Multidimensional 'better than' and Decision Making

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Overview

Central Theses

- **Multidimensionality of Value:** 'better than' and similar value comparatives are *multidimensional*.
- **Separation Thesis:** Natural properties of dimensions do not directly determine the *value* we attribute to an item.
- **Lexicographic Aggregation:** Different aspects of 'better than' comparisons are aggregated lexicographically.

Why Is ‘better than’ Multidimensional?

- There is good linguistic evidence for it: Stojanovic (2015), McNally (2017), Sassoon (2013, 2017).
- It is intuitively plausible to assume that if it is asserted that ‘a is overall better than b (all things considered)’ such a verdict is often based on multiple evaluations of the items a and b under considerations, which are sometimes also called ‘criteria’, ‘features’, or ‘attributes’.
- Usually, an item a is better than an item b in some aspects, but not in others, and there is a weighing or outranking of these aspects to determine which item is better.

Condorcet Case (e.g. Schumm 1987)

How should we aggregate the following comparisons?

- $a \succ_1 b$, $b \sim_1 c$
- $b \succ_2 c$, $a \sim_2 c$
- $c \succ_3 a$, $c \sim_3 b$

With unidimensional overall ‘better than’ such problem couldn’t occur. But they do occur all the time. The problem here is that every comparison in every single aspect is perfectly rational, but the aggregation seems to lead to non-transitive overall betterness.

Value Dimension $\neq$ Value Aspect

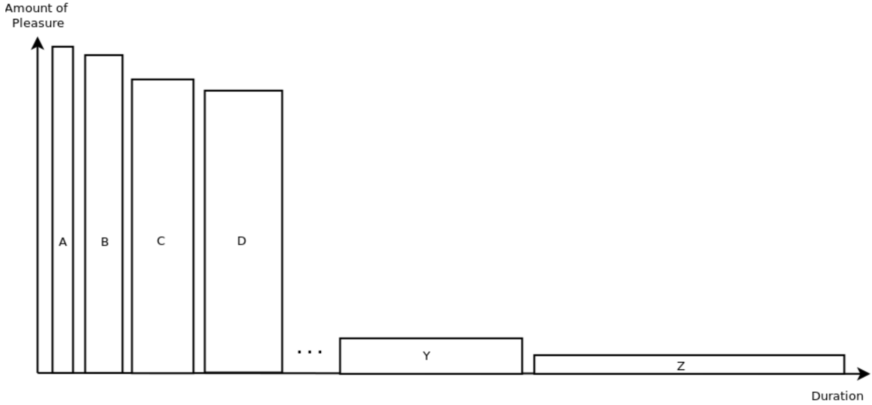
Dimension: The range of natural properties of an aspect of ‘better than’. Aspect: The evaluation of particular properties within that range.

- Dimensions often have an associated natural ordering. For example: the number of polite acts per week (lower to higher), low sodium levels per time unit to high sodium levels, less pleasure to more pleasure, etc.
- If the number of polite acts per week increases, then at some point politeness turns into creepiness.
- Low sodium level can lead to hyponatremia (bad for health, life threatening), higher sodium levels are good for health, and then even higher sodium levels become bad for health because of an increased risk of cardiovascular disease.
- Pleasure can be good from an hedonic perspective, but turn into debauchery at some level.

Lexicographic Thresholds

- In the previous examples a value turns into disvalue—or, at least, less value—within the given dimension, or vice versa.
- This happens when a threshold is reached.
- Klocksiem (2016) tackles Temkin's Spectrum Arguments with thresholds.

A Spectrum Argument (Individual Hedonic)



$B \succ A$, $C \succ B$, $D \succ C$, ... But: $A \succ Z$

Larry Temkin

Rachels on Badness (Individual Hedonic)

Based on Rachels (1998), cf. Temkin (2012: Ch. 3&4):

A: one year of extreme agony
B: 100 years of slightly less agony than A
C: 300 years of slightly less agony than B
$\vdots$
Z: millions of years of extremely mild pain (e.g. pinprick, mosquito bite)
$A \succ B \succ C \succ D \succ \ldots$ But: $Z \succ A$

Parity

Trichotomy Thesis

'better than', 'equally good', and 'worse than' are the only types of overall value comparison.

Chang (2002): This is not the case. Sometimes we judge two items on a par, and this is neither 'equally good' nor 'noncomparable (tout court)'.

Examples: The creativity of Michelangelo and Mozart may be on a par, a life as a lawyer vs. life as a clarinetist, etc. All kind of 'hard cases' of comparisons.

Small Improvements Argument

A small difference between two evaluatively very different items does not necessarily lead to a different assessment of parity.

1.	$a \not\succ b$, $b \not\succ a$	w.r.t. V
2.	$a^+ \succ a$	w.r.t. V
3.	$a^+ \not\succ b$	w.r.t. V
4.	$a \not\sim b$, $a \not\succ b$, $b \not\succ a$	w.r.t. V

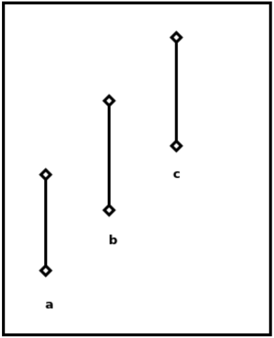
From 1: $a \sim b$ or $a \perp b$.

From 2, 3: $a \not\sim b$.

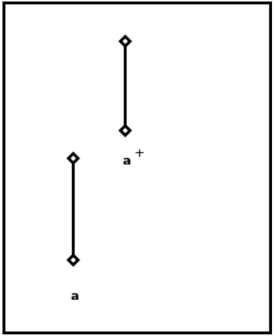
Hence: $a \perp b$ under the assumption of the Trichotomy Thesis

Is Parity Rough Equality?

Representation of rough equality: semiorder or interval order (Luce 1956; Vincke et al. 1997).



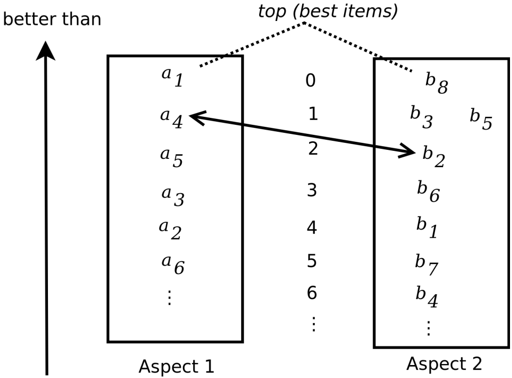
Example 1



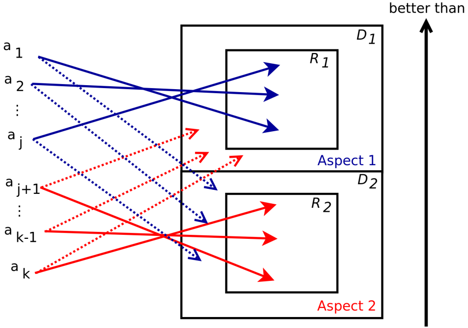
Example 2

$a \sim b$, $b \sim c$, $a \nsim c$, $c \succ a$ and $a^+ \succ a$. Chang's problem: If we improve a to a^+, then it seems she wants it to be possible for a^+ to remain on a par with a. But notice that we could have $a^+ \succ a$, $a \sim b$, $a^+ \sim b$.

Multidimensional Parity



Dealing With Spectrum Cases



Assuming that Aspect 1 is lexicographically preferred to Aspect 2, the outcome of aggregating them is:

$$a_j \succ a_{j-1} \succ a_{j-2} \succ \cdots \succ a_1 \succ a_k \succ a_{k-1} \succ \cdots \succ a_{j+1}$$

Value Structures

Definition (Evaluative Structure)

Let $E = \langle A_n, \mathcal{D}, \mathcal{S}, \mathcal{R} \rangle$, where A_n is a finite set of n aspects $\{1, 2, 3, \ldots, n\}$, $\mathcal{D}$ a set of aspect domains indexed by A, where $D_i \subseteq \mathcal{D}$ for any $i \in A$, $\mathcal{S}$ a set of sets of comparable items indexed by A, where $S_i \in \mathcal{S}$ and $S_i \subseteq D_i$ for any $i \in A$, and R is a set of i binary relations R_i over S_i. Given all that, E is an *evaluative structure* if the following conditions hold:

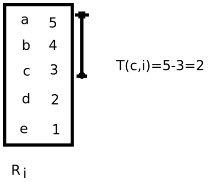
1. For all $x \in \bigcup \mathcal{D}$, there is at least one $S_i \in \mathcal{S}$ and $i \in A$ such that $x \in S_i$.
2. For all $i \in A$, $|S_i| \geq 2$.

Top Distance

$L_i(x)$ is a ranking function based on R_i.

Definition (Top Distance)

The function $T : A \rightarrow \mathbb{N}$ for a set of aspects A calculates the top distance of $x \in S_i$ within aspect i: $T(x, i) := [\max_y L_i(y)] - L_i(x)$.



Parity

Definition (Aspect Parity)

Two items $a, b \in \mathcal{D}$ are on a par with respect to two aspects $i, j \in A$ iff. (i) a and b are aspectually exclusive with respect to i, j, and (ii) $|T(a, i) - T(b, j)| \leq \delta$ for constant $\delta > 0$.

The constant δ represents a threshold for the rough equality of top distances. It may also be defined relative to the aspects under consideration, although this is not assumed here. Given the above definitions, complete parity is a straightforward generalization.

Definition (Parity)

Within a value structure two items $a, b \in \mathcal{D}$ are on a par iff. for every distinct pair $i, j \in A$ of aspects such that $a \in S_i$ and $b \in S_j$, a and b are aspectually on a par with respect to i, j.

Aggregation: Canonical Utility

- Based on Borda Rank with Averaging Ties (but reversed in comparison to many common formulations).
- The sum of all ranks is given by Euler's famous formula:

$$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1) \tag{1}$$

- Writing down the correct definition of $v_i(x)$, the Averaging Borda Rank of item x in aspect i, is surprisingly unwieldy and annoying.

Definition (Canonical Ordinal Utility Function)

For each aspect i, ordinal utility is defined as the averaging Borda rank for comparable items $x \in S_i$:

$$u_i(x) := \frac{2v_i(x)}{|S_i|(|S_i|+1)} \tag{2}$$

Maximum at a Level

Definition (Utility and Maximum at Aspect Level)

Let each aspect i at aspect at aspect level k have some weight w_i such that $w_i > 0$ and the sum of all w_i at k is 1. The *utility of an item* x at aspect level k given by '$\succeq$' is

$$u^k(x) := \sum_{i \in Eq(k)} w_i u_i(x) \tag{3}$$

The *maximum utility* at an aspect level k is defined as:

$$M(k) := \max_x u^k(x) \tag{4}$$

Ordinal Lexicographic Aggregation

Definition (Mixed Lexicographic/Additive Aggregation)

For all $x \in D$ such that there is a highest aspect level k at which some $u_i(x)$ is defined, the aggregate utility $u : D \to \mathbb{R}$ is

$$u(x) := u^k(x) + M(k-1) + M(k-2) + \cdots + M(1). \qquad (5)$$

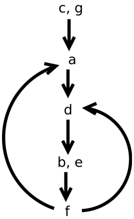
If there is no $u_i(.)$ defined at x for any $i \in A$, then $u(x)$ is undefined at point x.

What About Cardinal 'better than'?

- Hard to argue against cardinal utility if the underlying dimension already supports it, e.g. monetary value.
- Requires no major changes:
  - Replace Borda Rank by the cardinal utilities.
  - Re-use the same top distance concept for parity.
  - Hence, the underlying scale is an interval scale.
  - Aggregation remains the same.
- But: Aggregation of mixed cardinal and ordinal utilities cannot be justified.

Remark: Acyclicity Not Needed For Rational Choice

Minimal conditions for decision making are laid out by Hansson (2001):



Weak Eligibility

An item a is weakly eligible iff. no other item b ($a \neq b$) is strictly better than a.

Top Transitivity

If a is weakly eligible and $a \sim b$, then b is weakly eligible, too.

Imposed on a *particular* value relation, these conditions ensure that a rational decision can be made.

Summary & Conclusions

- Theory D gives particular types of explanations of Spectrum Cases and parity.
- Lexicographic ordering of aspects can occur even within the same dimension.
- Spectrum Cases suggest lexicographic aggregation, but there is no final knockdown argument for or against such rationality principles.
- Overall betterness remains transitive in the proposed theory.
- Neither acyclicity nor transitivity are needed for decision making; there are good reasons for and against transitivity.

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