m,k,r} X_{k,r} e^{j2\pi\nu(r-n+i+u)} + N_{u,n}
= \left( \frac{1}{N} \sum_{i=0}^{N-1} e^{j2\pi\nu i} \right) \left( \sum_{m=1}^{M} \sum_{k=1}^{K} H_{m,u,n} w_{m,k,n} X_{k,n} \right)
+ \frac{1}{N} \sum_{i=0}^{N-1} \sum_{m=1}^{M} \sum_{r=0}^{K} \sum_{k=1}^{K} H_{m,u,r} w_{m,k,r} X_{k,r} e^{j2\pi\nu(r-n+i+u)} + N_{u,n},
\] (43)

Additional interference

Comparing (35) and (43), we can see that in OFDM massive MIMO the CFO generates an additional interference term due to users’ interfering adjacent sub-carriers.

Since the users know the channel statistics, we can write \( \hat{X}_{u,n} \) as

\[
\hat{X}_{u,n} = \sum_{m=1}^{M} \mathbb{E}\{w_{m,u,n} H_{m,u,n}\} X_{u,n} + \Xi_{u,n},
\] (44)
where $\Xi_{u,n}$ is the effective noise over the $n$-th sub-carrier of the $u$-th user which is defined as

$$
\Xi_{u,n} = \sum_{m=1}^{M} w_{m,u,n} H_{m,u,n} \left( \frac{1}{N} \sum_{i=0}^{N-1} e^{j2\pi r_{u,i}} \right) X_{u,n} - \sum_{m=1}^{M} E\{ w_{m,u,n} H_{m,u,n} \} X_{u,n}
+ \frac{1}{N} \sum_{r=0,r\neq n}^{N-1} \sum_{i=0}^{N-1} \sum_{m=1}^{M} (w_{m,u,r} H_{m,u,r} e^{j2\pi (r-n+i) / N}) X_{u,r}
+ \frac{1}{N} \sum_{k=1,k\neq u}^{K} \sum_{r=0}^{K-1} \sum_{i=0}^{N-1} \sum_{m=1}^{M} (w_{m,k,r} H_{m,u,r} e^{j2\pi (r-n+i) / N}) X_{u,r}
+ N_{u,n}.
$$

(45)

The power of $\Xi_{u,n}$ will be

$$
P_{\Xi_{u,n}} = E\left\{ \sum_{m=1}^{M} w_{m,u,n} H_{m,u,n} \left( \frac{1}{N} \sum_{i=0}^{N-1} e^{j2\pi r_{u,i}} \right) X_{u,n} - \sum_{m=1}^{M} E\{ w_{m,u,n} H_{m,u,n} \} X_{u,n} \right\}^2
+ \frac{P}{N} \sum_{r=0}^{N-1} \sum_{i=0}^{N-1} \sum_{m=1}^{M} (w_{m,u,r} H_{m,u,r} e^{j2\pi (r-n+i) / N})^2
+ \frac{P}{N} \sum_{k=1,k\neq u}^{K} \sum_{r=0}^{K-1} \sum_{i=0}^{N-1} \sum_{m=1}^{M} (w_{m,k,r} H_{m,u,r} e^{j2\pi (r-n+i) / N})^2
+ \sigma^2.
$$

(46)

Hence, we can derive the SINR for the $n$-th sub-carrier of the $u$-th user as follows

$$
SINR_{u,n} = \frac{P \sum_{m=1}^{M} E\{ w_{m,u,n} H_{m,u,n} \}}{P_{\Xi_{u,n}}}. 
$$

(47)

By using the MF precoding, the SINR is obtained as

$$
SINR_{u,n}^{MF} = \frac{M}{K + 2M \left( 1 - \frac{1}{N} \sum_{i=0}^{N-1} \cos\left( \frac{2\pi r_{u,i}}{N} \right) \right) + \frac{\sigma^2K}{P}}.
$$

(48)

For the ZF precoding, the SINR is calculated as

$$
SINR_{u,n}^{ZF} = \frac{1}{2 \left( 1 - \frac{1}{N} \sum_{i=0}^{N-1} \cos\left( \frac{2\pi r_{u,i}}{N} \right) \right) + \frac{\sigma^2K}{P(M-K)}}.
$$

(49)

Since

$$
E\{ X_{u,i} \Xi_{u,i} \} = PE\{ w_{m,u,n} H_{m,u,n} \} \left( \frac{1}{N} \sum_{i=0}^{N-1} e^{j2\pi r_{u,i}} - 1 \right),
$$

we can conclude that the desired signal and the effective noise are uncorrelated for small $\varepsilon_u$. Thus, considering Theorem 1 in [24], the achievable sum-rate of the users for MF and ZF precoding for small CFO are

$$
R_{OFDM}^{MF} = \sum_{u=1}^{K} \sum_{n=0}^{N-1} \log_2 \left( 1 + SINR_{u,n}^{MF} \right),
$$

(50)

Where $R_{OFDM}^{MF}$ is the sum-rate of the non-synchronized OFDM massive MIMO system.

$$
R_{OFDM}^{ZF} = \sum_{u=1}^{K} \sum_{n=0}^{N-1} \log_2 \left( 1 + \frac{2}{1 - \frac{1}{N} \sum_{i=0}^{N-1} \cos\left( \frac{2\pi r_{u,i}}{N} \right)} \right).
$$

(51)

Therefore, if $M$ tends to infinity, the sum-rate in the non-synchronized OFDM massive MIMO system is limited to

$$
R_{OFDM}^{ZF} = \sum_{u=1}^{K} \sum_{n=0}^{N-1} \log_2 \left( 1 + \frac{1}{2 \left( 1 - \frac{1}{N} \sum_{i=0}^{N-1} \cos\left( \frac{2\pi r_{u,i}}{N} \right) \right)} \right).
$$

(52)

This is due to the fact that the interference term generated by the CFO in (48) and (49) does not vanish as $M$ tends to infinity. Consequently, the achievable sum-rate does not increase unlimitedly.

To compare this value with that of the non-synchronized SC-FDP massive MIMO system, we note that when $M$ tends to infinity, (23) and (24) are simplified to

$$
R_{FDP}^{ZF} = \sum_{u=1}^{K} \sum_{n=0}^{N-1} \log_2 \left( 1 + \frac{1}{2 \left( 1 - \frac{1}{N} \sum_{i=0}^{N-1} \cos\left( \frac{2\pi r_{u,i}}{N} \right) \right)} \right).
$$

(53)

Since $1 - \cos\left( \frac{2\pi r_{u,i}}{N} \right) = 0$ for $i = 0$, the sum-rate of the non-synchronized SC-FDP massive MIMO system increases unlimitedly as the number of BS antennas tends to infinity.

Comparing (53) and (52) indicates that, unlike the synchronized case where the achievable sum-rate of SC-FDP and OFDM are identical, in non-synchronized case where the users know the channel statistics there is a fundamental difference between the enumerated massive MIMO systems. As can be verified, in SC-FDP by increasing $M$ the achievable sum-rate increases. In OFDM, however, for very large number of antennas the achievable sum-rate is bounded by (52). This difference is of vital importance in distributed massive MIMO systems where due to having several independent local oscillators we have to use inexpensive ones in which frequency stability is a challenge and suffer further from the frequency offset.

3) Massive MIMO with Non-linear Power Amplifier: In this subsection, we investigate the achievable sum-rate in the downlink of the OFDM massive MIMO system with non-linear power amplifier. The $i$-th sample of the power amplifier input signal of the $m$-th antenna is

$$
y_{m,i} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=1}^{K} w_{m,k,n} X_{k,n} e^{j2\pi r_{m,i} / N}.
$$

(54)

For large $N$, the distribution of (54) can be approximated by Gaussian. Thus, we use the Bussgang Theorem to derive the sum-rate of this system. According to the Bussgang theorem, the output signal of the power amplifier of the $m$-th antenna over the $n$-th sub-carrier is [25]

$$
\tilde{y}_{m,n} = \alpha \sum_{k=1}^{K} w_{m,k,n} X_{k,n} + N_{m,n}^{ap},
$$

(55)

where $N_{m,n}^{ap}$ is the distortion signal which is independent of the signal and is modeled by a zero mean Gaussian process.
with variance of $\sigma_{ap}^2$. Consequently, the received signal at the $n$-th sub-carrier of the $u$-th user is

$$\hat{X}_{u,n} = \alpha \sum_{m=1}^{M} E\{w_{m,u,n}H_{m,u,n}\}X_{u,n} + \Xi_{u,n},$$

(56)

where $\Xi_{u,n}$ is the effective noise defined as

$$\Xi_{u,n} = \alpha \sum_{m=1}^{M} w_{m,u,n}H_{m,u,n} - \sum_{m=1}^{M} E\{w_{m,u,n}H_{m,u,n}\}X_{u,n} + \alpha \sum_{k=1}^{K} \sum_{m=1}^{M} w_{m,k,n}H_{m,u,n}X_{k,n} + \sum_{m=1}^{M} H_{m,u,n}N_{m,n} + N_{u,n} + \alpha \sum_{k=1}^{K} \sum_{m=1}^{M} w_{m,k,n}H_{m,u,n}X_{k,n} + \sum_{m=1}^{M} H_{m,u,n}N_{m,n} + N_{u,n}.$$

(57)

Henceforth, we can obtain the SINR of the $u$-th user as follows

$$SINR_{u,n} = \frac{E\left\{\left(\alpha \sum_{m=1}^{M} E\{w_{m,u,n}H_{m,u,n}\}X_{u,n}\right)^2\right\}}{P_{\Xi_{u,n}}},$$

(58)

where $P_{\Xi_{u,n}}$ is the power of the effective noise defined as

$$P_{\Xi_{u,n}} = P\text{Var}\left(\sum_{m=1}^{M} w_{m,u,n}H_{m,u,n}\right) + P \sum_{k=1}^{K} \sum_{m=1}^{M} E\left\{w_{m,k,n}H_{m,u,n}\right\}^2 + M\sigma_{ap}^2 + \sigma^2.$$

(59)

If we use the MF precoding, we can calculate the SINR as follows

$$SINR_{u,n}^{MF} = \frac{M}{K + M\sigma_{ap}^2 + \sigma^2}.$$

(60)

Therefore, by using Theorem 1 of [24], the achievable sum-rate is

$$R_{OFDM}^{AP} = K\log_2\left(1 + \frac{P\alpha^2(M-K)}{MK\sigma_{ap}^2 + \sigma^2}\right).$$

(61)

For the ZF precoder, the SINR is obtained as

$$SINR_{u,n}^{ZF} = \frac{\alpha^2(M-K)}{MK\sigma_{ap}^2 + \sigma^2},$$

(62)

and the corresponding achievable sum-rate is

$$R_{ZF}^{AP} = K\log_2\left(1 + \frac{P\alpha^2(M-K)}{MK\sigma_{ap}^2 + \sigma^2}\right).$$

(63)

By comparing (32) and (61) and comparing (34) and (63), we can conclude that the effect of the power amplifier non-linearity on the sum-rate of both systems is similar when the number of users is large. As can also be seen, in the presence of the non-linear power amplifier the sum-rate of both systems does not increase unlimitedly as the number of the BS antennas tends to infinity and it is limited to

$$R_{SC-FDP}^{AP} = R_{OFDM}^{AP} = K\log_2\left(1 + \frac{P\alpha^2}{K\sigma_{ap}^2}\right),$$

(64)

for both MF and ZF precoders.

Figure 2: Histogram of the input signal of the power amplifier in SC-FDP for $K = 1$.

B. PAPR and Out-of-Band Radiation Issues

It is well known that in the absence of precoders, the PAPR of OFDM signals is significantly larger than that of single carrier systems based on the same constellation. This necessitates using expensive RF amplifiers to avoid in-band distortion and out-of-band radiation. This issue is more critical in massive MIMO systems where the number of RF amplifiers grows with $M$. In the following, we compare the PAPR and out-of-band radiation of OFDM and SC-FDP massive MIMO systems in the downlink where the system is equipped with precoder.

The PAPR of the signal transmitted from the $m$-th antenna of the BS is expressed as

$$PAPR_m = \frac{\max_{i}\left|y_{m,i}\right|^2}{\frac{1}{N} \sum_{i=0}^{N-1} \left|y_{m,i}\right|^2}.$$

In the OFDM massive MIMO system, $y_{m,i}$ is defined as (54). We know that for large $N$, the distribution of (54) can be approximated by Gaussian. Thus, the CCDF of the PAPR for the OFDM case can be computed as [26]

$$Pr(PAPR_m > \gamma_0) = 1 - \left(1 - e^{-\gamma_0}\right)^{2.8N}$$

(65)

For SC-FDP massive MIMO $y_{m,i}$ is defined as (25). We previously stated that the distribution of (25) is Gaussian for large number of users. So we expect that the SC-FDP and OFDM massive MIMO systems to have the same out-of-band radiation and PAPR performance for large number of users. For small $K$, however, we expect that the SC-FDP system to have a lower PAPR than the OFDM system. Fig. 2 and Fig. 3 present the histogram of $y_{m,i}$ in the SC-FDP massive MIMO system with $K = 1$ and $K = 10$, respectively. These figures indicate that the distribution of $y_{m,i}$ in SC-FDP for $K = 1$ can also be approximated as Gaussian, but this approximation is more accurate as the number of users increases. So, in the downlink we expect the PAPR of SC-FDP massive MIMO to be less than that of the OFDM when the number of users are small. Contrarily, as the number of users increases, the PAPR of the SC-FDP massive MIMO also increases and tends to that of the OFDM massive MIMO. The simulation results for the PAPR and out-of-band radiation provided in section V also confirm this point.
C. Complexity

In this section we will analyze the complexity of the transmitter and receiver of SC-FDP and OFDM massive MIMO systems in the downlink. At the transmitter side of SC-FDP massive MIMO, we have $K$ FFT and $M$ IFFT blocks to transmit the data to frequency domain and back to time domain, respectively. So in overall these blocks require $(K + M)\frac{N}{2} \log_2 N$ complex multiplications and $(K + M)N \log_2 N$ complex additions. The precoding is done at the transmitter in the frequency domain with linear precoders. Considering (2), to precode a data block of length $N$ we need $NMK$ complex multiplications and $NM(K - 1)$ complex additions. In OFDM, however, we have $M$ IFFT blocks at the transmitter and a FFT block at each receiver. So in the OFDM case we have $K$ FFT blocks which are located at the user side and not at the BS. Consistent with SC-FDP, in OFDM the precoding is done at the transmitter side in the frequency domain.

Thus, as presented in Table I, the overall complexity of the SC-FDP and OFDM massive MIMO systems have similar computational complexities. Both systems involve $M$ IFFT and $K$ FFT blocks in the downlink. They also use similar linear frequency domain precoding and detection schemes. However, in the SC-FDP massive MIMO all the computational load is concentrated in the BS and the computational complexity at the user terminal is less than that of the OFDM case. It is worth mentioning that in the multiple access forms of OFDM and SC-FDP, OFDMA and SC-FDMA, there is sub-carrier allocation and deallocation. Thus, we need an additional FFT and IFFT block at the receiver side in the SC-FDMA case too. Thus, the complexity of the SC-FDMA massive MIMO is higher than its multi-carrier counterpart.

V. Performance Results

In this section, we investigate the efficiency of the SC-FDP, and OFDM massive MIMO systems through comparative simulations. We consider a massive MIMO system whose data block length for SC-FDP and OFDM is $N = 128$ and the data is modulated as QPSK. We consider multipath channel with $L = 15$ paths with uncorrelated Rayleigh fading on the different multipath components and for different antennas. For the sake of simplicity, we considered the same exponential power delay profile for all links between transmit and receive antennas and we assume the users to have the same distance from the BS. The normalized delay spread of the channel is 3.

Fig. 4 and Fig. 5 present the rate per user for a fully synchronized massive MIMO system with the considered modulations versus the number of BS antennas for $\sigma_f^2 = 0.01$ and $\sigma_f^2 = 1$, respectively. The rate is presented for different number of users and for both MF and ZF precoders. As expected, by increasing the number of BS antennas and decreasing the number of users, the rate per user increases. Fig. 4 and Fig. 5 indicate that at high SNR the rate per user for the ZF precoder is higher than that of the MF precoder. However, at low SNR for small $\rho = \frac{M}{K}$, the rate per user for the MF precoder is higher than that of the ZF precoder. Fig. 6 demonstrate the sum-rate of the SC-FDP and OFDM massive MIMO systems with the MF and ZF precoders in the presence of carrier frequency offset for $K = 10$ and different number of BS antennas for $\sigma_f^2 = 0.01$. In this figure the CFO vector of the users is assumed $\epsilon = [0.01, 0.02, 0.03, 0.04, 0.05, 0.02, 0.03, 0.04, 0.04, 0.05]$. This figure confirm that the OFDM massive MIMO is more sensitive to carrier frequency instabilities than the SC-FDP massive MIMO system. As can be seen, in a non-synchronized massive MIMO system, the gap between the performance of the SC-FDP and OFDM is large in high SNR regime for the ZF precoder and large $M$.

![Figure 3: Histogram of the input signal of the power amplifier in SC-FDP for $K = 10$.](image-url)

![Table I: Number of arithmetic operations required for precoding a data block of length $N$ in a system with $K$ users and $M$ BS antennas and channel impulse response of length $L$.](table-url)

![Figure 4: Rate per user analysis for fully synchronized SC-FDP and OFDM massive MIMO systems with MF and ZF precoder for $\sigma_f^2 = 0.01$.](figure-url)
Figure 5: Rate per user analysis for fully synchronized SC-FDP and OFDM massive MIMO systems with MF and ZF precoder $\sigma^2_F = 1$.

Figure 6: Sum-rate of SC-FDP and OFDM massive MIMO systems with MF and ZF precoder in presence of carrier frequency offset for $K = 10$ and $\sigma^2_F = 0.01$.

Figure 7: Sum-rate of SC-FDP and OFDM massive MIMO systems with MF and ZF precoder versus CFO for $K = 10$, $M = 500$ and $\sigma^2_F = 0.01$.

Figure 8: Sum-rate of the SC-FDP and OFDM massive MIMO systems with MF and ZF precoder in presence of power amplifier non-linearity with $\alpha = 0.5$ and $\alpha = 0.8$ for $K = 10$, $\sigma^2_F = 0.01$ and $\sigma^2_{ap} = 0.01$.

to the upper bound derived in (52) as $M \to \infty$. As stated earlier, this is due to the fact that the interference caused by the CFO in the OFDM massive MIMO does not vanish as $M$ increases. In Fig. 7, the sum-rate of an SC-FDP and an OFDM massive MIMO system with $K = 10$, $M = 500$ and $\sigma^2_F = 0.01$ is presented versus CFO. This figure also confirms that OFDM massive MIMO is more sensitive to CFO than SC-FDP massive MIMO system. Fig. 8 shows the sum-rate of the SC-FDP and OFDM massive MIMO systems in the presence of a non-linear power amplifier for $K = 10$ and different number of BS antennas and attenuation factor $\alpha$. In this figure, we have assumed that $\sigma^2_{ap} = 0.01$ and $\frac{\sigma^2}{F} = 0.01$. As can be seen, the sum-rates of both systems are identical and do not increase unlimitedly as the number of BS antennas tends to infinity. However, they approach the upper bound derived in (64). Fig. 9 demonstrates the sum-rate of these two systems for the ZF and MF precoder versus the distortion noise variance when $K = 10$, $M = 100$, and $\frac{\sigma^2}{F} = 0.01$. Since the derived sum-rates for both systems are identical in the presence of non-linear power amplifier, we have only showed the curves for one system and removed the system type in the legend of Fig. 9. Fig. 8 and Fig. 9 show that the sum-rate of the system with the ZF precoder is higher than that of the MF precoder at high SNR. However, as the variance of the distortion noise increases and $\alpha$ decreases, the performance of these precoders get closer.

Fig. 10 presents the complementary cumulative distribution function (CCDF) of the PAPR of the transmitted signal in the SC-FDP and OFDM massive MIMO systems when $M = 50$. The out-of-band radiation of these systems for different back-off values are presented for $K = 1$ and $K = 10$ in Fig. 11 and Fig. 12, respectively. The power amplifier non-linearity is modeled by the Rapp model. It is worth mentioning that for each of the considered cases, the PAPR for the ZF and MF precoders are similar. This statement is also valid for the power spectrum. Since the corresponding curves are similar when we use ZF or MF precoders, the curves presented in Fig. 10, Fig. 11, and Fig. 12 are valid for both ZF and MF precoders.

As can be seen, when the number of users is low the SC-FDP massive MIMO system has lower PAPR and consequently lower out-of-band radiation than those of the OFDM massive MIMO system. It is noteworthy that by increasing the number of users, the PAPR and out-of-band radiation of the SC-FDP system approaches to those of the OFDM massive MIMO. The
reason resides in the fact that as the number of users increases in the downlink, the distribution of the precoded signal can be well approximated by Gaussian. This is similar to the case of OFDM systems with large block lengths.

Both theoretical and simulation results have shown the superiority of massive MIMO systems using SC-FDP over the one using OFDM in the presence of CFO and power amplifier non-linearity.

VI. CONCLUSION

In this paper, we studied the performance of SC-FDP as a modulation scheme for the downlink of massive MIMO systems, in a single cell scenario. We derived the achievable sum-rates for this system with and without carrier frequency offset and in presence of power amplifier non-linearity. We then compare, analytically and through simulations, the performance of SC-FDP massive MIMO system with OFDM massive MIMO systems from different perspectives. Our analysis indicates that in the absence of CFO, the derived achievable sum-rates of SC-FDP and OFDM massive MIMO systems are identical. However in the presence of CFO, SC-FDP outperforms OFDM significantly, especially in large number of BS antennas. We also showed that this difference is due to the effect of the interference term caused by CFO in OFDM massive MIMO system which does not vanish as the number of BS antennas tends to infinity. We also show that for large number of users, the achievable sum-rates for OFDM and SC-FDP massive MIMO systems are identical in the presence of power amplifier non-linearity. The peak to average power ratio and out-of-band radiation analysis show the superiority of SC-FDP over OFDM in the massive MIMO systems for small number of users. However, this superiority fades as the number of users grow.

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